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Impedance and Collective Effects

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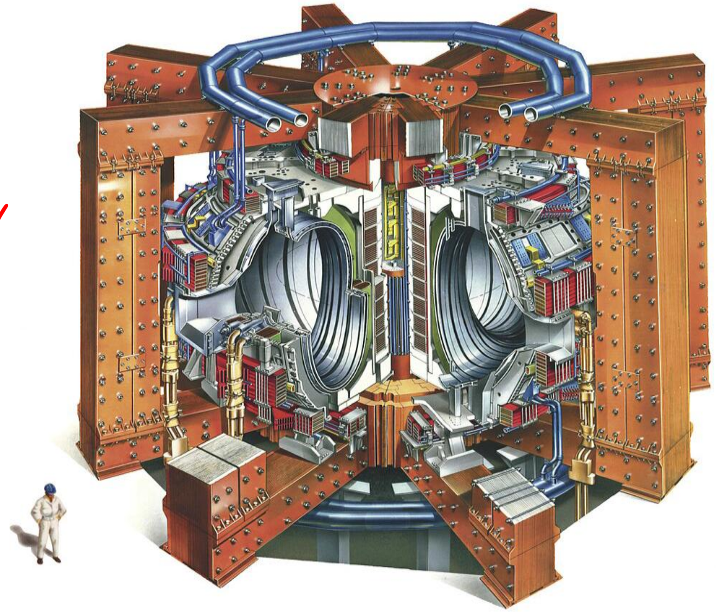
Introduction to Collective effects

Introduction to Collective effects

VS



Tokamak

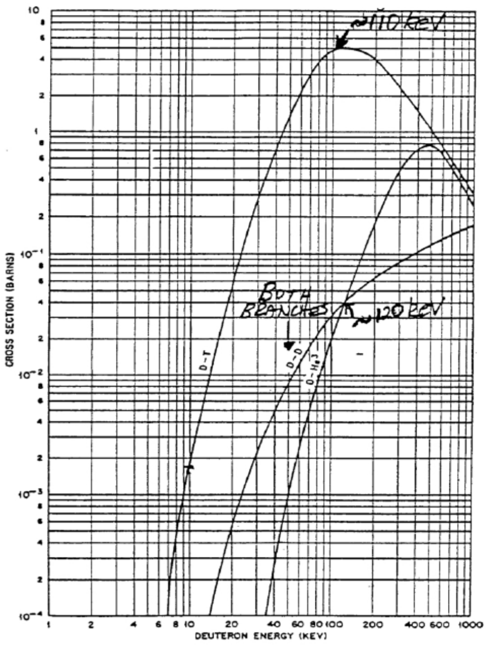
- The motion of particles is highly ordered.
- The motion of particles is （mainly） determined by external electromagnetic fields.
- Particles in an accelerator ~ $10^{1\sim14}$
- The self field may play role when particle number is high (Collective effects)
- The time of the stable motion can be ∞
- Energy up to 10TeV

Developed

- The motion of particles is in random.
- The motion of particles is majorly determined by self fields.
- Particles in a Tokamak ~ $10^{19\sim20}$ （10^{-3}mol）
- The time of the stable motion around 1000s at present
- Energy ~10keV non-relativistic

Developing

Once upon a time accelerator physics want to develop a fusion device based on accelerator.



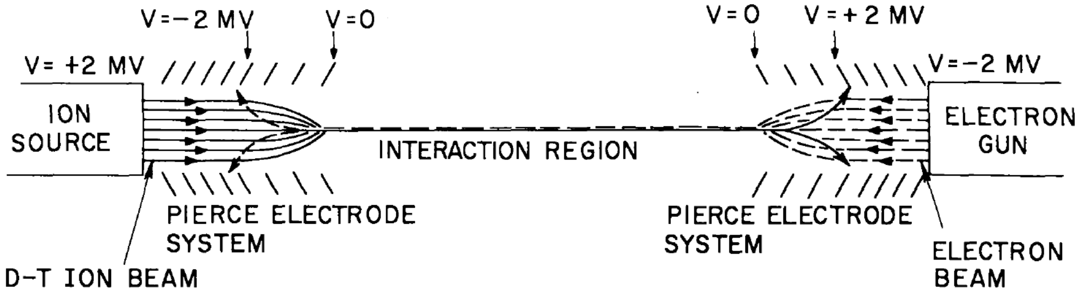
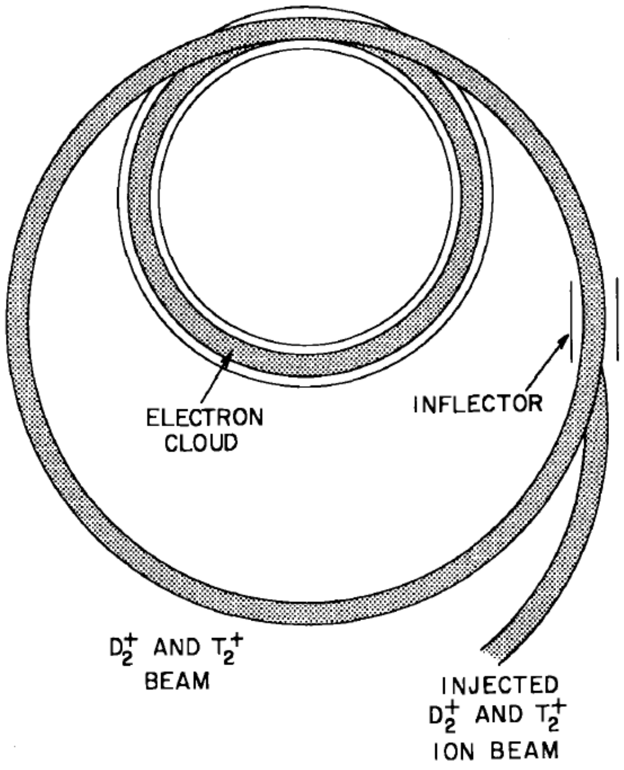


Fig. 1. Linear colliding beam system.



The maximu cross section D-T fusion @110 keV is

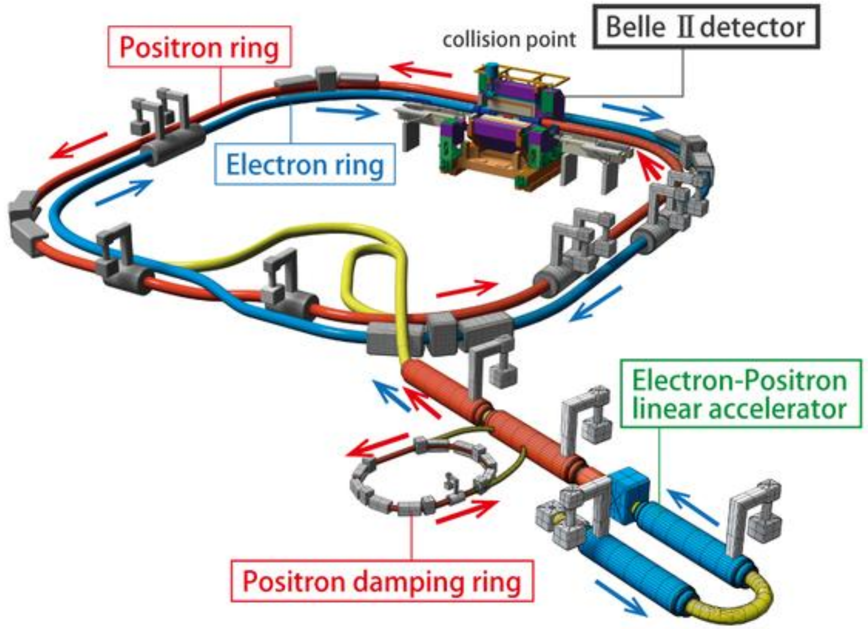
$\sigma_{D-T} = 5\ barn = 5 \times 10^{-28} m^2$

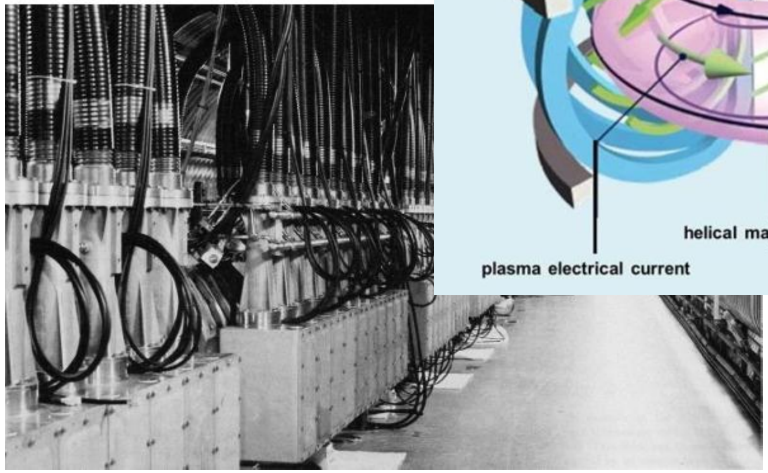
To achieve enough nuclear reaction the large amounts of particles(10^{14}) should be transversely focused to bellow 1μm. For such a low energy beam, space charge effects will be a big challenge for focusing.

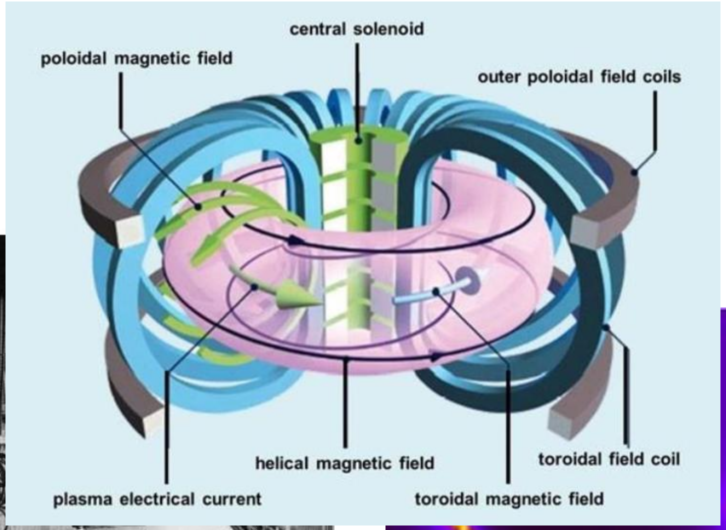
We will introduce space charge effects as one of major collective effects for low energy beam later

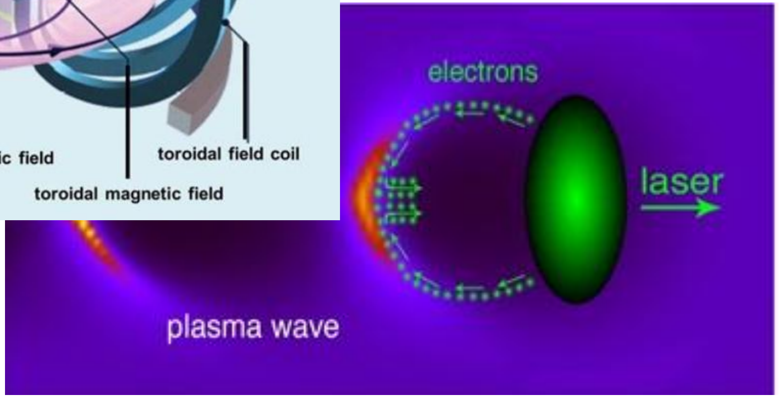
How far have we gone

Tokamak ~ million Amp









Synchrotron

KEK B factory
2.6 A @3.5GeV (average beam current)

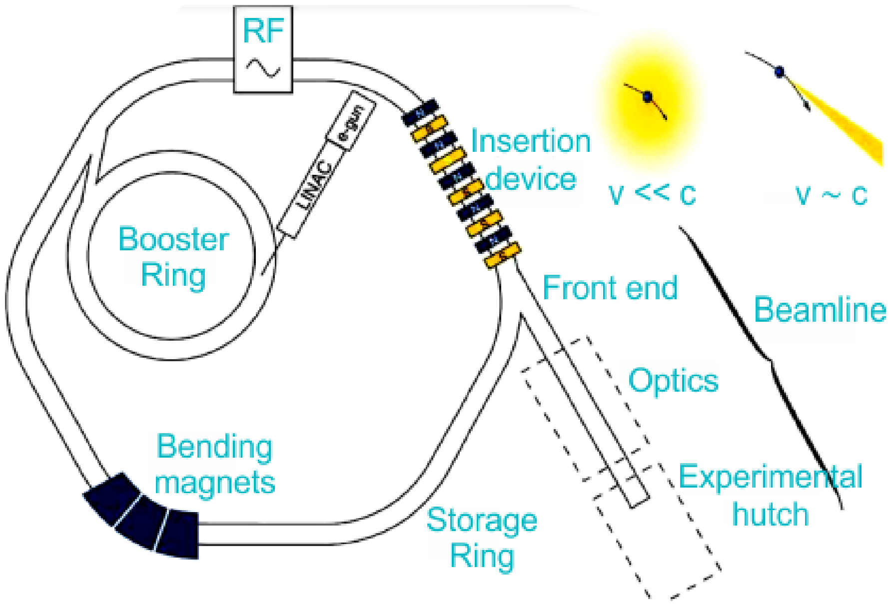
Induction linac

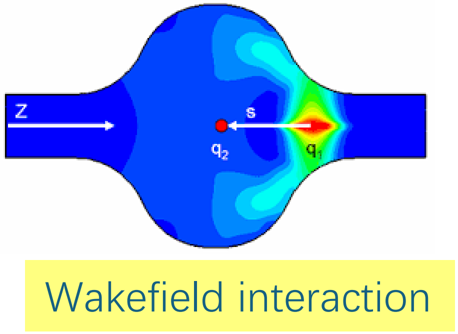
ATA LLNL
10kA @50MeV (peak current)
60ns pulse

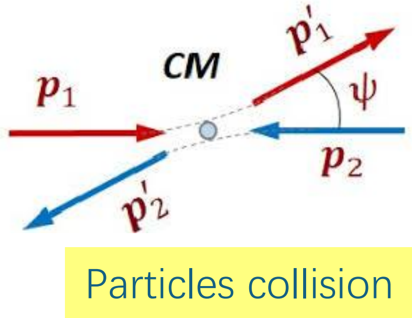
Plasma wake-field accelerator

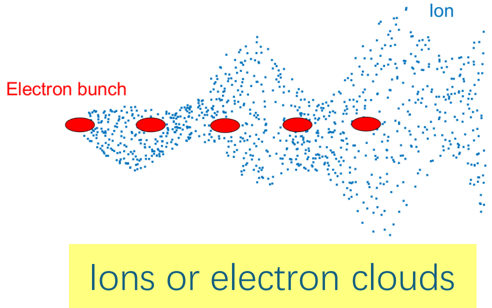
20–40 kA (peak current)
~30fs

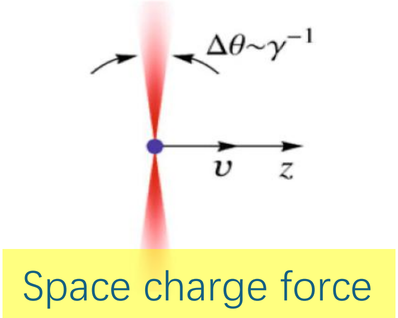
Introduction to Collective effects

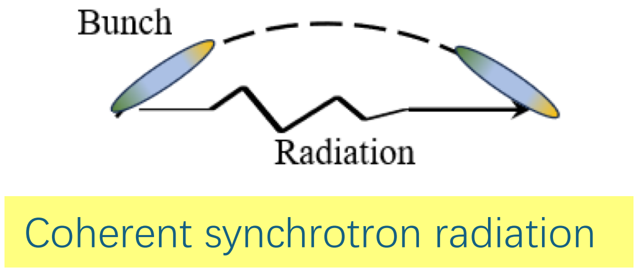


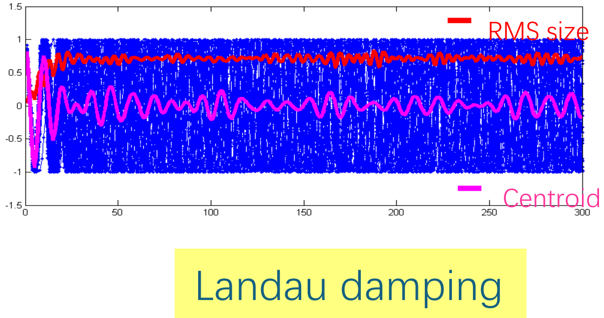












The behavior of Charged particles interact with **magnets, RF field** is usually called **single particle beam dynamics.**

Above-mentioned effects can be categorized to collective effects. They depend on charge density, peak current or average current.

Results: Beam performance degraded, charge reduction or even beam loss 。

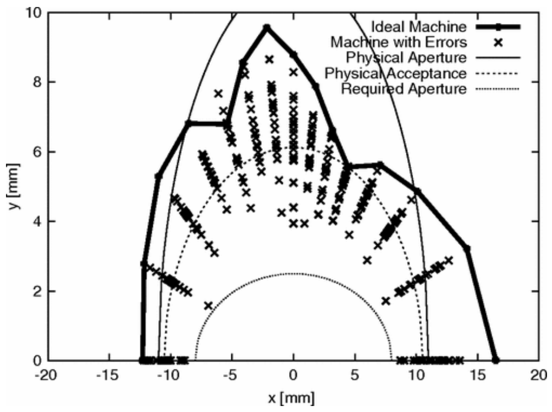
Introduction to Collective effects

Single particle beam dynamics is well established. One of the most important method is **factorization**.

$$X_{out} = M\ X_{in}$$

Accelerator design is only to work on ***M*** (lattice design), which significantly simplifies the problem. For any initial condition $X_{in}^{(1,2,3,\ldots)}$, there's no need to repeat the entire analysis—as long as the system remains in the **linear regime**.

For comparing, once we enter the nonlinear regime, the problem becomes much more complex. For example, studying the beam Dynamic Aperture requires extensive numerical computation.



There is no universal solution to collective effects.

They should be addressed individually by various methods.

Introduction to Collective effects

Over the years, accelerator physicists have observed, explained, and (mostly) cured several intricate instability mechanisms. An incomplete list follows below

- negative mass instability 1959
- resistive wall instability 1960
- Robinson instability 1964
- beam breakup instability 1966
- head-tail instability 1969
- microwave instability 1969
- Landau damping 1969
- beam-beam limit in colliders 1971
- potential well distortion 1971
- Sacherer formalism 1972
- anomalous bunch lengthening 1974
- transverse mode coupling instability 1980
- hose instability 1987
- coherent synchrotron radiation instability 1990
- sawtooth instability 1993
- electron beam-ion instability 1996
- electron cloud instability 1997
- microbunching instability 2005
- interplay of multiple instability mechanisms 2013

Alexander Wu Chao, Lectures on Accelerator physics, World Scientific, 1st edition, 2020, ISBN 9789811227967.

Instability

Coherent instability

Particles in a bunch oscillate coherently, can be treated as macro-particle.

Oscillate Amplitude increased

Incoherent instability

Particles move incoherently

Bunch distorted, beam size increased, beam energy increased.

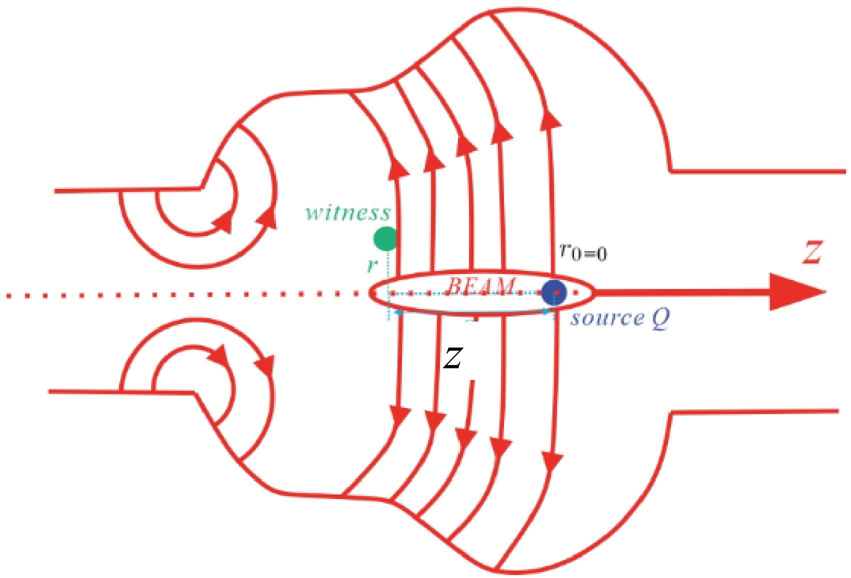
beam lifetime

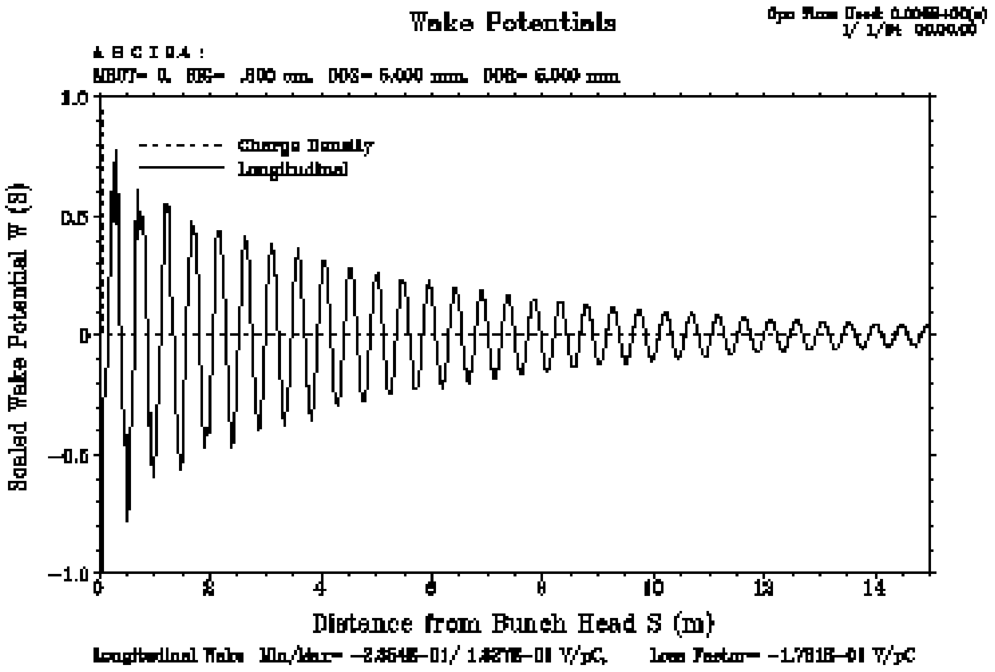
Particles loss when exceeding acceptance (Transverse acceptance or Momentum acceptance)

Beam current decay gradually

Wakefield & impedance

Wakefield & impedance





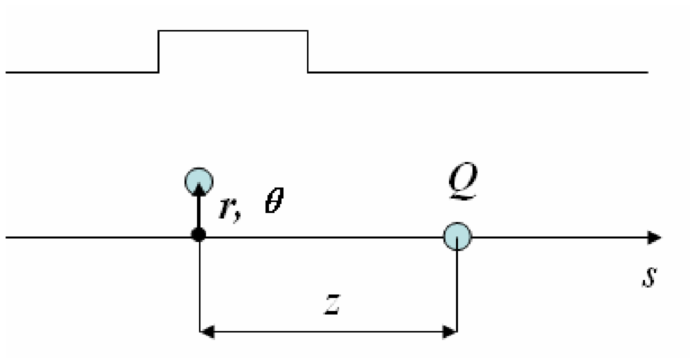
When a charge beam passing through a beam pipe which gets discontinuities or gets finite conductivity. The EM field will be excited, it is called wakefield.

Wakefield decay with time, it will disturb the motion of tail particles or the following bunches.

The wakefield is the main sources of beam instaiblity

Wakefield & impedance

Wakefield can be expressed by the wake function $W_{\parallel m}(r,z)$, $W_{\perp m}(r,z)$。 It an integral value.
Wake function is a Green function, where independent variable *"z"* denotes distance of witness beam following the source beam.
Wake function is a characterizes of vacuum chamber not the beam. Wakefield = Wake function convolution source charge distribution



$$W_{\perp m}(r,z) = -\left.\frac{\int_{-L}^{L}[E_{\perp m}(r,s,t) + \vec{V} \times \vec{B}_m(r,s,t)]dt}{r_0 Q}\right|_{s=vt-z}$$

$$W_{\parallel m}(r,z) = -\left.\frac{\int_{-L}^{L} E_{\parallel m}(r,s,t)dt}{Q}\right|_{s=vt-z}$$

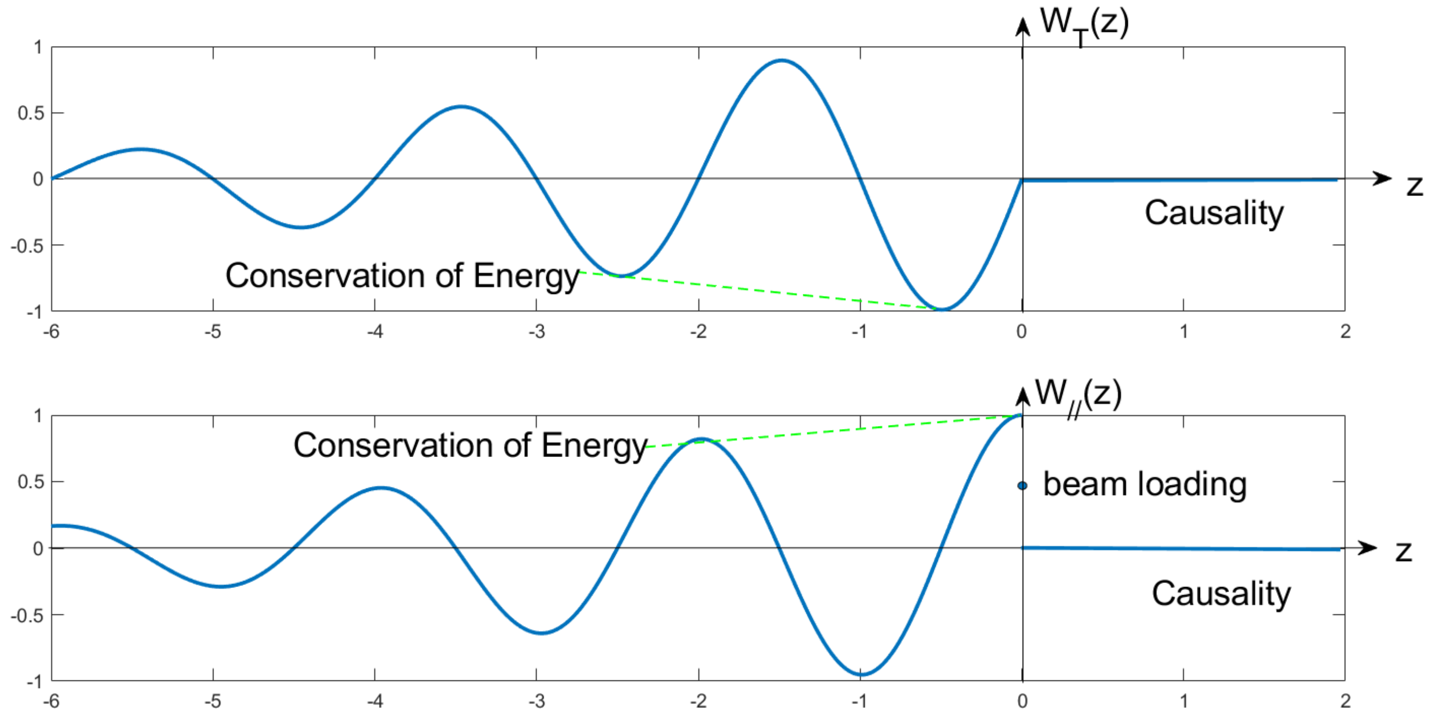
Wake function

$$w_{\parallel}(z) = \int_{z}^{\infty} -e\rho(\tilde{z})W_{\parallel}(\tilde{z}-z)\mathrm{d}\tilde{z}$$

Wakefield (wake potential)

Wakefield & impedance

Wake function properties



$$W_{\parallel m}(z>0)=W_{\perp m}(z>0)=0$$ Causality

$$W_{\parallel m}(z=0^-)\geq 0\,,\qquad W_{\perp m}(z=0^-)=0$$

$$W_{\parallel m}(z=0^-)>\left|W_{\parallel m}(z<0)\right|$$ Conservation of Energy

$$W_{\parallel m}(z=0)=\frac{1}{2}W_{\parallel m}(z=0^-)$$ Beam loading

$$\frac{\partial W_\perp}{\partial z}=\frac{1}{r_0}\nabla_\perp W_\parallel$$ Panofsky-Wenzel Theory

Panofsky-Wenzel Theory

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\nabla \times \vec{F} = q\nabla \times \vec{E} + q\nabla \times \left(\vec{v} \times \vec{B}\right)$$

$$= -q\frac{\partial \vec{B}}{\partial t} + q\vec{v}\left(\nabla \cdot \vec{B}\right) - q\left(\vec{v} \cdot \nabla\right)\vec{B}$$

$$= -q\frac{\partial \vec{B}}{\partial t} - qv\frac{\partial}{\partial z}\vec{B}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla_s \times \Delta\vec{p}(x,y,s) = -q\int_{-\infty}^{\infty}\left[\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)\vec{B}(x,y,z,t)\right]_{z=vt-s} dt = -q\int_{-\infty}^{\infty}\frac{d}{dt}\vec{B}(x,y,vt-s,t)dt = 0$$

$$\frac{\partial}{\partial s}\Delta\vec{p}_\perp = -\vec{\nabla}_\perp \Delta p_z$$

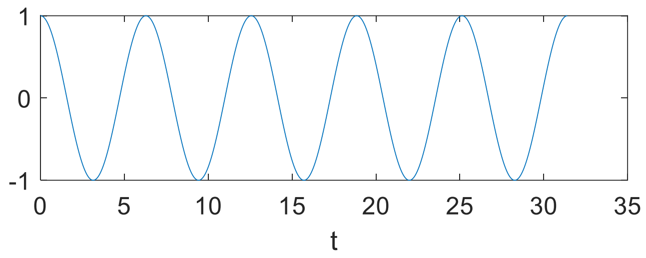
For a axial symmetry structure, we can get Panofsky-Wenzel Theory of wake field.

$$\frac{\partial}{\partial s}\overrightarrow{w_t} = \frac{1}{r_0}\nabla_\perp w_l$$

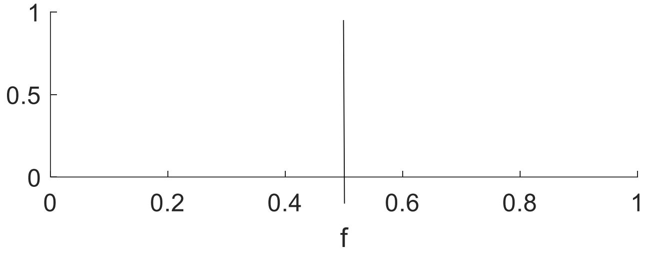
Wakefield & impedance

Wakefield ----》 Tracking。
Impedance ----》 Analysis。

Time domain



Frequency domain



Simulation codes： CST Microwave Studio, GdfidL, ABCI, Mafia,

Impedance is the Fourier transform of the wake function.

$$Z_{\parallel}(\omega) = \frac{Z_0 c}{4\pi} \int_{-\infty}^{+\infty} W_{\parallel}(z) e^{-i\frac{\omega z}{c}} \frac{dz}{c}$$

$$Z_{\perp}(\omega) = i\frac{Z_0 c}{4\pi} \int_{-\infty}^{\infty} W_{\perp}(z) e^{-i\frac{\omega z}{c}} \frac{dz}{c}$$

$$Re(Z_{\parallel}(\omega)) = Re(Z_{\parallel}(-\omega))$$

$$Im(Z_{\parallel}(\omega)) = -Im(Z_{\parallel}(-\omega))$$

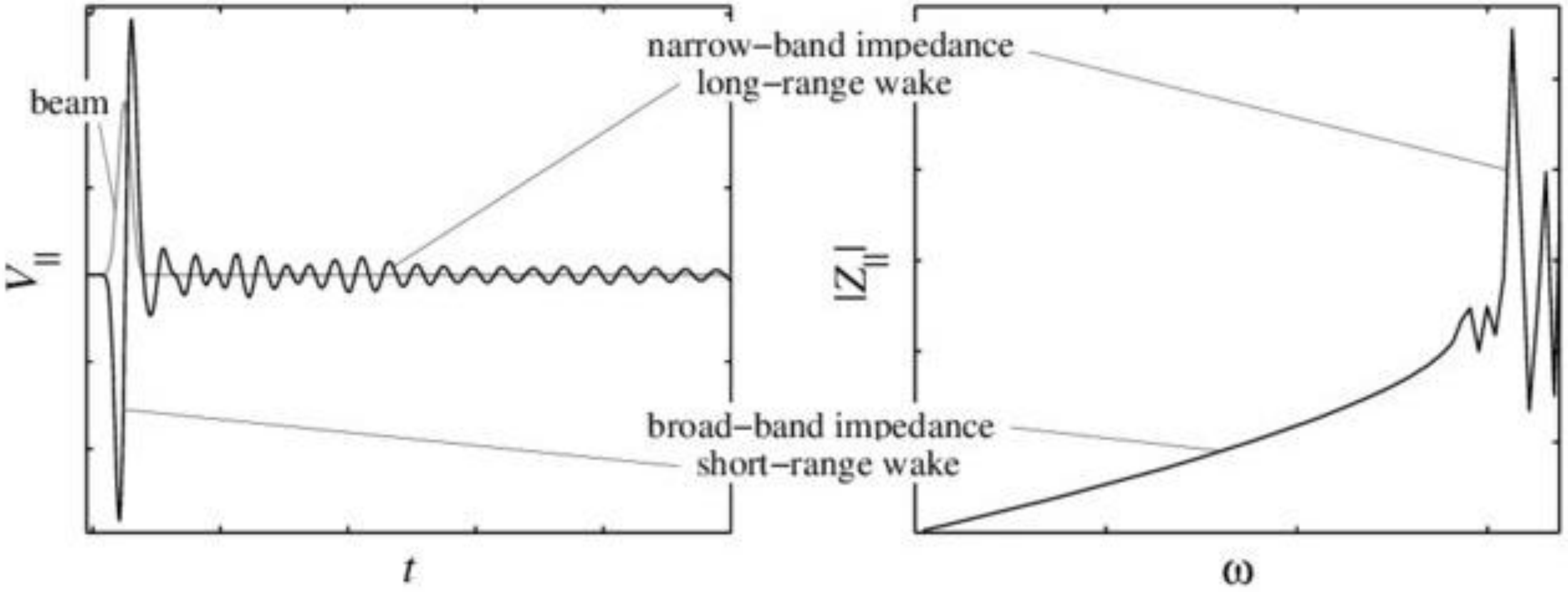
$$Re(Z_{\perp}(\omega)) = -Re(Z_{\perp}(-\omega))$$

$$Im(Z_{\perp}(\omega)) = -Im(Z_{\perp}(-\omega))$$

$$Z_{\parallel}(\omega) = \frac{\omega}{c} Z_{\perp}(\omega)$$

Panofsky-Wenzel Theory for impedance

Wakefield & impedance



Narrow-band impedance： High Q cavity like structure, long range wakefield, resonance peak. Important for multi bunch instability.

Resonance model
$$Z_{\parallel}(\omega) = \frac{R_s}{1 + iQ(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r})}$$
ω_r Resonance frequency

Broad-band impedance: Low Q structure, short range wakefield, spectrum is broad. Important for single bunch instability.

Effective broad-band impedance
$$\left|\frac{Z_{\parallel}}{n}\right|_{eff} = \frac{\int \left|\frac{Z_{\parallel}}{n}\right| h_m dn}{\int h_m dn}$$
$$h_m = \frac{1}{\Gamma\left(m + \frac{1}{2}\right)} (n\omega_0 \sigma_{z0}/c)^{2m} e^{-(n\omega_0\sigma_{z0}/c)^2}$$
$$n = \omega/\omega_0$$

Parasitic energy loss

When a beam passing through a beam pipe, a wakefield is generated, beam energy will loss due to impedance.

$$\Delta E = -k^{\parallel} q^2$$

Where $k^{\parallel}$ is the loss factor, q is bunch charge.

$$k^{\parallel} = \frac{1}{\pi}\int_0^{\infty} d\omega Re[Z_0^{\parallel}(\omega)]|\tilde{\rho}(\omega)|^2$$

$$\tilde{\rho}(\omega) = e^{-\omega^2\sigma_t^2/2}$$

Unlike impedance, the loss factor depends on the bunch length—the shorter the bunch, the higher the loss factor. The associated energy loss will deposit onto the beam pipe, causing it to heat up. The energy loss increases quadratically with the bunch charge it should be seriously considered for high charge bunch.

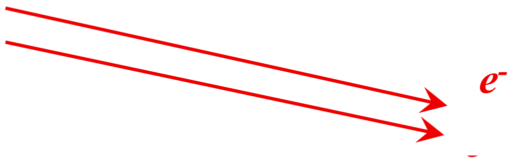
Space charge effect

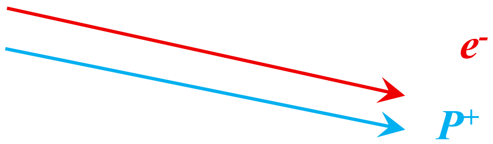
Space charge effect

The **space charge force** describes the interaction between particles their distance is greater than the Debye length.

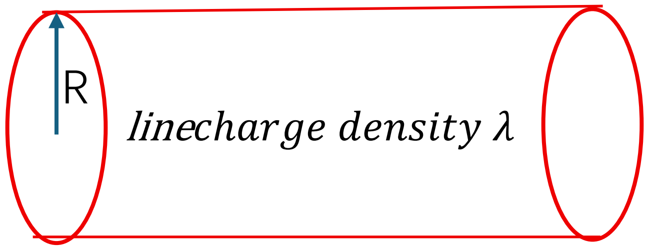
Debye length $\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n_0 e^2}} \approx 69\sqrt{\frac{T(K)}{n_0\,(\mathrm{m}^{-3})}}\mathrm{m}$ T is “Temperature” of the bunch, n_0 particle density in the bunch

If particles distance is smaller than the Debye length, the interaction is described by **Scattering**.





The Lorentz force： $F = e(\vec{E} + \vec{v} \times \vec{B})$



$$E_r = \frac{Q/L}{2\pi\epsilon_0 r}$$

$$\frac{Q}{L} = \frac{\lambda * \frac{2\pi r^2}{2\pi R^2} * L}{L}$$

$$E_r = \frac{\lambda r}{2\pi\epsilon_0 R^2}$$ For r<R

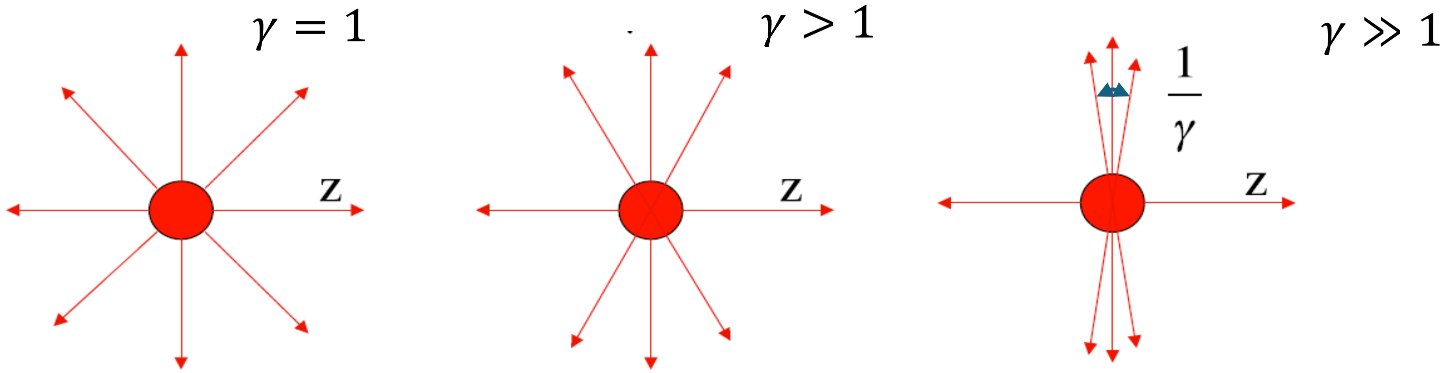
Coulomb's law of line charge

Space charge effect

When two same type particles move in same direction, the electric field force repulse each other, the magnetic force attract each other.

$$F_M = -\beta^2 F_E$$

When beam is ultra relative $(\beta \approx 1,\ \gamma \gg 1)$, electric and magnetic forces will cancel each other, thus space charge force is seriously considered when beam is at low energy.

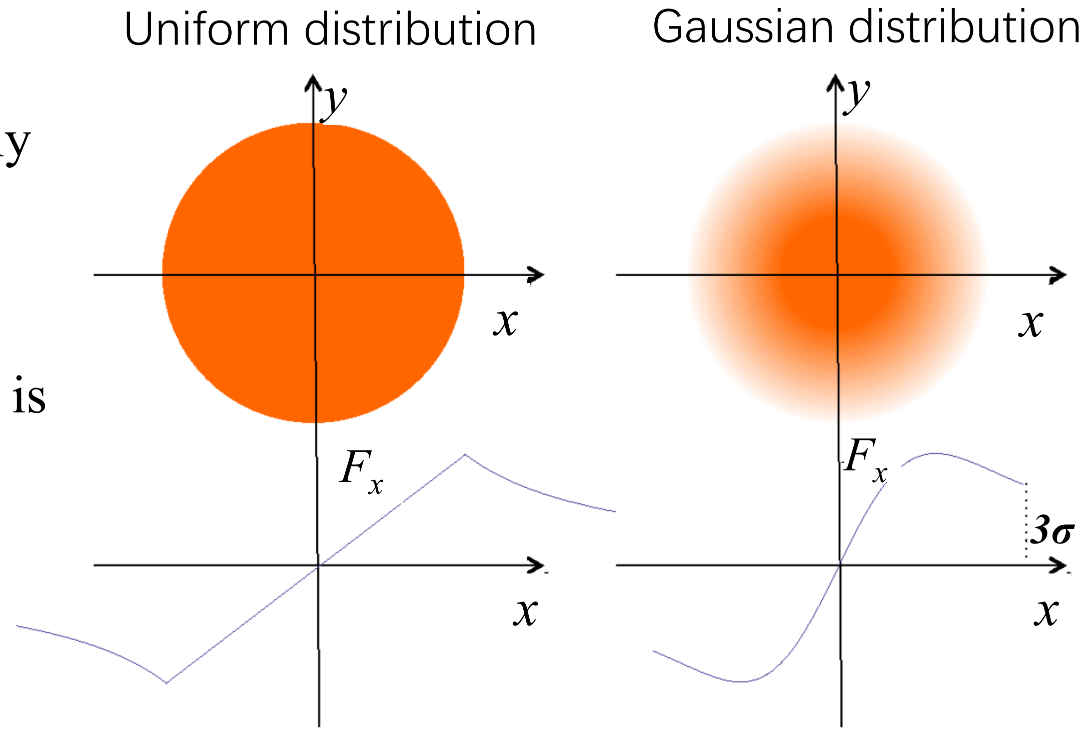


$$E_x = \frac{q}{4\pi\varepsilon_0}\frac{x\gamma}{(x^2+y^2+z^2\gamma^2)^{3/2}} \qquad E_y = \frac{q}{4\pi\varepsilon_0}\frac{y\gamma}{(x^2+y^2+z^2\gamma^2)^{3/2}} \qquad E_z = \frac{q}{4\pi\varepsilon_0}\frac{z\gamma}{(x^2+y^2+z^2\gamma^2)^{3/2}}$$

$$B_x = -\frac{\beta}{c}E_y \qquad B_y = \frac{\beta}{c}E_x \qquad B_z = 0$$

Space charge effect

For a transverse uniformly distributed beam， The space charge force is linearly increased with radius inside the beam, it is very similar to a defocusing force.



For a Gaussian distribution, the force is highly nonlinear.

Bassetti-Erskine given the space charge force for the first order approximation:

$$E_x \approx \frac{e\lambda}{2\pi\varepsilon_0}\frac{x}{\sigma_x(\sigma_x+\sigma_y)}$$

$$E_y \approx \frac{e\lambda}{2\pi\varepsilon_0}\frac{y}{\sigma_y(\sigma_x+\sigma_y)}$$

$$F_{x,y} = \frac{e}{\gamma^2}E_{x,y}$$

*The space charge force is **inverse proportional** to $\sigma^2_{x,y}$ and γ^2, and proportional to λ, the space charge force will be very large when beam size is very small, beam energy is very low and charge line density is large.*

$e\lambda$ where is line charge density in longitudinal coordinate, $\sigma_{x,y}$ is transverse RMS bunch size.

Space charge effect

The space charge force will cause tune shift, emittance growth.

Tune shift can be estimated by the defocus force:

$$\Delta\nu_y = \frac{1}{2\pi}\oint \beta_y k_y \, ds$$

$$k_y = -\frac{2r_e\lambda}{\beta^2\gamma^3\sigma_y(\sigma_x+\sigma_y)}$$

The normalized emittance growth when the beam passing through a drift L is:

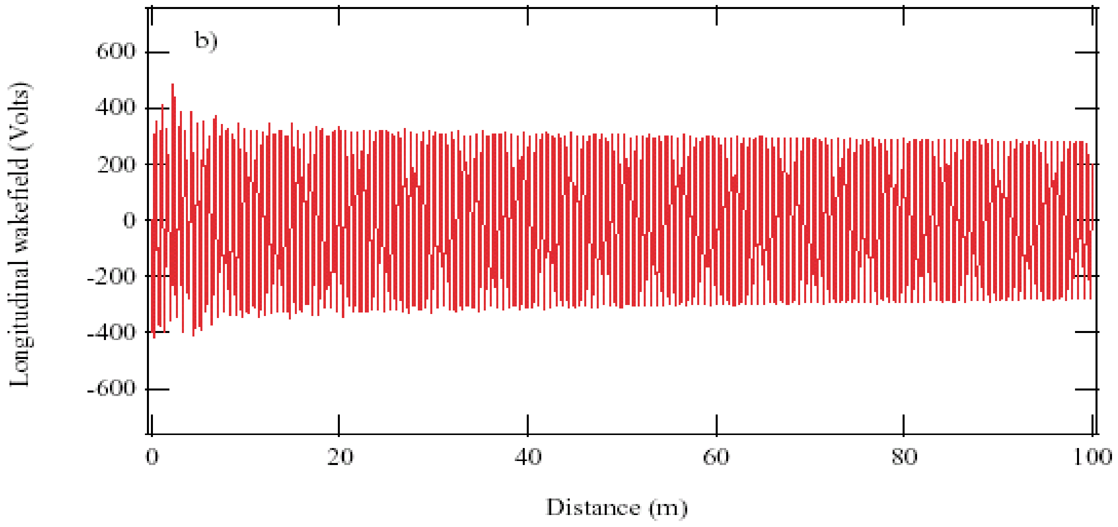
$$\Delta\varepsilon_{x,n} \approx \frac{eIs}{16\pi\varepsilon_0 \mathrm{m}_0 c^3\gamma^2\beta^2} G\left(\frac{\gamma L}{b}\right)$$

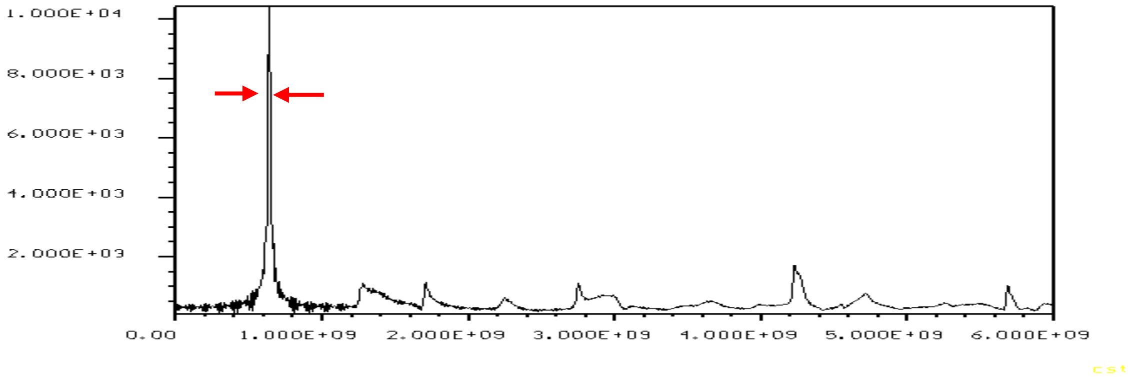
I is bunch peak current, s *is* longitudinal coordinate, L drift length, G is geometric factor, b is pipe radius.

A more accurate method is using simulation code such as PARMELA to evaluate space charge effect.

Coupled bunch instability

Coupled bunch instability





If wakefield decay slowly, the wakefield will interact with following bunches or even bunches in following turns， This interaction will cause coupled bunch instability.

The slowly decayed wakefield is named long range wakefield which hold high quality factor Q

The impedance is narrow banded.

Narrow band impedance is produced by cavity like structure or resistive wall.

Coupled bunch instability

Transverse coupled bunch instability

$$\ddot{y}_n(t) + \omega_\beta^2 y_n(t) = -\frac{r_0 c}{\gamma T_0}\sum_{k=-\infty}^{\infty}\sum_{m=0}^{M-1} N a_m W_\perp\left(-kC - \frac{m-n}{M}C\right) \times y_m\left(t - kT_0 - \frac{m-n}{M}T_0\right) \qquad (1)$$

Where ω_β is betatron oscillate frequency, n denotes the bunch index to be investigated, m denotes bunch excite wakefield. M is the harmonic number of the ring, C is the circumference of the ring, γ is relative energy, T_0 is revolution period, N is number of electrons in a bunch, a is shaping factor.

Assume bunch oscillation gets form:

$$y_n(t) = \tilde{y}_n e^{-i\Omega t}, \quad \Omega \approx \omega_\beta \qquad (2)$$

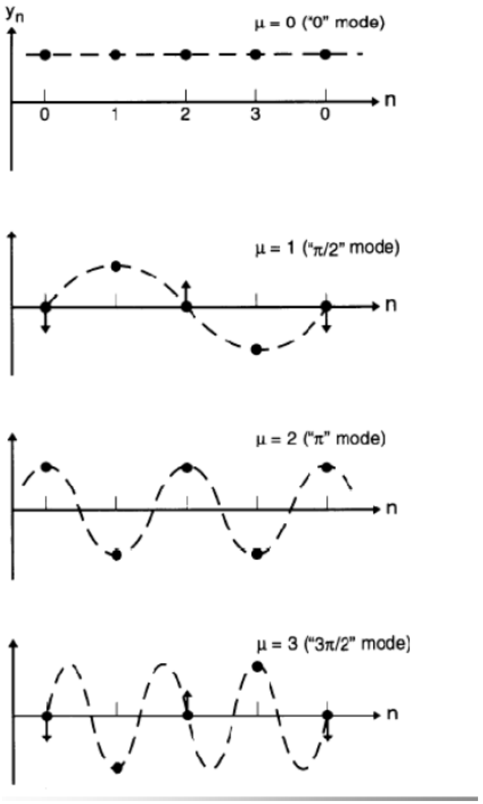
Substituting Eq.(2) into Eq.(1), With Fourier transform we get:

$$(\Omega - \omega_\beta)\tilde{y}_n = -i\frac{N r_0 c}{2\gamma T_0^2 \omega_\beta}\sum_{m=0}^{M-1} a_m \tilde{y}_m \sum_{p=-\infty}^{\infty} Z_\perp(p\omega_0 + \omega_\beta)\, exp\left(-i2\pi p\frac{m-n}{M}\right) \qquad (3)$$

Coupled bunch instability

Rewrite (3),

$$(\Omega-\omega_\beta)\tilde{y}_n=-i\frac{Nr_0c}{2\gamma T_0^2\omega_\beta}\sum_{m=0}^{M-1}a_m\tilde{y}_m\sum_{p=-\infty}^{\infty}Z_\perp(p\omega_0+\omega_\beta)\,exp\left(-i2\pi p\frac{m-n}{M}\right) \qquad (3)$$



It can be simplified by mode separation

If there are M bunches evenly filled in the ring, the motion can be separated into M modes, the μ^{th} mode get following form:

$$\tilde{y}_n^\mu \propto e^{2\pi i\mu n/M}, \quad \mu=(0,1,2\ldots M\text{-}1) \qquad (4)$$

Substituting Eq.(4) into Eq.(3), we get:

$$\Delta\omega_n^\mu=\Omega_n^\mu-\omega_\beta=-i\frac{Nr_0c}{2\gamma T_0^2\omega_\beta}\sum_{p=-\infty}^{\infty}\sum_{k=0}^{M-1}\sum_{m=0}^{M-1}Z_\perp[(pM+k)\omega_0+\omega_\beta]\,a_m\,exp[-2\pi i\frac{(m-n)(k-u)}{M}] \qquad (5)$$

Beam motion is combination of M modes or in other words M modes is the Fourier expansion of the beam motion. Any mode unstable represents beam is unstable. Real part of $\Delta\omega_n^\mu$ means tune shift, imaginary part will be the growth rate.

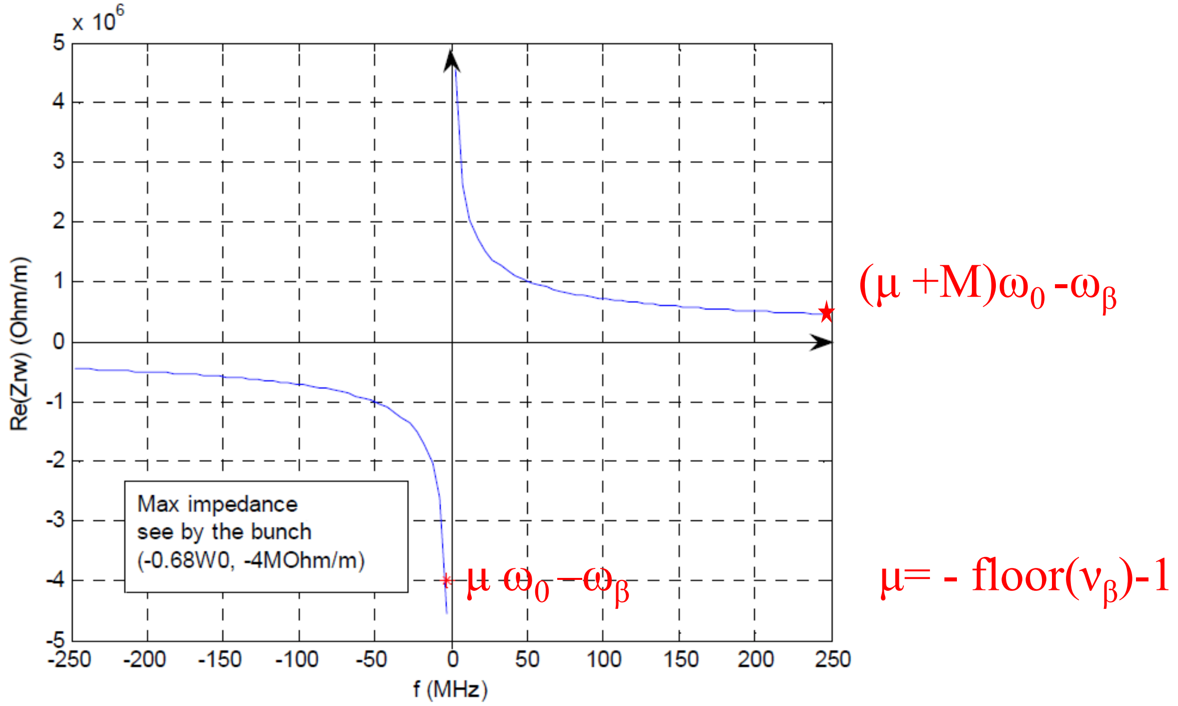
Coupled bunch instability

If ring is uniformly filled $a_m = costant$, only k=μ terms left in Eq.(5), all the other terms equals 0. Eq.(5) can be further simplified:

$$\Delta\omega_n^\mu = \Omega_n^\mu - \omega_\beta = -i\frac{a_m M N r_0 c}{2\gamma T_0^2 \omega_\beta}\sum_{p=-\infty}^{\infty} Z_\perp[(pM+u)\omega_0+\omega_\beta] \qquad (6)$$

Take resistive wall impedance as an example, where b is the radius of beam pipe, σ is the conductivity of the metal.

$$Z_\perp(\omega) = \frac{1}{\pi b^3}\sqrt{\frac{2\pi}{\sigma|\omega|}}[sgn(\omega)-i]$$



If sum of real part of impedance in Eq.(6) <0, it will cause instability. Where $\omega_\beta=\nu_\beta\omega_0$, it can be deduced that fraction part of ν_β<0.5 will benefit resistive wall instability.

Coupled bunch instability

Longitudinal coupled bunch instability can be done in a similar way:

$$\Delta\Omega = \Omega - \omega_s = i\frac{\eta Q_e}{2E_0T_0^2\omega_s}\sum_{p=-\infty}^{\infty} p\omega_0 Z_{\parallel}(p\omega_0) - (p\omega_0 + \Omega)Z_{\parallel}(p\omega_0 + \Omega) \quad (7)$$

Where η is slip factor, Q_e is bunch charge, ω_s is synchrotron ocsilliation angular frequency, Real part of the impedance will contribute to instability. $\mathrm{Re}(Z_{\parallel}(p\omega_0))$ is an even function， first part in sum is 0 (for uniformly filled ring) . Only second term with sideband contribute to instability.

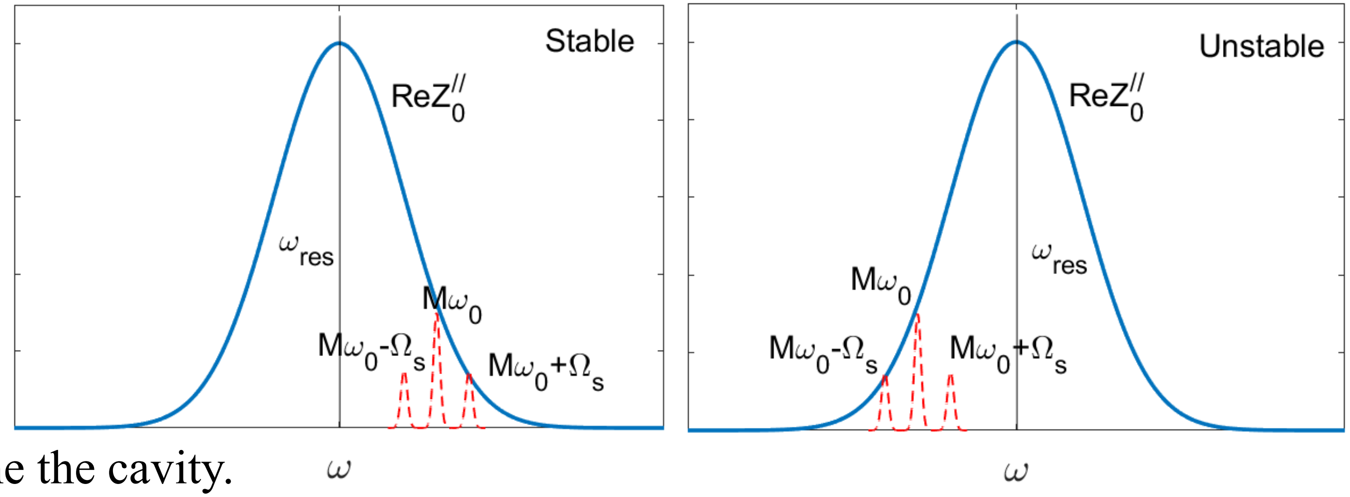
Narrow band longitudinal impedance is mainly from HOM of RF cavity. It becomes dangerous when f_{HOM} close to $p\omega_0$. When number of HOMs is limited, the cavity can be detuned to avoid instability. However, when there are large number of HOMs, it is difficult to do so, the only way is to damp HOM to reduce the impedance.。

The resonance frequency of RF cavity accl mode $\omega_{rf} \approx M\,\omega_0$ in order built up enough RF voltages feed by RF power. To efficiency build up RF voltage, the impedance of the accl mode should be large. The accl mode will be dangerous, the instability is called Robison instability, the criterion can be expressed:

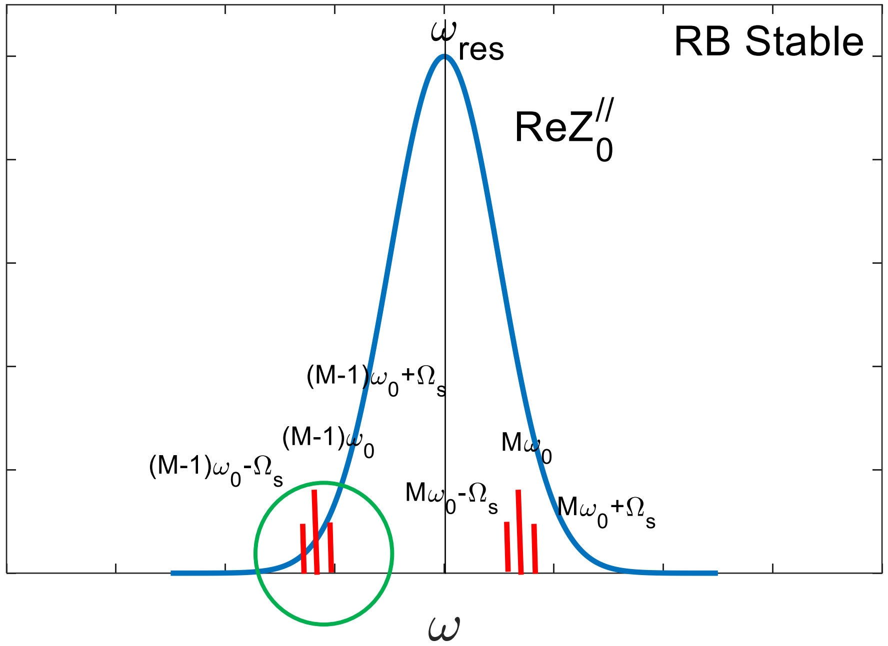
$$\alpha_s \approx \frac{N r_e \eta M\omega_0}{2\gamma T_0^2\omega_s}\left\{Re\left[Z_0^{//}(M\omega_0 + \Omega_s)\right] - Z_0^{//}(M\omega_0 - \Omega_s)\right]\right\}$$

The only way to avoid Robinson instability is to detune the cavity.

Robinson instability for $\eta > 0$



Coupled bunch instability



For some cases, when cavity Q is not so high and ring circumference is large, the spectrum width is comparable to revolution frequency. The -1 mode frequency can still overlap with impedance, may cause -1 mode instability.

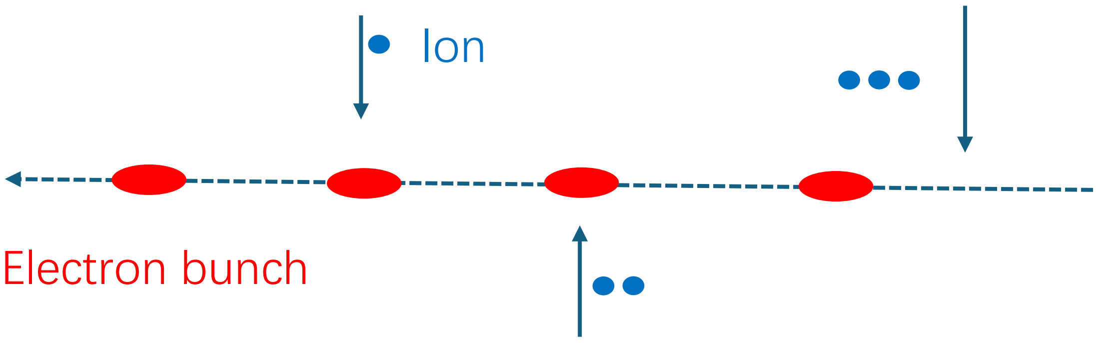
Then arrive at a conclusion that larger ring will be more sensitive to the longitudinal coupled bunch instability.

Ion instability

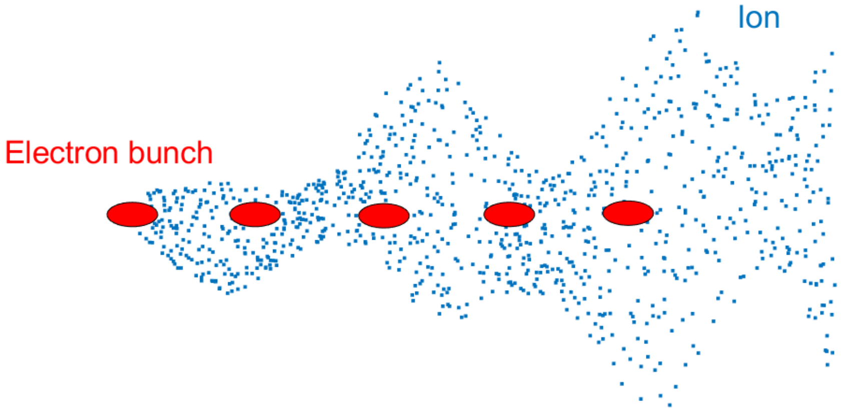
Ion instability

Ion production： Residual gas molecules can be ionized by the beam collision or by the synchrotron radiation photodecomposition.
The ionized electrons will be expelled away form closed orbit and get no effects on beam.

Ion instability
- Ion trap — Ions accumulate in multi turns.
- fast ion instability — Ions accumulate along bunch train, drift away after train.



Ion trap



Fast ion

Ion instability

Ion trap

When an electron bunch passes through, the ions will experience an attractive force proportional to their transverse distance from the beam center. Therefore, it can be treated as a focusing lens.

$$k_y = \frac{r_p N_e}{A\sigma_y(\sigma_x + \sigma_y)}$$

Where A is molecular mass number, r_p classical radius of the proton, N_e electron number in the bunch, $\sigma_{y,x}$ transverse electron beam RMS beam sizes. If electron bunch is uniformly filled in the ring, the ion will receive a periodic focus which can be represented by transfer matrices:

$$\begin{pmatrix} y \\ y' \end{pmatrix}_j = \begin{bmatrix} 1 & S \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}_i = \begin{bmatrix} 1-kS & S \\ -k & 1 \end{bmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}_i$$

If ion motion is stable, it will be trapped around closed orbit. For the stable motion, the absolute value of the trace of the matrix <2, gives out Eq.(8)

$$0 < kS < 4$$

$$A > \frac{r_p N_e S}{4\sigma_y(\sigma_x + \sigma_y)} \qquad (8)$$

Which means the molecular mass number greater than Eq.(8) will be trapped. As more and more ions are trapped, the beam will be unstable. However, the molecular in pipe is mainly H_2 (A=2), CO/N_2 (A=28), Ar (A=40), CO_2 (A=44) , they are easily escaped.

Ion instability

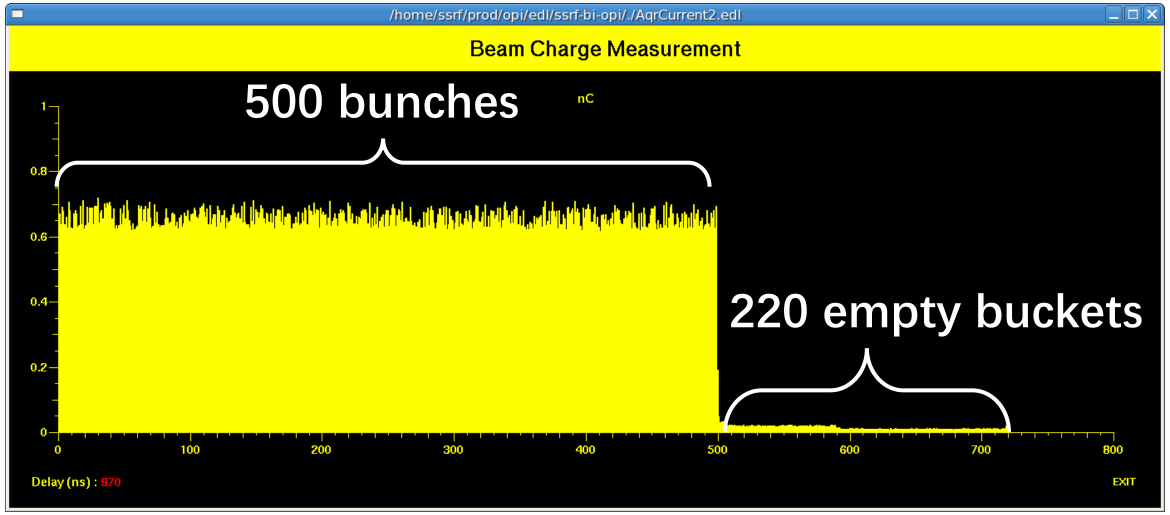
Ion trap

From Eq.(8) it can be found, lower the transverse beam size, larger the beam current will prevent ion trapping.

Lower the vacuum pressure will reduce ion trapping numbers.

Besides above solutions, there is another method to prevent ion trapping: Non-uniform filling

Non-uniform filling will break the periodic focus lattice, making ion motion unstable. A common filling pattern is a bunch train followed by empty buckets(ion clearing gap) letting ion drift away during the gap.



One of the filling patterns in SSRF storage ring.

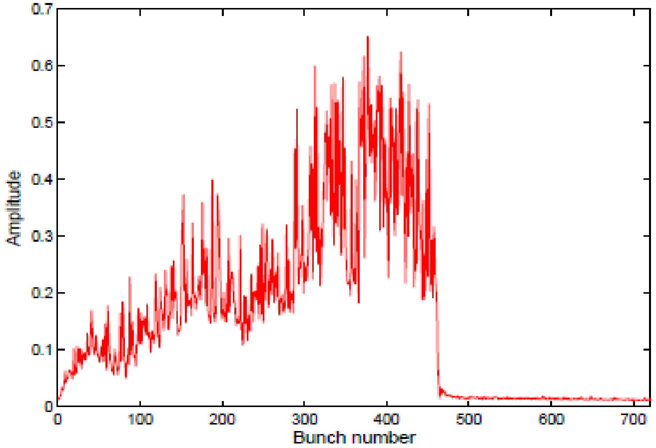
Ion instability

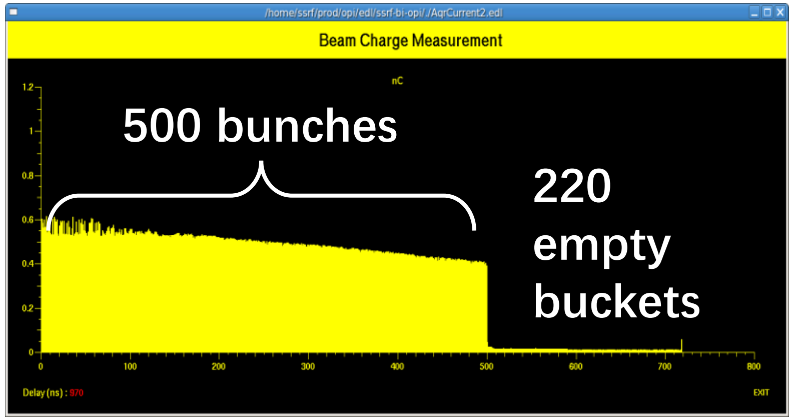
Fast ion instability

Ion instability can develop even when there is an ion clearing gap if the vacuum pressure is not sufficiently low. The ions produced by the bunches in the front of the train will affect the motion of the tail bunches. This phenomenon is known as fast ion instability, and its growth rate can be expressed as follows:

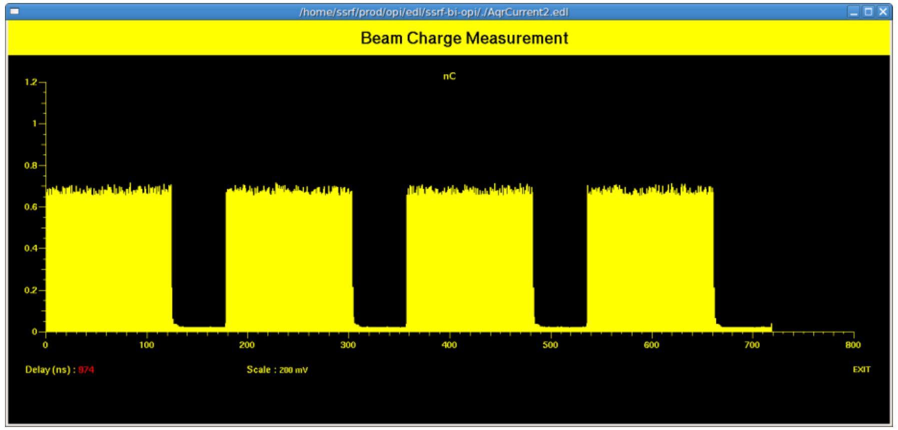
$$\frac{1}{\tau_{fi}} \approx \frac{n_{gas}\sigma_i r_p^{1/2}}{A^{1/2}} \frac{N_e^{3/2} n_b^2 r_e L_{sep}^2 c}{\gamma[\sigma_y(\sigma_x+\sigma_y)]^{3/2}}$$

Where n_{gas} is density of residual gas molecular, σ_i cross section of ionization， n_b bunches number in the bunch train， L_{sep} is the bunch separation in the train.





The fast ion instability in SSRF storage ring, the oscillation amplitude increased along the bunch train， When collimator closed, the tail bunches loss charge.

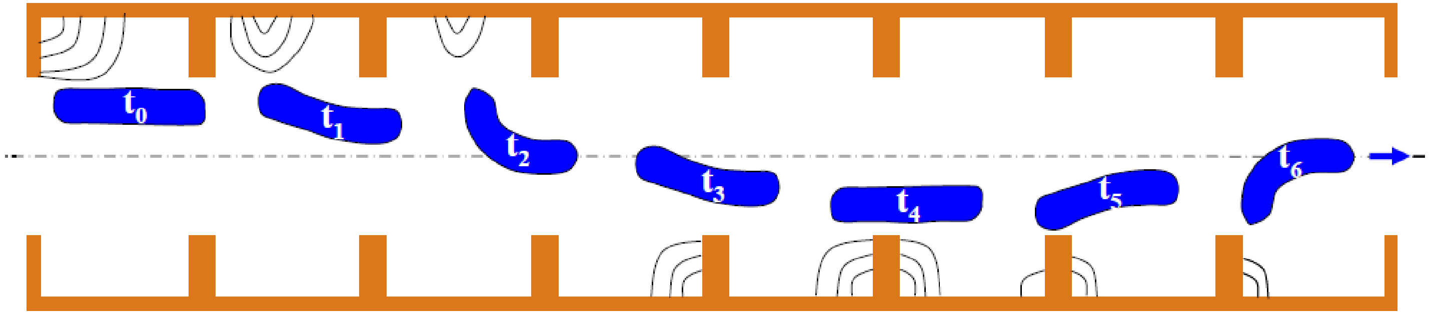


Multi bunch train filling to suppress fast ion instability.

Single bunch instability

Single bunch instability

a、Beam Breakup （BBU）



- Usually happens in linac, where particles in head and tail of the bunch do not exchange.
- When bunch charge is high, wakefield is great.
- The launch beam is off-axis.

➢ The particles at the tail of the bunch feel the wake generated by the particles at the head. This causes the transverse oscillation amplitude of the tail to increase, deforming the bunch into a banana shape and eventually leading to beam breakup.

a、Beam Breakup （BBU）

Two particle model: Head particle #1, tail particle #2, separation distance z, The equation of motion of #2:

$$y_2'' + k_\beta^2 y_2 = -\frac{N r_e W_\perp(z)}{2\gamma L}\hat{y}_1 \cos(k_\beta s)$$

where k_β is the wave-number of β oscillation in linac, L is the length of accelerator cavity, $W_\perp(z)$ is the transverse wake function of an accelerator cavity, $\hat{y}_1$ oscillation amplitude of head particle. The motion of #2 can be written:

$$y_2 = \hat{y}_1\left[\cos(k_\beta s) - \frac{N r_e W_\perp(z)}{4 k_\beta \gamma L} s \cdot \sin(k_\beta s)\right] \qquad (9)$$

The first term in right hand of Eq.(9) is a free betatron oscillation. The second term is relative to wakefield. After accelerate length L_0, the amplitude increased by a factor:

$$\mathrm{A} = -\frac{N r_e W_\perp(z)}{4 k_\beta \gamma L} L_0 \qquad (10)$$

Eq. (10) without considering energy increment. When energy increment (γ_i-> γ_f) is considered, the adiabatic damping will play role:

$$\mathrm{A} = -\frac{N r_e W_\perp(z)}{4 k_\beta \gamma_f L} L_0 \ln\left(\frac{\gamma_f}{\gamma_i}\right)$$

Single bunch instability

a、Beam Breakup （BBU）

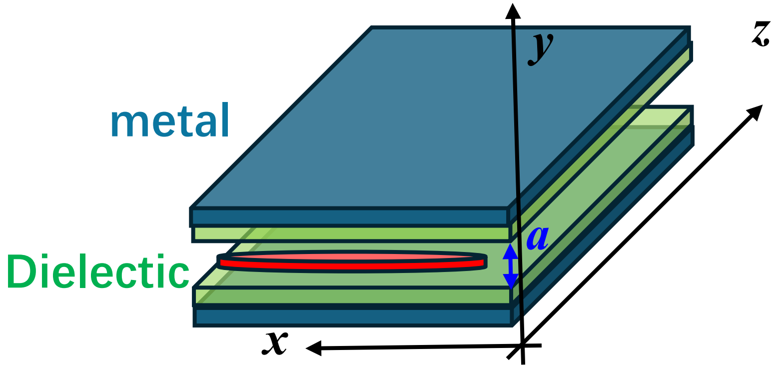
- When BBU should be seriously treated？ When acceleration structure is small such as X band accelerator, dielectric wakefield accelerator who pursue high gradient acceleration （$W_{\parallel} \propto 1/b$， $W_{\perp} \propto 1/b^3$）．
- How to fight against BBU
  - Stronger focus， increasing k_β.
  - More efficient acceleration, better adiabatic damping.
  - Better alignment, the injection beam is closer to center.
  - Add the BNS damping mechanism (Balakin-Novokhatsky-Smirnov)
  - Slab-symmetric structure + flat beam(for dielectric wakefield)

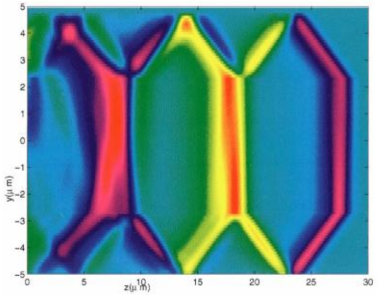
BNS damping: let head particle a tune shift with tail particle.

$$y_2'' + \left(k_\beta + \Delta k_\beta\right)^2 y_2 = -\frac{N r_e W_\perp(z)}{2\gamma L}\hat{y}_1 \cos(k_\beta s)$$

$$y_2 = \hat{y}_1 \cos\left(k_\beta s + \Delta k_\beta s\right) + \hat{y}_1 \frac{N r_e W_\perp(z)}{4 k_\beta \Delta k_\beta \gamma L}\left[\cos\left(k_\beta s + \Delta k_\beta s\right) - \cos(k_\beta s)\right]$$

slab-symmetric structure + flat beam





$\sigma_x >> a; \sigma_x >> \sigma_y$

$W_{\parallel}$ becomes uniform, Panofsky-Wenzel Theory, $W_\perp$ greatly suppressed

Single bunch instability

b、Head tail instability

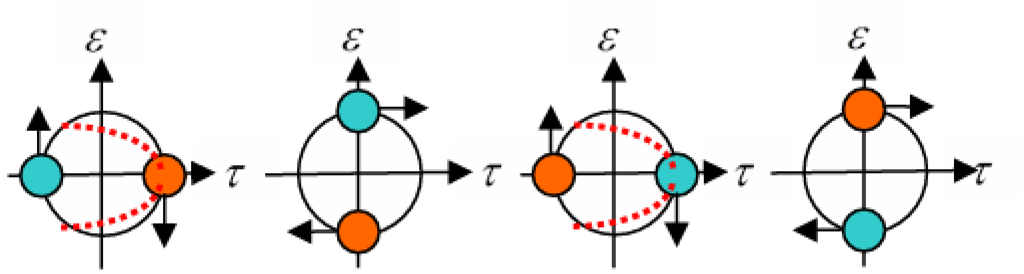
- Unlike in a linac, bunch in a storage ring will exchange it particles between head and tail caused by synchrotron oscillation.
- Synchrotron oscillation will cause energy variation, when chromaticity ≠0, the betatron tune will be variated.

Two particle model

$$y_2'' + [\frac{\omega_\beta(\delta_2)}{c}]^2 y_2 = -\frac{N r_e W_\perp(z)}{2\gamma C} y_1$$

$$\omega_\beta(\delta_2) = \omega_\beta[1 + \frac{\xi \hat{z} \omega_s}{c\eta}\cos\left(\frac{\omega_s s}{c}\right)]$$

C is ring circumference, ξ is chromaticity, ω_s is synchrotron oscillation angular frequency.



$$\begin{bmatrix}\tilde{y_1}\\ \tilde{y_2}\end{bmatrix}_{\pi c/\omega_s} = \begin{bmatrix}1 & 0\\ i\Upsilon & 1\end{bmatrix}\begin{bmatrix}\tilde{y_1}\\ \tilde{y_2}\end{bmatrix}_0$$

$$\begin{bmatrix}\tilde{y_1}\\ \tilde{y_2}\end{bmatrix}_{2\pi c/\omega_s} = \begin{bmatrix}1 & i\Upsilon\\ 0 & 1\end{bmatrix}\begin{bmatrix}\tilde{y_1}\\ \tilde{y_2}\end{bmatrix}_{\pi c/\omega_s}$$

$$\Upsilon = \frac{\pi N r_e W_\perp(z) c^2}{4\gamma C \omega_\beta \omega_s}[1 + i\frac{4\xi \hat{z} \omega_\beta}{\pi c \eta}]$$

b、Head tail instability

The transfer matrix for a whole synch period

$$\begin{bmatrix} 1 & 0 \\ i\Upsilon & 1 \end{bmatrix}\begin{bmatrix} 1 & i\Upsilon \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\Upsilon^2 & i\Upsilon \\ i\Upsilon & 1 \end{bmatrix}$$

The eigenvalue of the matrix is

$$\lambda_\pm = e^{\pm i\Upsilon}$$

The growth rate can be found

$$\alpha_\pm = \tau_\pm^{-1} = \mp \frac{N r_e W_\perp(z) c \xi \hat{z}}{2\pi\gamma C\eta}$$

When $\xi/\eta > 0$时， “+” mode is stable, while “-” mode is unstable. Solving the Vlasov equation can get more accurate solutions， which shows we have overestimated the growth rate of “-” mode by the simplified model which can be damped by synchrotron radiation and Landau damping. For ring η>0， we can choose working point $\xi > \sim 0$.

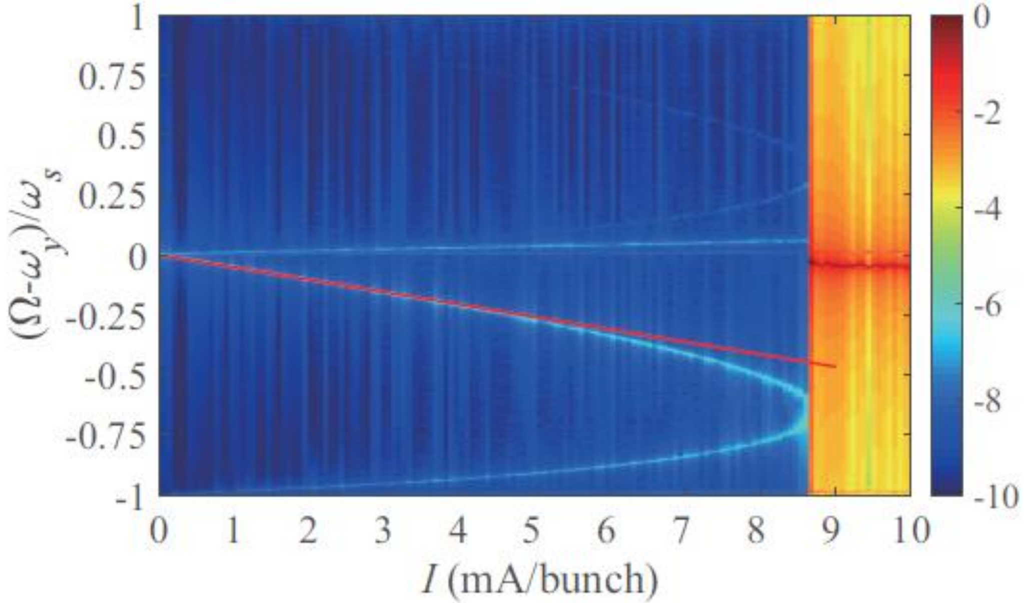
Single bunch instability

c、Transverse mode coupling instability（TMCI）

Though head tail instability can be avoided when $\xi > \sim 0$， when bunch current further increased, another instability TMCI will occur. Transverse mode 0 and -1 mode frequency will be shifted, when their frequency merged, turbulence will happen, which will cause instability, the threshold can be written:

$$I_{th} \approx \frac{4\omega_s \omega_\beta b^3 E_0[eV]}{\pi c^2 R(Z/n)}$$

R is average radius of the ring， $Z(\omega)/n$ longitudinal broad band impedance, $n = \omega/\omega_0$, ω_0 is revolution frequency.



UTEF ring TMCI simulation result

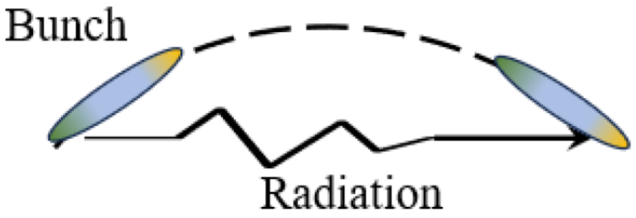
Single bunch instability

d、CSR instability

wakefield

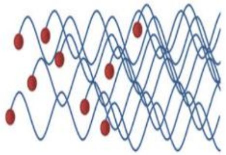
CSR

When the radiation wavelength is close to or longer than the bunch length, coherent synchrotron radiation (CSR) will be emitted. In a dipole, electrons move on a curved path, and the CSR will catch up with the leading electrons. As the electrons get transverse momentum, they will interact with the CSR.

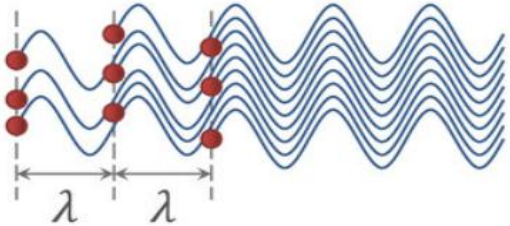


In a storage ring where $\eta \neq 0$ (R56 ≠0), energy modulation will cause density modulation, creating micro-bunches. These micro-bunches will enhance CSR, leading to an instability. This process is very similar to FEL, except with a **longer wavelength** and in an uncontrolled way.

Why?



Incoherent radiation



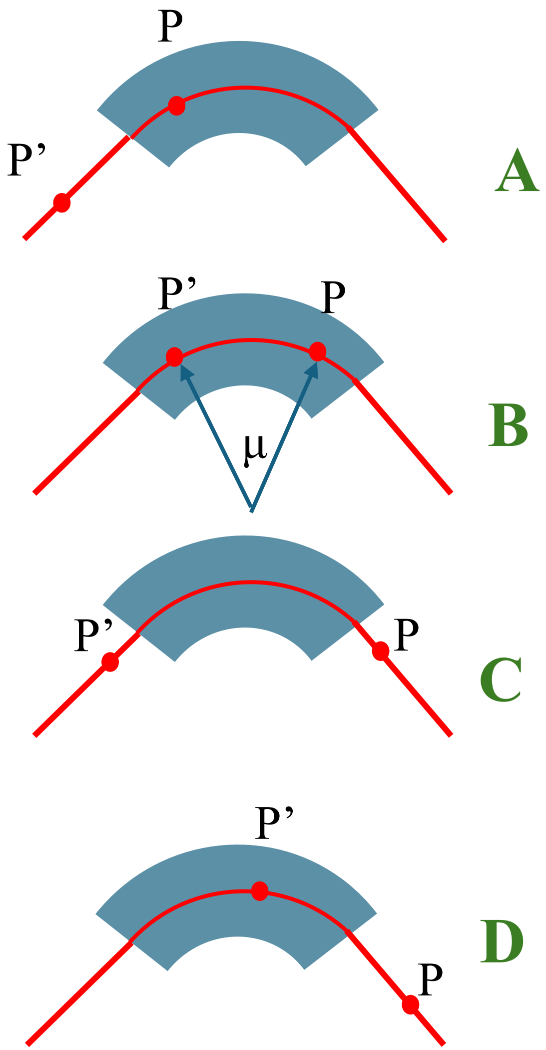
Coherent radiation

If above process happens at the place where gets dispersion, transverse emittance will be increased.

CSR can be shield by vacuum chamber.

Single bunch instability

d、CSR instability



CSR can also be investigated by “wakefield” &“impedance”， Unlike EM field, it effect leading particles.

3D model can describe CSR more accurate, it is complex. We usually using1D model, 1D model will somewhat overestimate the problem.

P’ is the macro-particle emit CSR， P is the macro-particle effected by CSR， There are four cases A, B, C, D as shown in left， Among which **B** is most important， We take B for example to calculate CSR wake function， it is shown in Eq.(11)， where R bending radius， z is distance between P’ and P.

$$w = -\frac{Z_0 c}{4\pi}\frac{2}{3^{4/3}R^{2/3}z^{4/3}} \qquad (11)$$

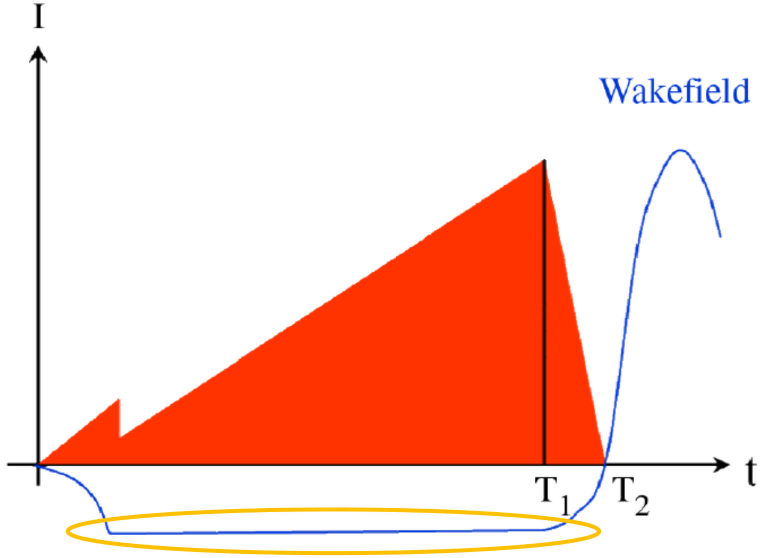
CSR wake is the convolution wake function with bunch charge density.

$$\mathcal{W}(z) = \int_{-\infty}^{\infty} w(z-z')\lambda(z')dz' \qquad (12)$$

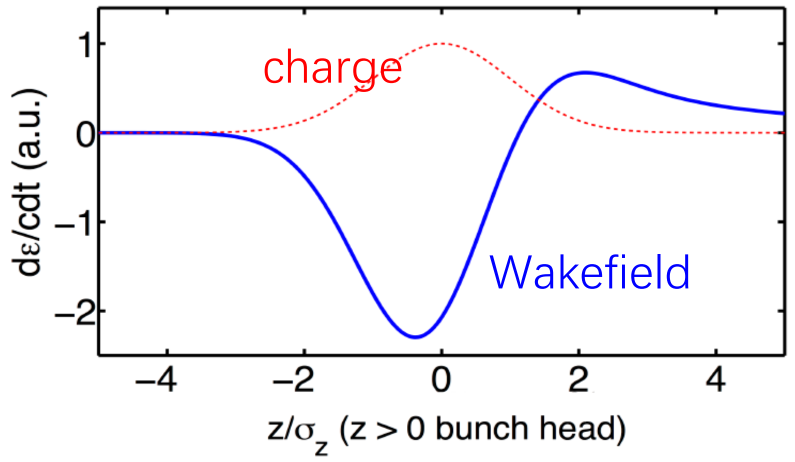
It can be found in Eq.(11) that w is a monotonically increasing function which is far from EM field. Eq.(12) gets singularity, needs P.V. integration.

Single bunch instability

d、CSR instability



For EM wake, it is Cos like or combination of Cos waves, Specified longitudinal distributions can be found to make the wakefield flattened (covering most part of the bunch) which will benefit the instability.



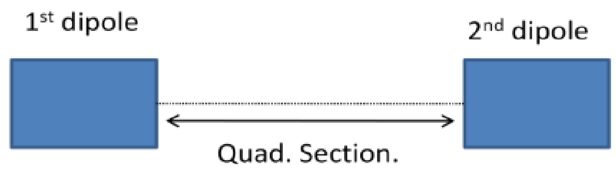
However, for CSR wake, it is pointless to find a situation like left side. It means CSR will inevitably increase the energy spread.

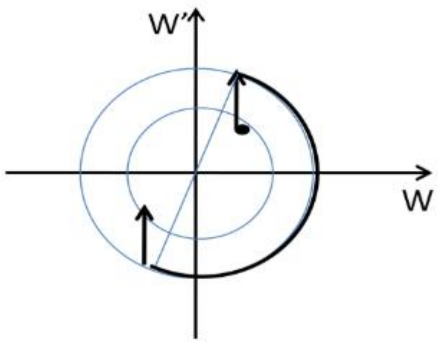
How to suppress CSR instability?

- In a linac, suppress CSR effect usually refers to control the transverse emittance growth.
- In a storage ring suppress CSR instability is to control the development of micro-bunch.

d、CSR instability

In a linac, such as driving for an FEL, the peak current is very high, CSR is a serious problem. Any bending should be carefully treated. Chicane is used to suppress the bunch length, where CSR will degrade the beam performance.





The energy modulated by CSR is different to the energy spread heated by ISR. ISR is a random process, the energy spread increment mapping to transverse plane causing an emittance growth is inevitable unless dispersion is free. However, energy modulation caused by CSR is predictable, by a wise Lattice design the emittance growth can be cancelled between dipoles.

Single bunch instability

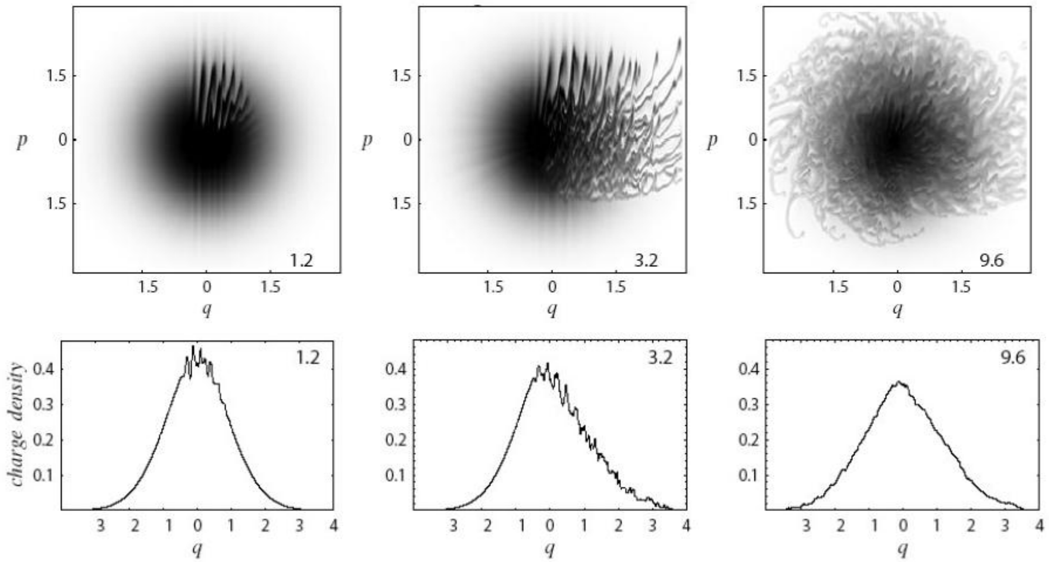
d、CSR instability

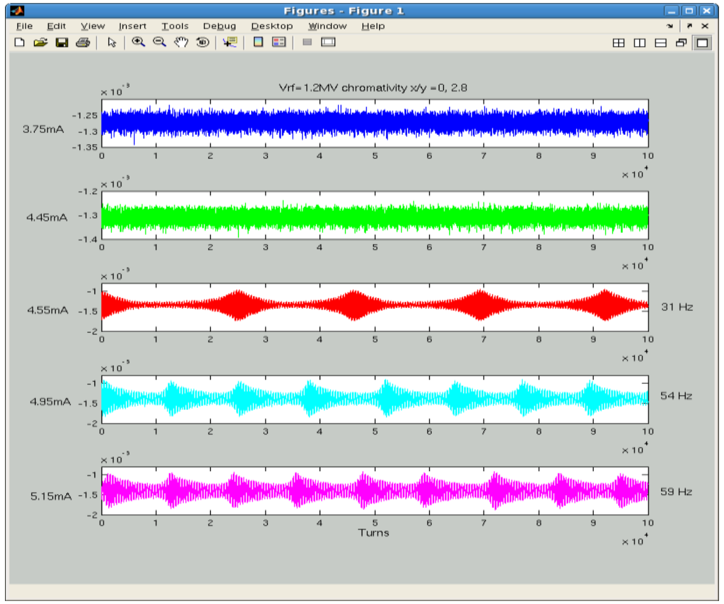
CSR instability threshold in a storage ring can be written （Stupakov & Bane）:

$$I_{th} = \frac{I_A \gamma |\eta| \sigma_{z0}^{1/3} \sigma_{\varepsilon 0}^2}{\rho^{1/3}} \left[0.5 + 0.12 \frac{\sigma_{z0} \rho^{1/2}}{b^{3/2}}\right]$$

Where σ_{z0} is initial bunch length， $\sigma_{\varepsilon 0}$ is energy spread， ρ is bending radius， I_A is Alfven current=17.045kA。

Increase energy spread is powerful method to suppress CSR instability.





Most of the ring present CSR bursts as CSR instability happens.

Single bunch instability

d、CSR instability

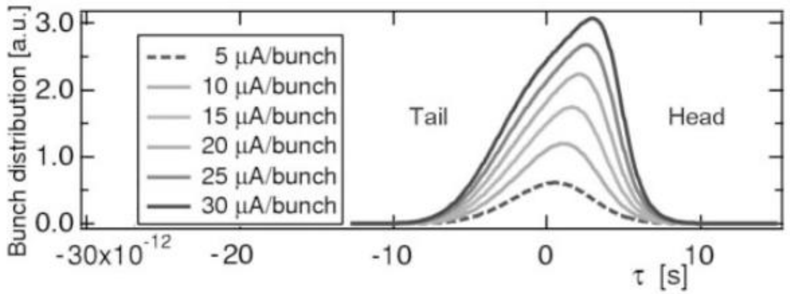


Figure 2: Calculated equilibrium distributions using the shielded SR wake. Case of the BESSY II in the configuration for CSR production.

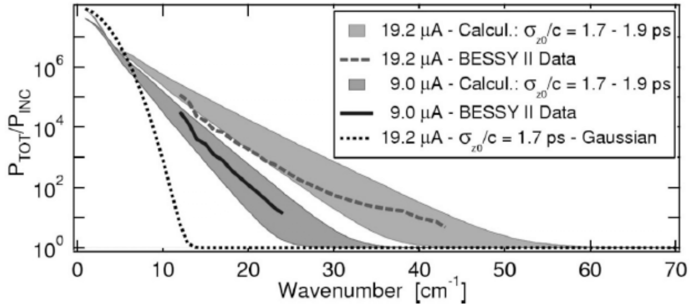
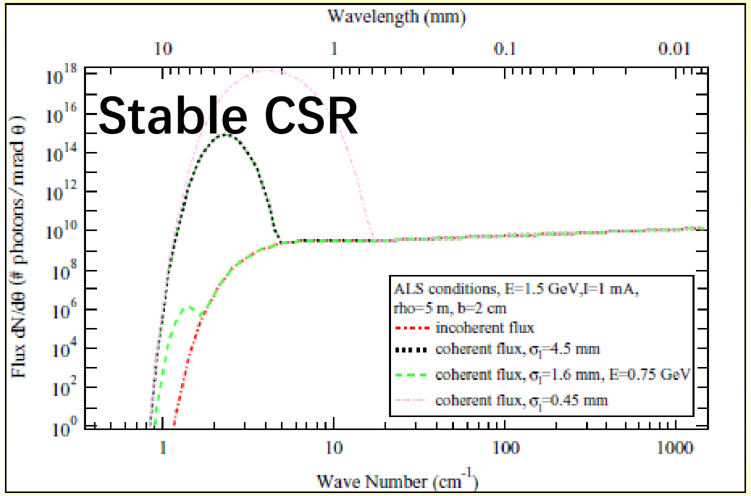


Figure 3: CSR gain as a function of the wavenumber. The BESSY II data for two different currents per bunch are compared with the shielded SR calculation and with the curve for a Gaussian distribution of the same length.



Few machine can provide stable CSR such as BESSY II, there are 1 ~ 2 weeks annually operating in low alpha mode, to create high flux THz radiations.

e、Microwave instability

The instability caused by the short range longitudinal EM wakefield is called Microwave instability.（Some literature also call CSR instability to Microwave instability）.

Microwave instability and CSR instability is very similar，but still some differences：

Microwave instability	CSR instability
Tail particles are affected	Leading particles are affected
Wakefield all around ring	CSR wake appear in Bend
Wakefield proportional to Q	Wakefield proportional to Q^2

The instability criterion can be gotten by solving Vlasov equation on bunch longitudinal distribution $\Psi(z,\delta;s)$：

$$\frac{\partial\Psi}{\partial s}+z'\frac{\partial\Psi}{\partial z}+\delta'\frac{\partial\Psi}{\partial\delta}=0 \qquad z'=-\eta\delta \qquad \delta'=\frac{qV_{RF}}{E_0C}\sin\left(\phi_{RF}-\frac{\omega_{RF}}{c}z\right)-\frac{U_0}{E_0C}+\frac{qV_{\parallel}(z)}{E_0C}$$

However the mathematics is difficult.

e、Microwave instability

Keil-Schnell-Boussard criterion is given here:

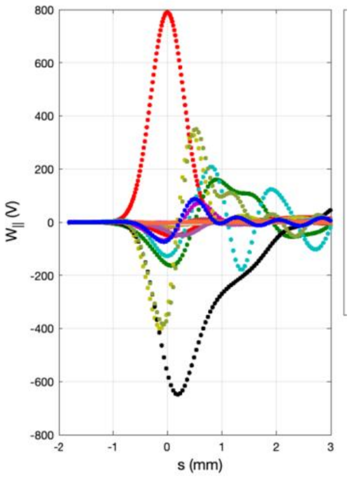
$$I_{th}=\frac{(2\pi)^{3/2}\eta E_0[eV]\sigma_{z0}\sigma_{\varepsilon0}^2}{C\left|\frac{Z}{n}\right|_{eff}}$$

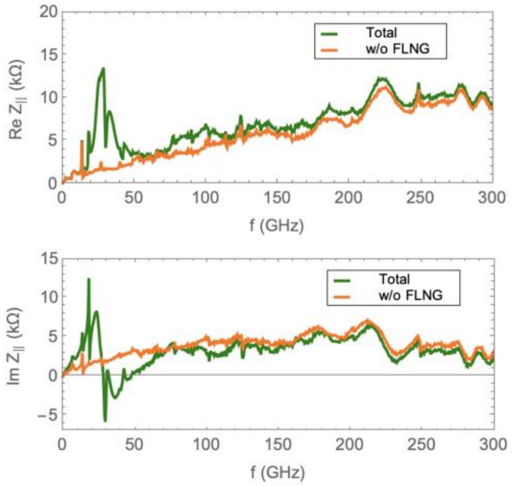
$\left|\frac{Z}{n}\right|_{eff}$ is effective longitudinal broad band impedance.

$$\left|\frac{Z}{n}\right|_{eff}=\frac{\int\left|\frac{Z}{n}\right|h_m dn}{\int h_m\, dn}\qquad n=\frac{\omega}{\omega_0}$$

$$h_m=\frac{1}{\Gamma\left(m+\frac{1}{2}\right)}(n\omega_0\sigma_{z0}/c)^2 e^{-(n\omega_0\sigma_{z0}/c)^2}$$

m usually to be 1.

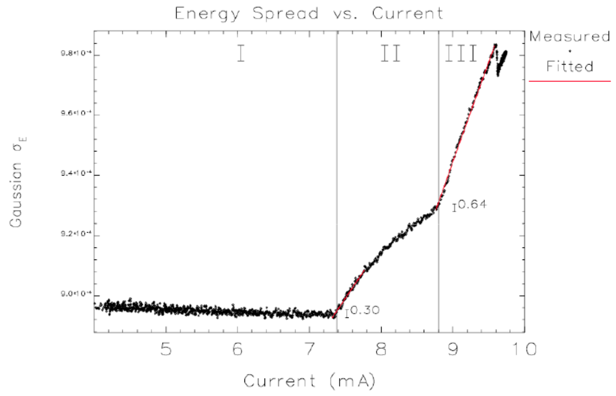




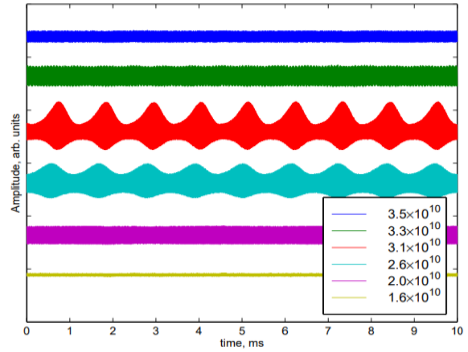
Steady energy spread growth is more likely to be observed

$$\sigma_\varepsilon=\sigma_{\varepsilon0}+k(I-I_{th})^{1/3}$$

Sometimes it is called SPEAR 1/3 low.



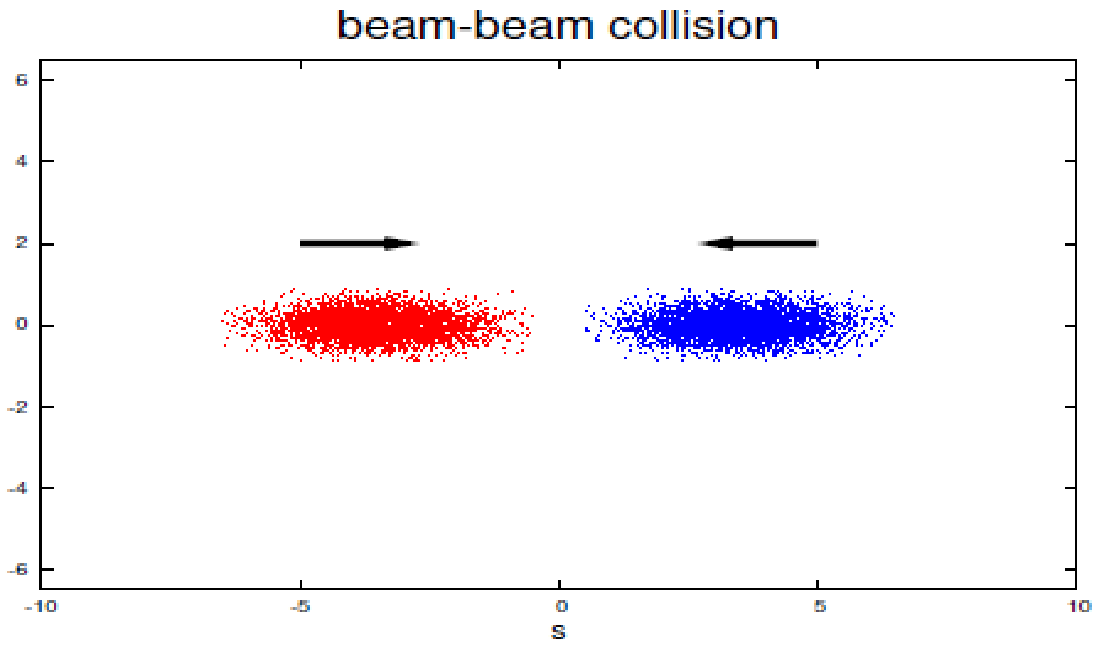
Y.-C. Chae et al, "Measurement of the longitudinal microwave instability in the APS storage ring", Proceedings of the 2001 Particle Accelerator Conference, Chicago (2001).



Microwave burst in SLC damping rings (9.9GHz)

Beam-Beam effects

Beam-Beam effects



Beam-beam effect is a particular phenomenon in colliders when beams get collision.

When beams moves collinearly in the same direction, the space charge force vanishes.

When beam moves in opposite direction, electric and magnetic force added up, providing additional focusing force （for different charged beams） .

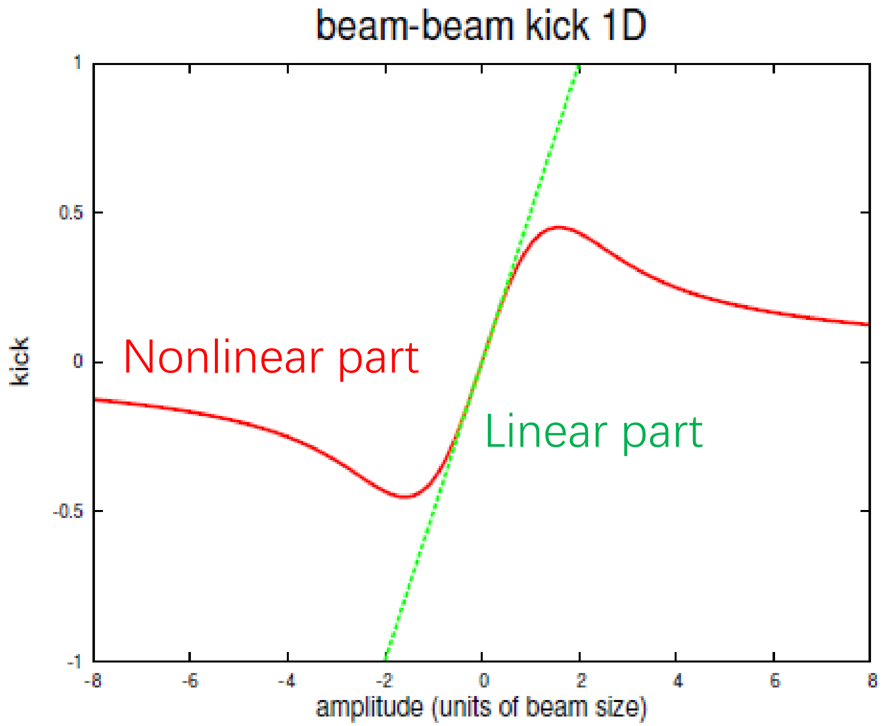
Beam-Beam effects

The transverse kick of round beam head on collision can be expressed:

$$\Delta r' = \frac{1}{mc\beta\gamma}\int_{-\frac{\Delta t}{2}}^{+\frac{\Delta t}{2}} F_r(r,s,t)dt$$

Where Δt is the interaction time of two bunches $F_r(r,s,t) = -\frac{Ne^2(1+\beta^2)}{\sqrt{(2\pi)^3}\epsilon_0 r\sigma_s}\left[1-exp\left(-\frac{r^2}{2\sigma^2}\right)\right]\cdot\left[exp\left(-\frac{(s+vt)^2}{2\sigma_s^2}\right)\right]$

The effect of Nonlinear part needs tracking.



For round beam, beam-beam parameters can be defined

$$\xi = \frac{Nr_0\beta^*}{4\pi\gamma\sigma^2}$$

For flat beam, beam-beam parameters can be defined

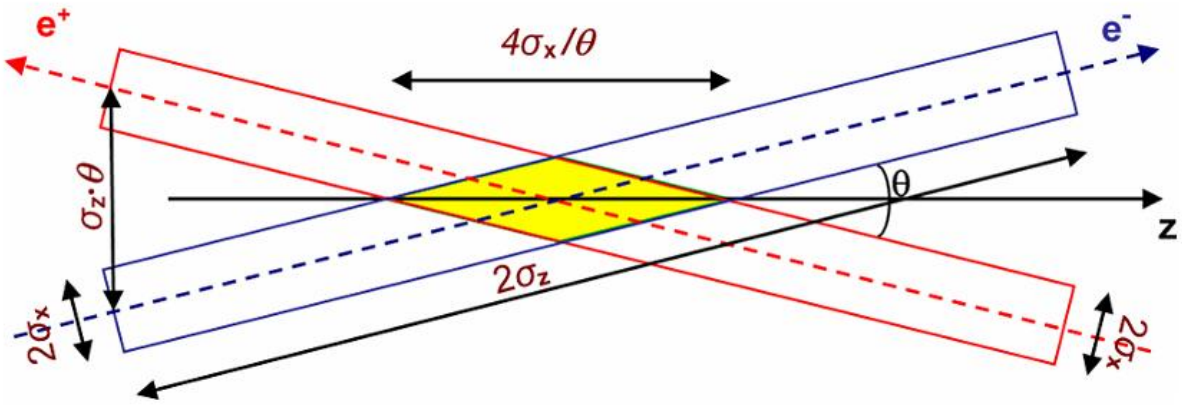
$$\xi_{x,y} = \frac{Nr_0\beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x+\sigma_y)}$$

Beam-Beam effects

For small ξ and Q (transverse tune) not too close to 0.0 and 0.5, Tune shift caused by beam-beam effect:

$$\Delta Q \approx \xi$$

Beam-beam effect distorts the linear optics, drive even order resonance; as the tune shift increases, the luminosity gain becomes less efficient with increasing beam current.



One way to reduce the beam-beam effect is to collide beams with large Piwinski angle.

$$\xi_y \propto \frac{N\beta_y^*}{\sigma_x^*\sigma_y^*\sqrt{1+\varphi^2}} \qquad \xi_x \propto \frac{N\beta_x^*}{\sigma_x^*\sigma_y^*(1+\varphi^2)} \qquad \varphi = \frac{\sigma_z}{\sigma_x^*}\tan\left(\frac{\theta}{2}\right)$$

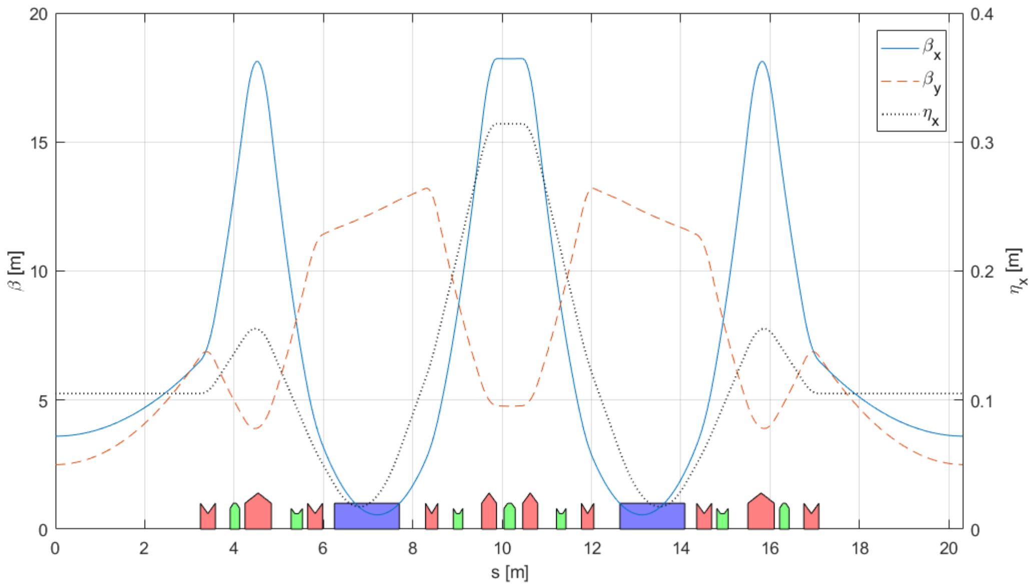
A large Piwinski angle may reduce luminosity; however, the weakened beam-beam effect allows for an even smaller $\beta_y{}^*$, which can significantly enhance luminosity.

IBS effects & Touschek lifetime

IBS effects & Touschek lifetime

When distance between particles is shorter than Debye length, th beam scattering (IBS) and Touschek effect play roles.

A small angle scattering will cause a small fraction of momentum scattering take place where the dispersion nonzero, the scattering v scattering.
It is very similar to incoherent synchrotron radiation (ISR): IBS is considered as electrons scatted by photons (magnetic field).



IBS – ISR also get difference

- IBS takes place all around the ring, ISR takes place only where exists 'photons' (bending magnets, undulator, wiggler, or a real laser Compton scattering). It is more complex to optimize emittance growth caused by IBS because $\mathcal{H}$ function should be optimized all around the ring. While for ISR, only $\mathcal{H}$ function at bending magnet should be considered.
- IBS probability of occurrence changes all around the ring according to Twiss parameters (beam size), ISR is fixed as long as bending magnet is fixed.
- The natural emittance caused by ISR is the balance between quantum excitation and radiation damping. IBS will not create damping, as energy spread, emittance growth, beam size increased which alleviate IBS, the balance is more complexed.

IBS effects & Touschek lifetime

IBS causing emittance and energy spread growth can be calculated by Piwinski formula

$$\frac{1}{T_x} \approx 2\pi^{3/2}(\log)A\left\langle \frac{\mathcal{H}_x\sigma_H^2}{\varepsilon_x}\left(\frac{1}{a}g\left(\tfrac{b}{a}\right)+\frac{1}{b}g\left(\tfrac{a}{b}\right)\right)-a\,g\left(\tfrac{b}{a}\right)\right\rangle$$

$$\frac{1}{T_y} \approx 2\pi^{3/2}(\log)A\left\langle \frac{\mathcal{H}_y\sigma_H^2}{\varepsilon_y}\left(\frac{1}{a}g\left(\tfrac{b}{a}\right)+\frac{1}{b}g\left(\tfrac{a}{b}\right)\right)-b\,g\left(\tfrac{a}{b}\right)\right\rangle$$

$$\frac{1}{T_\delta} \approx 2\pi^{3/2}(\log)A\left\langle \frac{\sigma_H^2}{\sigma_\delta^2}\left(\frac{1}{a}g\left(\tfrac{b}{a}\right)+\frac{1}{b}g\left(\tfrac{a}{b}\right)\right)\right\rangle$$

$$g(\omega)=\sqrt{\frac{\pi}{\omega}}\left[P_{-1/2}^{0}\left(\frac{\omega^2+1}{2\omega}\right)\pm\frac{3}{2}P_{-1/2}^{-1}\left(\frac{\omega^2+1}{2\omega}\right)\right]$$

$$\mathcal{H}_x=\gamma_x\eta_x^2+2\alpha_x\eta_x\eta_{px}+\beta_x\eta_{px}^2 \qquad \mathcal{H}_y=\gamma_y\eta_y^2+2\alpha_y\eta_y\eta_{py}+\beta_y\eta_{py}^2$$

$$\frac{1}{\sigma_H^2}=\frac{1}{\sigma_\delta^2}+\frac{\mathcal{H}_x}{\varepsilon_x}+\frac{\mathcal{H}_y}{\varepsilon_y}$$

$$A=\frac{r_e^2cN_0}{64\pi^2\gamma^4\varepsilon_x\varepsilon_y\sigma_z\sigma_\delta}$$

$$a=\frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_x}{\varepsilon_x}} \qquad b=\frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_y}{\varepsilon_y}}$$

IBS effects & Touschek lifetime

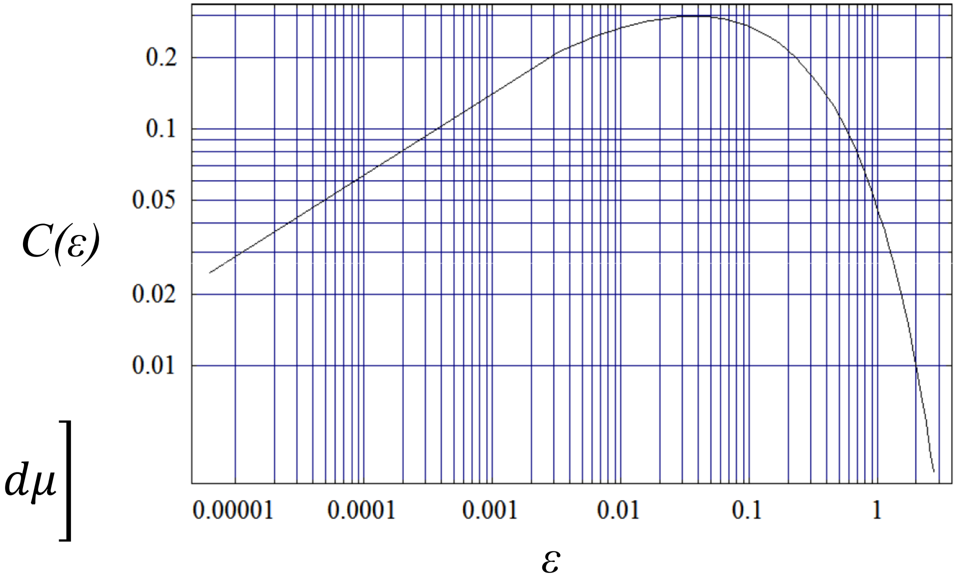
When two electrons scatter with a large amount of momentum exchange, both will exceed the momentum acceptance of the ring and get lost. The beam current will decay due to particle loss. This process is called the Touschek lifetime.

For a Gaussian beam, at ultra relative energy , the Touschek lifetime can be calculated as follows:

$$\frac{1}{\tau_T} = \frac{r_e^2 cN}{8\pi\sigma_x\sigma_y\sigma_z}\frac{1}{\gamma^2\delta_{acc}^3}C(\varepsilon)$$

$$\varepsilon = \left(\frac{\delta_{acc}\beta_x}{\gamma\sigma_x}\right)^2$$

$$C(\varepsilon) = \sqrt{\varepsilon}\left[-\frac{3}{2}e^{-\varepsilon} + \frac{\varepsilon}{2}\int_{\varepsilon}^{\infty}\frac{ln\mu}{\mu}e^{-\mu}d\mu + \frac{1}{2}(3\varepsilon - \varepsilon\cdot ln\varepsilon + 2)\int_{\varepsilon}^{\infty}\frac{e^{-\mu}}{\mu}d\mu\right]$$



Where N is electrons in a bunch, δ_{acc} is momentum acceptance of the ring.

Touschek lifetime is proportional to ~**cubic(^2.**)** of momentum acceptance and **cubic of beam energy**. For low energy ring Touschek lifetime is a dominant lifetime.

Increasing the momentum acceptance and stretching bunch length is the way to increase Touschek lifetime.

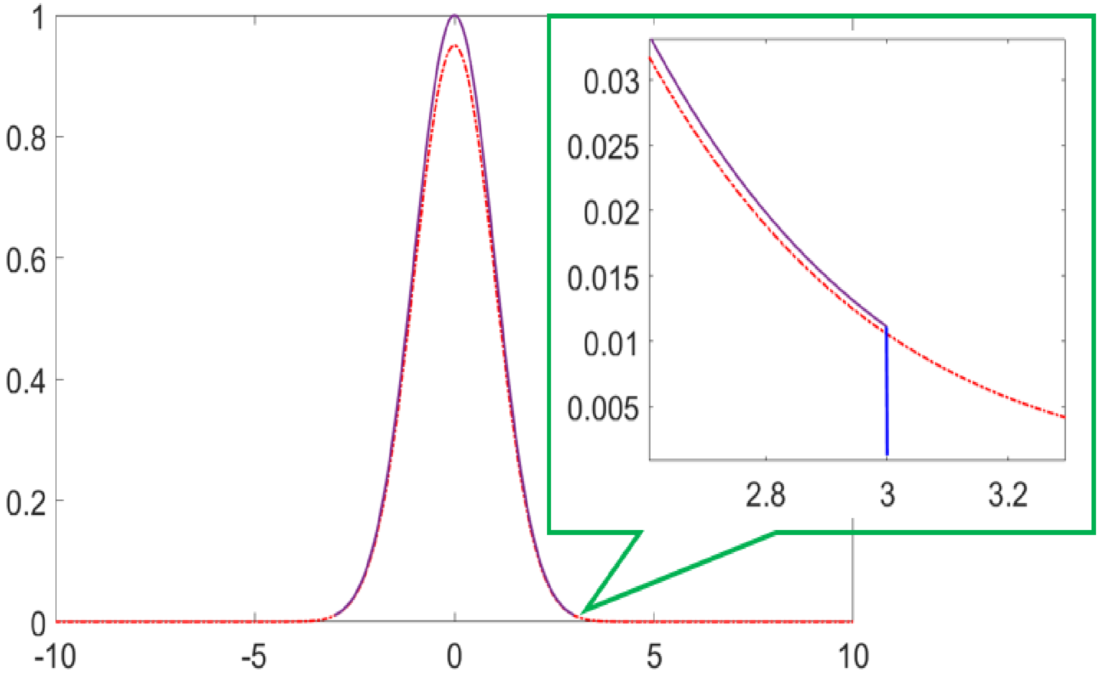
Beam lifetime

There are various mechanisms that cause gradual electron loss, leading to an exponential decay of the beam current. This phenomenon is described by the term 'beam lifetime'

$$I(t) = I_0 e^{-t/\tau}$$

Where τ is the beam lifetime. This 'lifetime' differs from the lifetime of a single particle; it is a concept of collective effect. It describes how long a bunch composed of a large number of electrons will exist in the ring.

Quantum lifetime



For an electron storage ring, due to quantum excitation, electrons have a Gaussian distribution in 6-D phase space. In engineering, it is generally considered that the density beyond 3σ is close to zero, but it is not strictly zero. In physical analysis, this cannot be ignored. If there is a blocker at 3σ as shown in the left figure (indicated by the blue vertical line), particles greater than 3σ will be lost, meaning the Gaussian distribution (purple curve) is truncated. However, due to quantum excitation, electrons will re-balance to a new Gaussian distribution curve (indicated by the red dashed line), meaning some electrons will migrate beyond 3σ space. These outward-migrating electrons will continue to be lost when they encounter the blocker. The continuous loss of electrons leads to beam quantum lifetime

$$\frac{1}{\tau_q} = \left[\tau_i \left(\frac{\sigma_i}{A_i}\right)^2 e^{\frac{1}{2}\left(\frac{A_i}{\sigma_i}\right)^2}\right]^{-1}$$

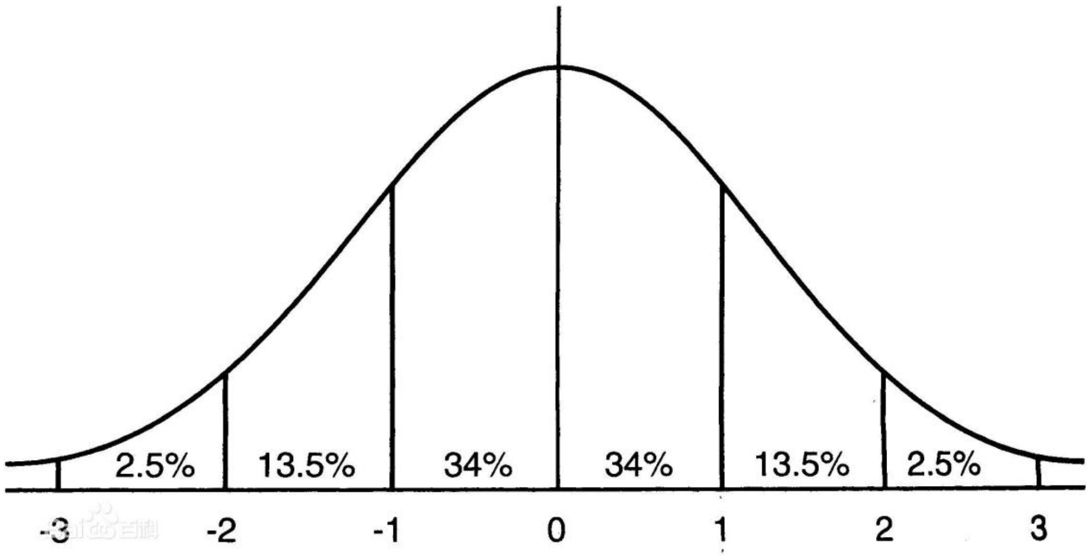
Where i can be x,y,s, τ_i is damping time, σ_i is beam size, A_i is the acceptance (aperture).

τ_q changes drastically with the aperture ,when $\frac{A_i}{\sigma_i} > 6$ quantum lifetime will be hours

Quantum lifetime

For a storage ring with damping time τ=20ms, the quantum lifetime caused by horizontal aperture is as shown bellow

Aperture X/σ_x	4	5	6	7
τ_q	3.7s	215s	10.1 h	4952h



A rough estimation: Aperture X = $2\sigma_x$

The fraction of particles whose amplitudes exceed 2 sigma, is 4.5%。 To decay to 1/e charge, should lost 63.21% of initial charge. 63.21/4.5≈14

14*20ms=280ms

Beam lifetime

The presence of residual molecules in the vacuum pipe can cause electrons to be scattered by these molecules and subsequently lost. This phenomenon is known as gas scattering lifetime.

- The Coulomb scattering of electrons on the residual gas nuclei can cause the transverse oscillation amplitude of the electrons to increase beyond the transverse acceptance, resulting in particle loss. This beam lifetime, known as gas elastic scattering lifetime, can be calculated using the following formula: :

$$\frac{1}{\tau_E} = 2\pi N_g c\, r_e^2 Z(Z+1)\ \gamma^{-2}]\frac{<\beta>}{\varepsilon_\beta}$$

Where N_g is gas density, Z is atomic number of the gass, $<\beta>$ is averaged beta function, ε_β is transverse acceptance.

- Electrons also interact with the residual gas nuclei, undergoing bremsstrahlung. When the energy of the scattered electrons exceeds the energy acceptance, the particles are lost. This process is referred to as the gas inelastic scattering lifetime:

$$\frac{1}{\tau_B} = \frac{4r_e^2 N_g c}{137} Z(Z+\xi)[\frac{4}{3} ln(1/\delta_{acc}) - \frac{5}{8}]\, ln(183Z^{-\frac{1}{3}}) \qquad \xi = \ln\left(1440Z^{-\frac{2}{3}}\right)/\ln(183Z^{-\frac{1}{3}})$$

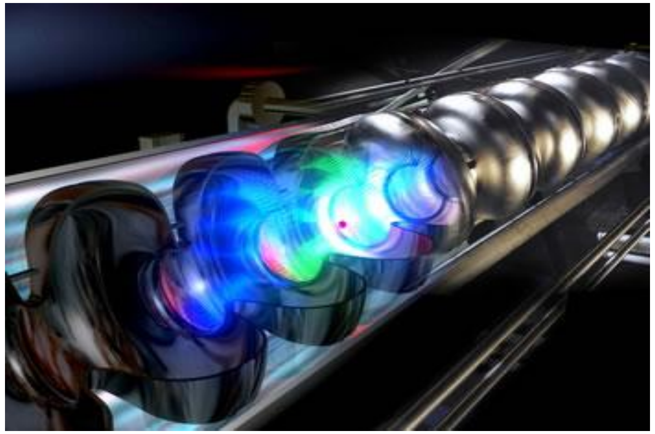
The overall beam lifetime can be calculated:

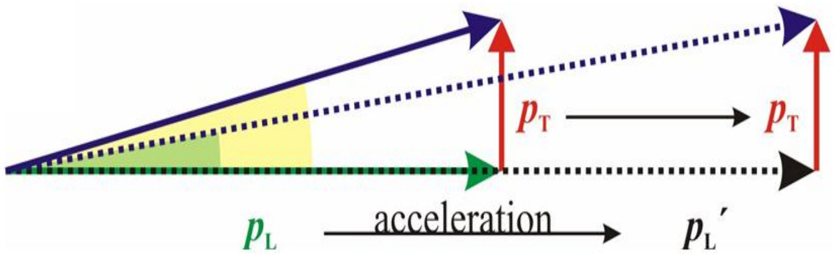
$$\frac{1}{\tau_{lifetime}} = \frac{1}{\tau_q} + \frac{1}{\tau_T} + \frac{1}{\tau_E} + \frac{1}{\tau_B}$$

Damping

Damping

1、 Adiabatic damping：





Consider a transverse motion when beam is accelerated

$$\frac{d}{dt}(m\gamma\dot{y}) = e\beta c B_x \quad with \frac{d}{dt} = \beta c \frac{d}{ds}$$

$$\Rightarrow \beta c(m\beta c\gamma y')' = e\beta c B_x$$

$$\Rightarrow y'' + \frac{(\beta\gamma)'}{\beta\gamma} y' + K_y y = 0 \quad (1)$$

$$y' = \frac{dy}{ds}, \quad K_y = \frac{eB_x}{m\beta c\gamma} \quad \gamma' = \frac{eE_z}{mc^2} = g$$

$$y'' + \frac{\gamma g}{\gamma^2 - 1} y' + K_y y = 0 \quad (2)$$

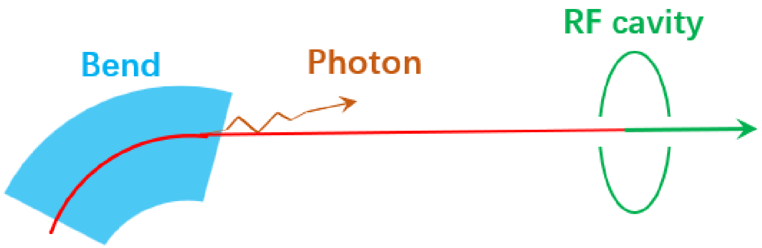
$$y = \frac{\tilde{y}}{\sqrt{\beta\gamma}}$$ Replace y with $\tilde{y}$

$$\tilde{y}'' - \frac{1}{2}\left[\frac{(\beta\gamma)''}{\beta\gamma} - \frac{\left((\beta\gamma)'\right)^2}{2(\beta\gamma)^2}\right]\tilde{y} + K_y \tilde{y} = 0 \quad (3)$$

The amplitude of $\tilde{y}$ is constant, $y \propto \frac{1}{\sqrt{\beta\gamma}}$

In a linac, the geometry emittance will be reduced when beam is accelerated, $\epsilon \propto \frac{1}{\beta\gamma}$, which is not convenient to describe beam performance, usually normalized emittance $\epsilon_N \propto \epsilon\beta\gamma$ is used in a linac

2、Radiation damping:



The radiation direction is parallel with electron trajectory. which takes away transverse momentums, however the energy compensated by RF cavity is absolute longitudinal. This will bring damping mechanism.

$$\overrightarrow{P'} = \vec{P}(1 - \frac{dP}{P_0})$$

$$P'_y = P_y\ (1 - \frac{dP}{P_0})$$

$$\frac{d\varepsilon_y}{dt} = -\frac{\varepsilon_y}{T_0}\oint\frac{dP}{P_0} \approx -\frac{U_0}{E_0 T_0}\varepsilon_y$$

$$\tau_y = 2\frac{\mathrm{E}_0}{U_0}T_0$$

2、Radiation damping:

$$P_\gamma(GeV/sec) = \frac{cC_\gamma}{2\pi}\frac{E^4}{\rho^2}$$ Synchrotron radiation

$$U_{rad} = U_0 + D\epsilon \qquad D = \left(\frac{dU_{rad}}{d\epsilon}\right)_0$$ For first order expansion

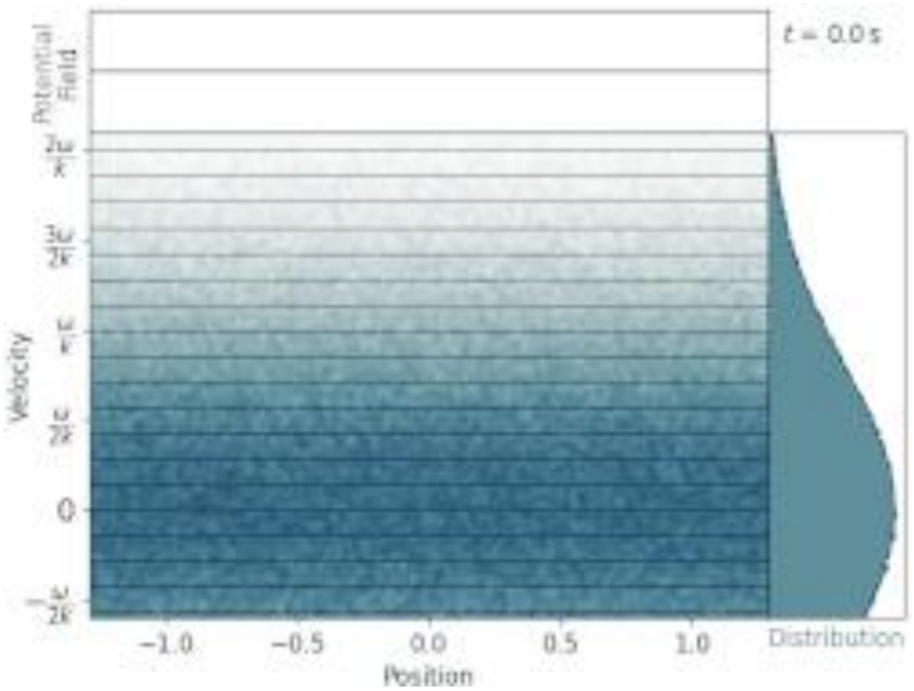
$$\frac{d^2\tau}{dt^2} + \frac{D}{T_0}\frac{d\tau}{dt} + q\frac{\eta_p \dot{V}_0}{E_0 T_0}\tau = 0$$ The longitudinal motion

It is a damped oscillation equation, the solution is $\tau = A_0 e^{-\alpha_\epsilon t}\cos(\Omega t - \theta_0)$

$$\tau_\epsilon = 1/\alpha_\epsilon = \frac{2T_0}{D}$$

The damping effect in the longitudinal plane arises from the properties of synchrotron radiation: particles with higher energy lose more energy, while those with lower energy lose less. This results in $D > 0$.

3. Landau damping：



Landau damping, named after its discoverer, Soviet physicist Lev Davidovich Landau (1908–68), is the effect of damping (exponential decrease as a function of time) of longitudinal space charge waves in plasma or a similar environment.

In Wave–particle interactions

In the wave's frame of reference. Particles near the phase velocity become trapped and are forced to move with the wavefronts, at the phase velocity. Any such particles that were initially below the phase velocity have thus been accelerated, while any particles that were initially above the phase velocity have been decelerated. Because, for a Maxwellian plasma, there are initially more particles below the phase velocity than above it, the plasma has net gained energy, and the wave has therefore lost energy.

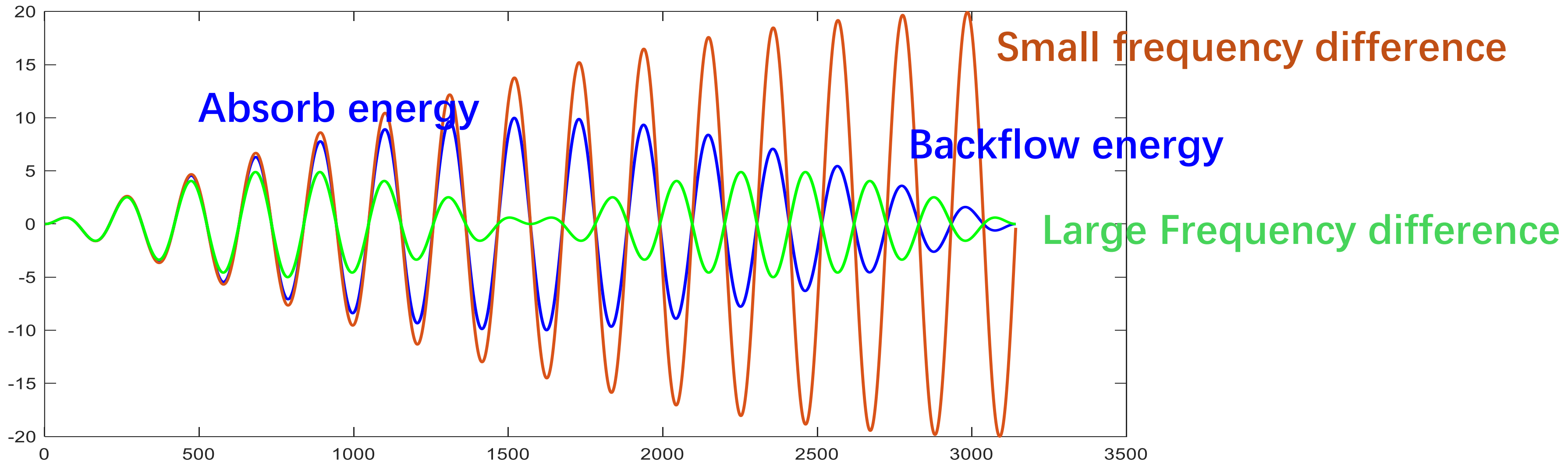
This provide a damping mechanism of the wave passing through the plasma.

3. Landau damping:

Wake field – beam interaction

We borrow this concept for accelerators: when a wakefield interacts with a beam, only particles whose oscillation frequency is close to the wakefield frequency can effectively interact with it. If a particle's oscillation frequency is far from the wakefield frequency, the energy exchange is and will soon resulting in backflow.

A fraction of the beam absorbs the wakefield while the beam as a whole will remain stable. Unlike the previous case, the energy of the wakefield originates from the beam itself. This means that the energy used to excite the wakefield by the entire beam will be absorbed by a fraction of the particles, effectively creating a damping mechanism."



3. Landau damping:

$$\ddot{x} + \omega^2 x = Acos(\Omega t)$$

$$x(t) = -\frac{A}{(\Omega^2-\omega^2)}[\cos(\Omega t) - \cos(\omega t)]$$

$$\langle x(t)\rangle = \int_{-\infty}^{\infty} x(t)\rho(\omega)d\omega$$

$$\langle x(t)\rangle \cong \frac{A}{2\omega_0}[\pi\rho(\Omega)\sin(\Omega t) + \cos(\Omega t)\,P.V.\int_{-\infty}^{\infty}\frac{\rho(\omega)}{\omega-\Omega}d\omega]$$

$$\mu = \frac{\omega_0 - \Omega}{\Delta\omega} \qquad f(\mu) = \Delta\omega P.V.\int_{-\infty}^{\infty}\frac{\rho(\omega)}{\omega-\Omega}d\omega \qquad g(\mu) = \Delta\omega\pi\rho(\Omega)$$

$$\langle x(t)\rangle = \frac{A}{2\omega_0\Delta\omega}e^{-i\Omega t}[f(\mu) + i\cdot g(\mu)] \qquad (13)$$

With tune shift (real and imaginary) caused by impedance, substitute them into Eq.(13) there will be a stable regime which is larger than that without Landau damping.

In practical, the tune spread provide damping rate $\alpha_{Landau} = \frac{1}{\sqrt{3}}\Delta\omega_{1/2}T_0$

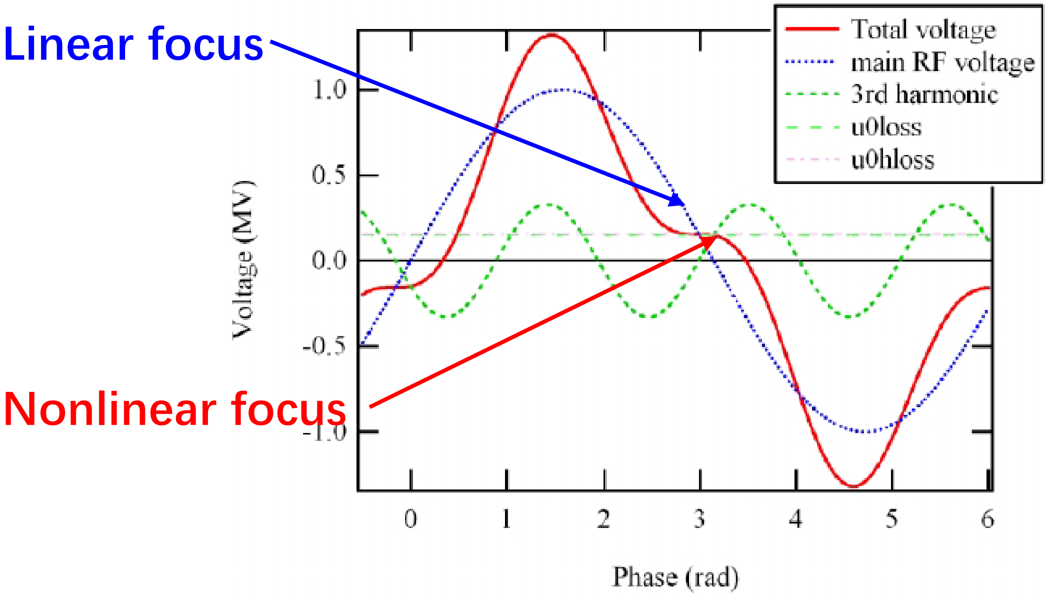
Where $\Delta\omega_{1/2}$ is the tune spread of Half-width-at-half-height

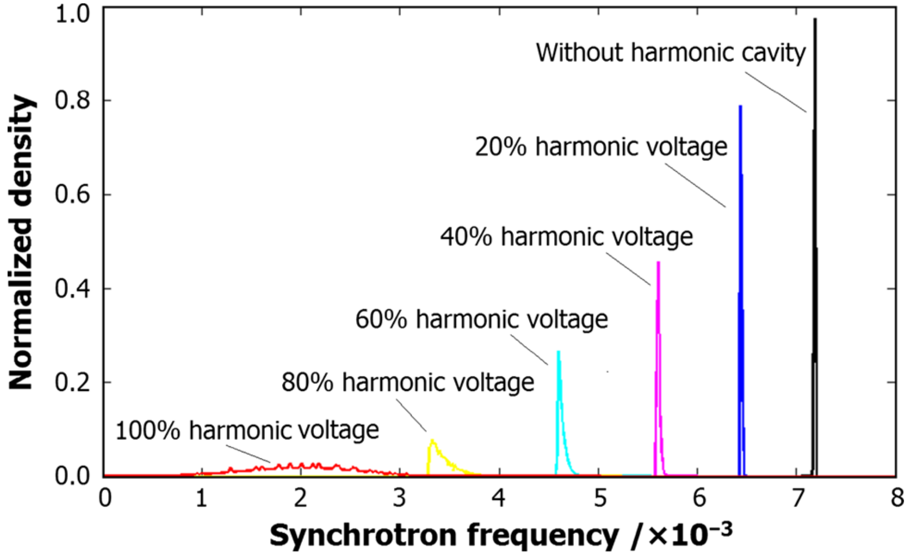
3. Landau damping：

Larger the tune spread, stronger the Landau damping.

Transverse tune spread:
- Octupoles (Amplitude dependent tune shift, should compromise with DA optimization).
- AC quadrupole/RF quadrupole.

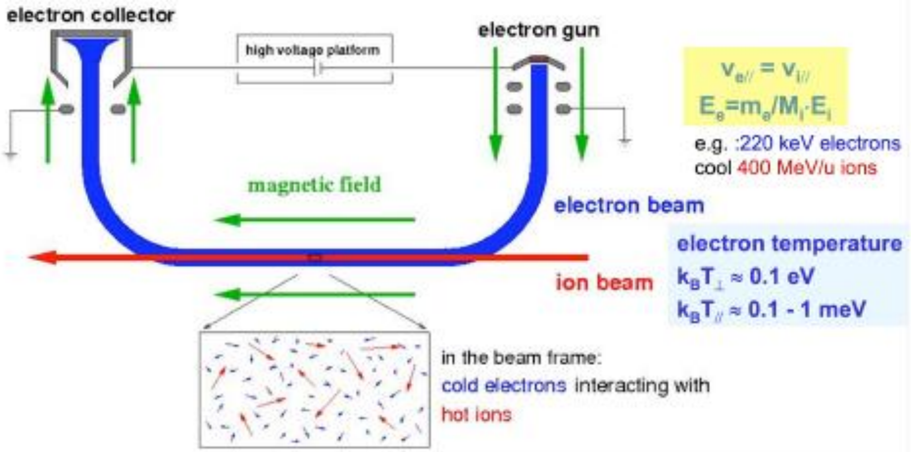
Longitudinal tune spread: High harmonic cavity （Landau cavity）



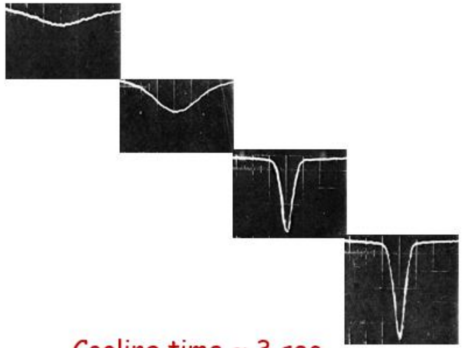


Damping

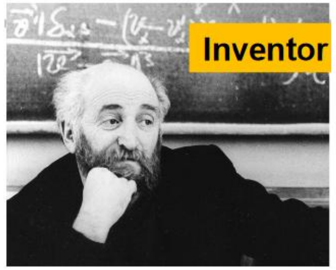
Electron cooling for proton or ion beams



Electron cooling was first tested in 1974 with 68 MeV protons at NAP-M storage ring at INP(Novosibirsk).



Cooling time ~ 3 sec



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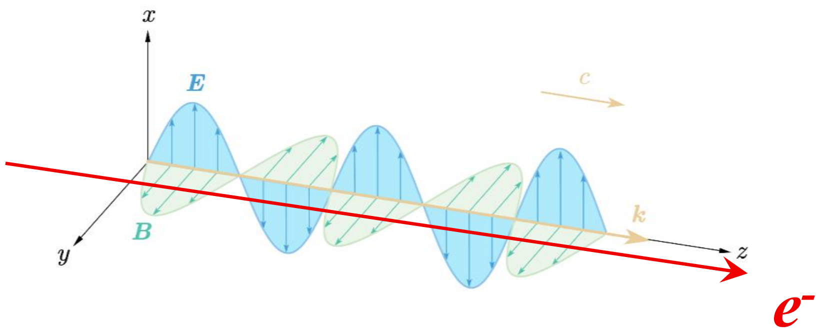
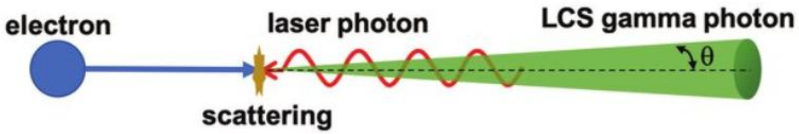
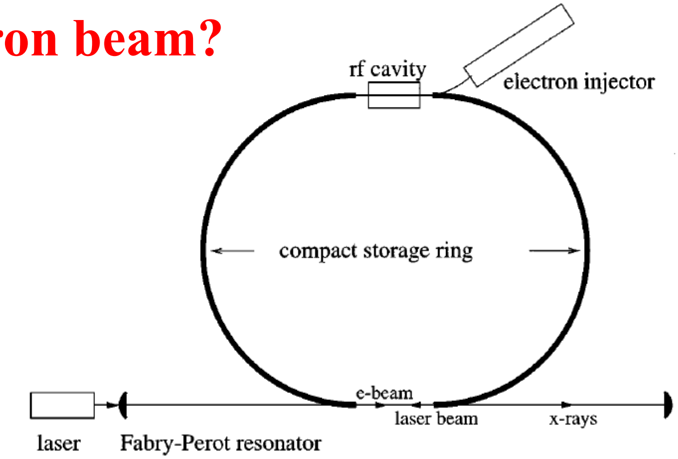
Electron cooling for proton or ion beams is so successful, that ···

Besides synchrotron radiation, is there any other mechanism to cool electron beam?



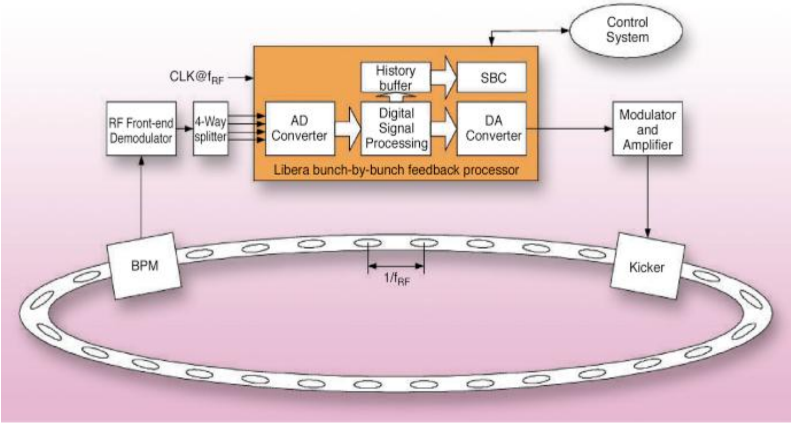




$$\sigma_{th} = 6.65 \times 10^{-29} m^2 = 0.665\ barn$$

PHYSICAL REV I EW LETTERS, 1998, Zhirong Huang and Ronald D. Ruth

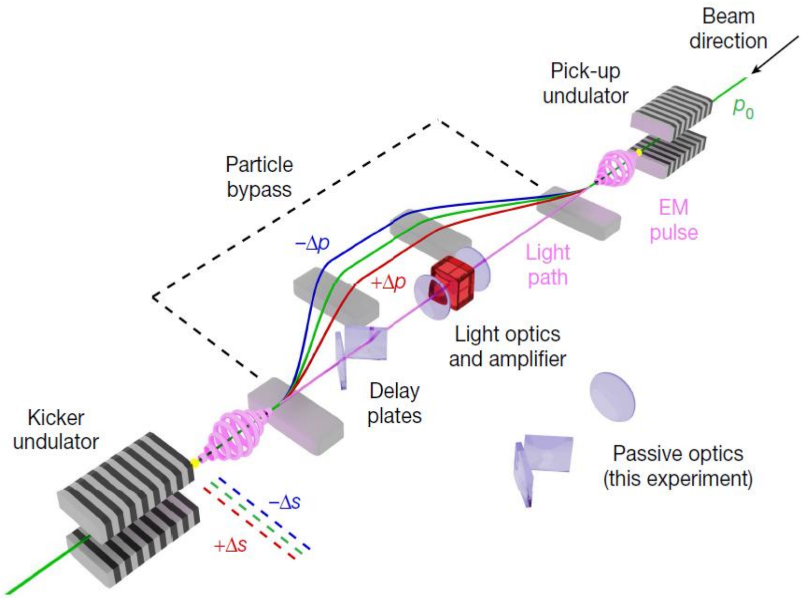
Some other damping mechanism



Feedback system

Longitudinal and transverse feedback are engineering techniques rather than physics, yet they play a crucial role in combating beam instabilities.

Longitudinal oscillations are relatively slow, so only a single BPM (pickup) is needed to calculate the required kick.
Transverse feedback was traditionally implemented as a mode-by-mode system. With advances in electronics, bunch-by-bunch feedback is now feasible. This allows the entire feedback process to be completed within one turn. As a result, two BPMs are required to determine the necessary kick — both its amplitude and phase.



Stochastic cooling

Transverse cooling is achieved by sensing the particle displacements in the pickup and applying a correcting signal at the kicker. Normally, the pickup and kicker are placed 90◦ apart in betatron phase so that a position displacement at the pickup will become an angular displacement at the kicker.

J. Jarvis et al., Nature | Vol 608 | 11 August 2022 | 287

Summary

- ✓ Coupled bunch instability: The spectrum of the impedance and the beam overlap? If **YES**, which side bands feels larger impedance?
- ✓ Single bunch instability: Beam evolution in phase space. Head-tail prevents instability growth, Otherwise BBU will be very powerful.
- ✓ Scattering: Cross section? Acceptance?
- ✓ Damping: The unsung hero, should not be neglected , the gifts of the Mather Nature.

To the end, we are so proud that most of the collective effects have been solved or understood.

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Thanks for listening!

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