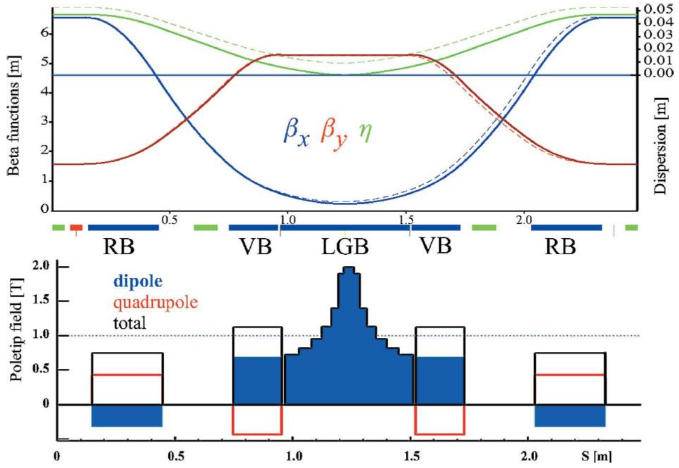
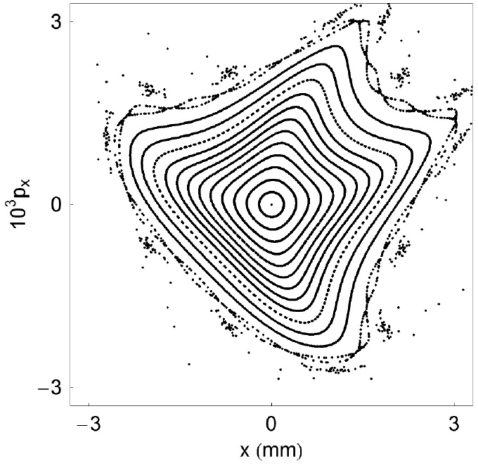
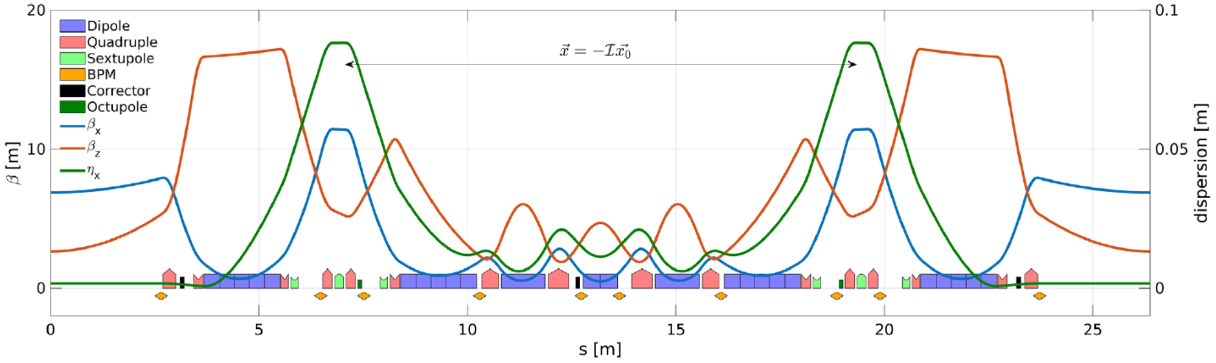
12th International Organization of Chinese Physicists and Astronomers Accelerator School
OCPA-2025
NAKHON RATCHASIMA, THAILAND 29 JULY - 7 AUGUST 2025







Lattice design

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National Synchrotron Radiation Laboratory

Nakhon Ratchasima, Thailand
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Lattice design is the art and science of arranging various magnets, including dipoles, quadrupoles, sextupoles and octupoles, into an accelerator, while carefully considering beam dynamics and performance specifications.

This course focuses on the lattice design of synchrotron light source storage rings, while much of the content is also applicable to lattice design for other types of accelerators.

Content

- Increasing radiation brightness and reducing electron beam emittance
- Dynamic aperture & momentum aperture
- Lattices: TME, LGB/RB cell, DBA, TBA, MBA
- Nonlinear cancellation: -I transformation & higher-order-achromat
- MBA lattices: conventional MBA & hybrid MBA
- Interplay of storage ring parameters
- Nonlinear analysis: frequency map analysis & resonance driving terms
- Optics design and nonlinear optimization algorithms: GLASS, MOGA, etc.
- Nonlinear optimization: analytical optimization & numerical optimization
- Lattice design of ultra-low-emittance synchrotron storage rings
- Summary of low-emittance lattice design philosophy

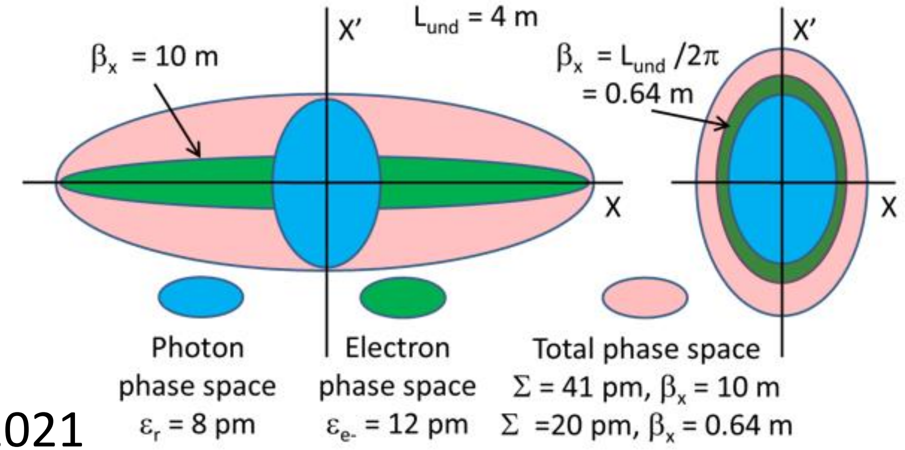
Increasing undulator radiation brightness

- Development of synchrotron light sources: toward higher radiation brightness
- Undulaltor brightness: the spectral flux per unit volume in transverse phase space
  - $B(\lambda) = \frac{Number\ of\ photons}{6D\ phase\ space\ volume} = \frac{photons/time}{(2D\ area)_x (2D\ area)_y (spectral\ bandwidth)} = \frac{F(\lambda)}{4\pi^2 \sigma_{Tx}\sigma_{Tx'}\sigma_{Ty}\sigma_{Ty'}}$

    with the flux $F(\lambda) \propto N_u I$ (N_u is the number of undulator periods, I is the beam current)
  - The total transverse phase space area: a convolution of the electron beam and photon phase spaces
- Increasing undulator brightness: reducing the total transverse phase space area
  - Reducing the electron beam emittance, which characterizes the phase space area occupied by the beam
  - Matching the beta functions in undulators, which characterize the partitioning of the emittance between size and divergence at the undulator position

Increasing undulator radiation brightness

- Reducing electron beam emittance
  - Emittance is determined by the balance between radiation damping and quantum excitation
    - Emittance reduction: reducing quantum excitation & enhancing radiation damping
  - When the electron beam emittance is reduced toward and beyond the photon emittance, maximum brightness can be achieved
    - Due to the wave nature, the photon has an intrinsic emittance (called diffraction-limited emittance), $\lambda/4\pi$ (λ is the radiation wavelength).
    - Fourth-generation synchrotron light sources, also called diffraction-limited storage rings (DLSRs), are being developed toward diffraction-limited emittances (typically 10-100 pm·rad or even lower).
- Matching beta functions in undulators
  - Phase space matching between electron and photon beams
  - Reducing beta functions of straight sections: optimal beta function $L_u/2\pi$ (L_u is the length of undulator)



C. Steier, MT Days, 2021

Reducing electron beam emittance

Emittance scaling law

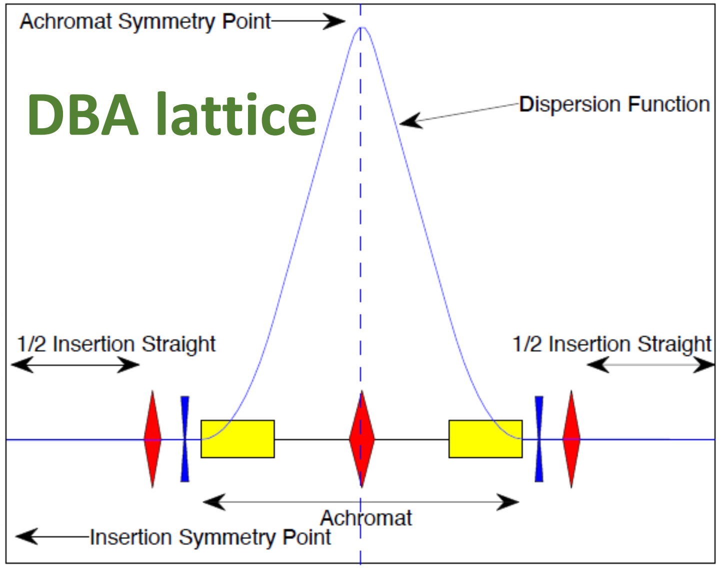
$$\varepsilon_x = 1.47 \times 10^{-9}(E[\text{GeV}])^2 \frac{I_5}{J_x I_2} \propto F(\text{lattice}) \frac{E^2}{{N_b}^3}$$

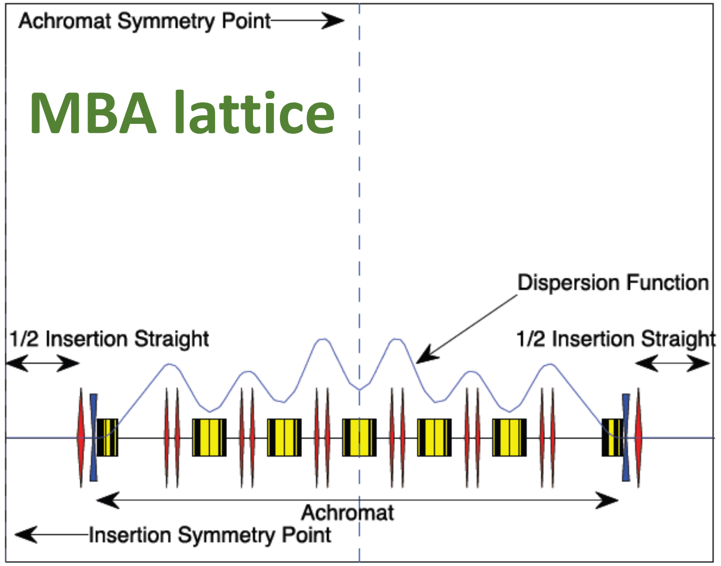
where E is the beam energy, I_5 and I_2 are radiation integrals (I_5 for quantum excitation, I_2 for radiation damping), J_x is the horizontal damping partition number, F(lattice) is a factor dependent on the lattice, N_b is the number of bending magnets (bends) in the storage ring.

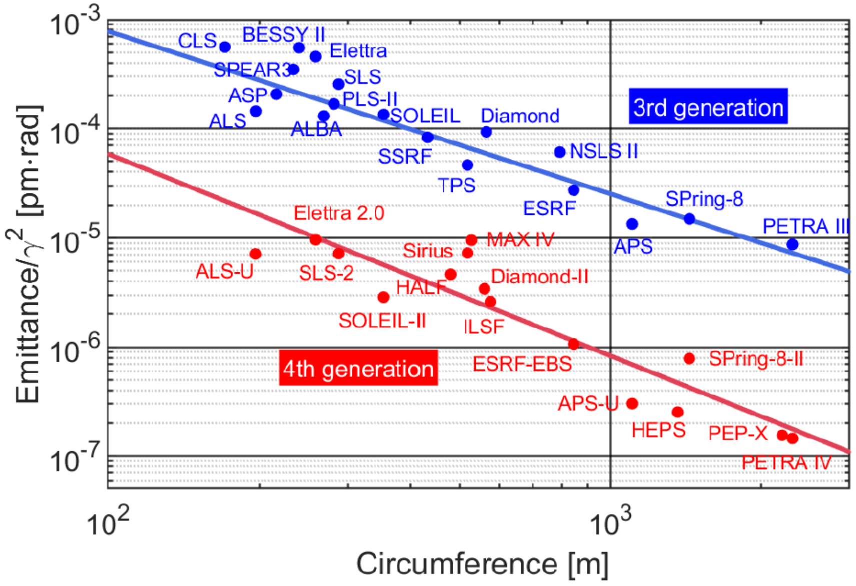
- Reducing *F*(lattice): enhancing the emittance reduction ability of bend unit cell
  - Use of combined-function bend (CB), longitudinal gradient bend (LGB) and reverse bend (RB); increasing the horizontal tune of unit cell
  - Development of bend unit cell: from conventional bend unit cell to LGB/RB unit cell
- Reducing beam energy (generally not used)
  - More difficult to produce hard X-ray (energy mainly determined by users)
  - More severe intra-beam scattering effect (significant emittance increase for ultra-low emittance storage rings)

Reducing electron beam emittance

- Increasing the number of bends: the most effective way for emittance reduction
  - Reduced dispersion in bends: better suppressing the quantum excitation
  - Fourth-generation synchrotron light sources: using MBA lattice instead of DBA lattice (MBA: multi-bend achromat, DBA: double-bend achromat)



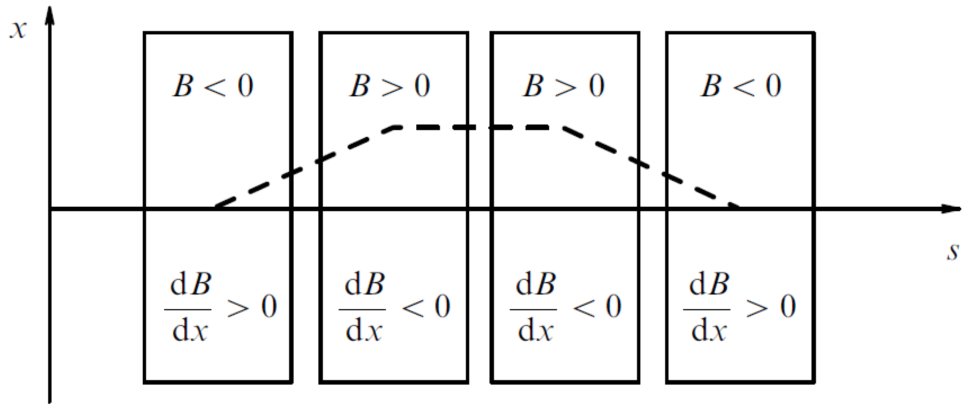




C. Steier, MT Days, 2021

Reducing electron beam emittance

- Other means
  - Installing damping wiggler in dispersion-free straight section: enhancing radiation damping for emittance reduction, also reducing damping times for suppressing the intra-beam scattering effect
    - Emittance reduction with damping wigglers: $\frac{\varepsilon_w}{\varepsilon_0} \approx \frac{U_0}{U_0+U_w}$, where U_0 is the energy loss per turn in bends, and U_w is the energy loss in damping wigglers
    - Advantage: reducing emittance and damping times without reducing momentum compaction factor
  - Installing Robinson wiggler in non-zero dispersion straight section: a sequence of magnets in which the field and the gradient change to keep their product negative
    - Reducing emittance by increasing J_x



Accepting particles with deviations in amplitudes and momentum

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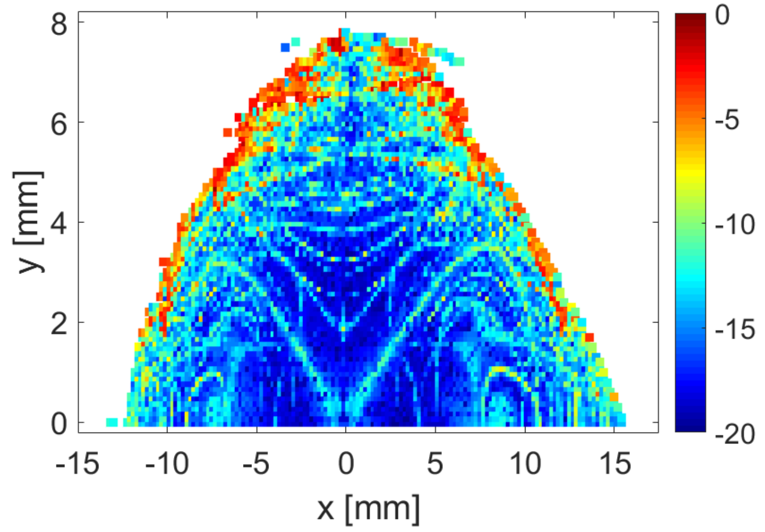
- Low beam emittance (small beam size) needs small storage ring acceptance?
  - In a storage ring, considering beam injection and scattering, the storage ring dynamics needs to “accept” particles with deviations in both transverse amplitudes (x, y) and momentum (δ), which are much larger than the beam transverse sizes and energy spread!
- The role of sextupoles in storage ring acceptance
  - The positive role of sextupoles: correcting chromaticities to suppress the head-tail instability and reduce the tune variation with momentum, thereby preventing off-momentum particles from crossing dangerous resonances and consequently increasing momentum acceptance.
  - The negative role of sextupoles: introducing nonlinear dynamics that usually limits both maximum transverse amplitudes and maximum momentum deviation.
  - Therefore, we need to optimize the nonlinear dynamics introduced by sextupoles, which are required for chromaticity correction.

Dynamic aperture

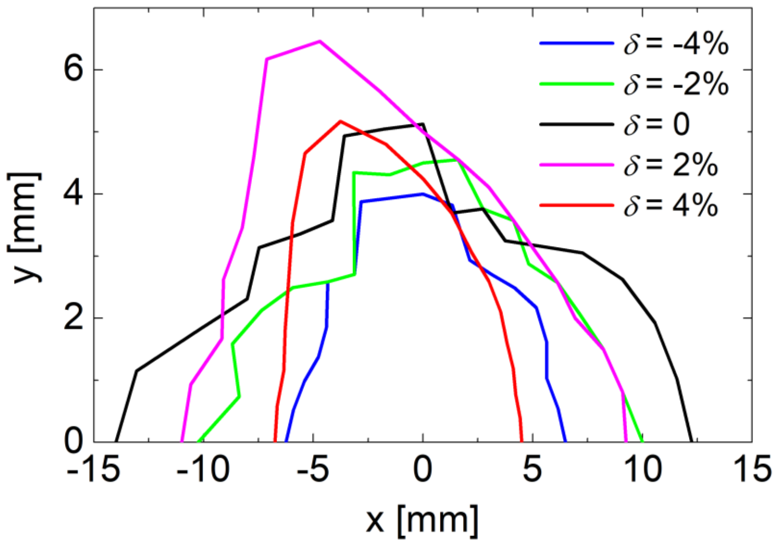
- Dynamic aperture (DA): maximum transverse amplitudes that particle can survive
  - Affecting injection efficiency and injection scheme: larger DA allows for higher injection efficiency and off-axis injection
  - The "quantity" of DA: characterized by DA size in (x, y)
  - The "quality" of DA: characterized by diffusion rates that are calculated using frequency map analysis (FMA)
    - Lower diffusion rate denoting more stable particle motion
    - DA with lower diffusion rates having larger error tolerance (i.e., more robust DA)
- Increasing DA
  - Reducing the strengths of sextupoles (placing sextupoles at high-dispersion locations) + nonlinear cancellation between sextupoles (properly choosing phase advances between sextupoles): minimizing nonlinear driving terms
  - Introducing harmonic sextupoles (located at dispersion-free positions, no contribution to chromaticity correction) & octupoles: control of resonance driving terms (RDTs) and amplitude dependent tune shifts (ADTSs)

Dynamic aperture

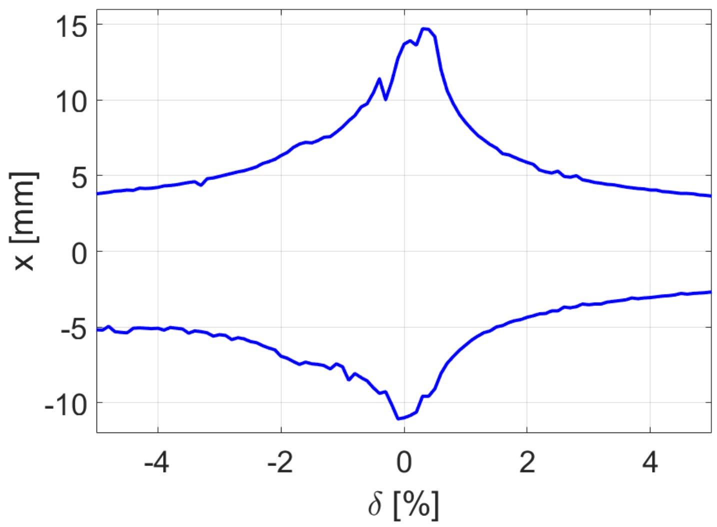
- Tracking DA (for on-momentum particle with δ = 0): starting from initial coordinates (x, x', y, y', z, δ) = (x, 0, y, 0, 0, 0) to search for maximum (x, y)
- DAs of on- and off-momentum particles
  - On-momentum DA: usually referred to simply as DA, affecting beam injection
  - Off-momentum DAs: affecting momentum aperture



(On-momentum) DA with blue color denoting lower diffusion rates



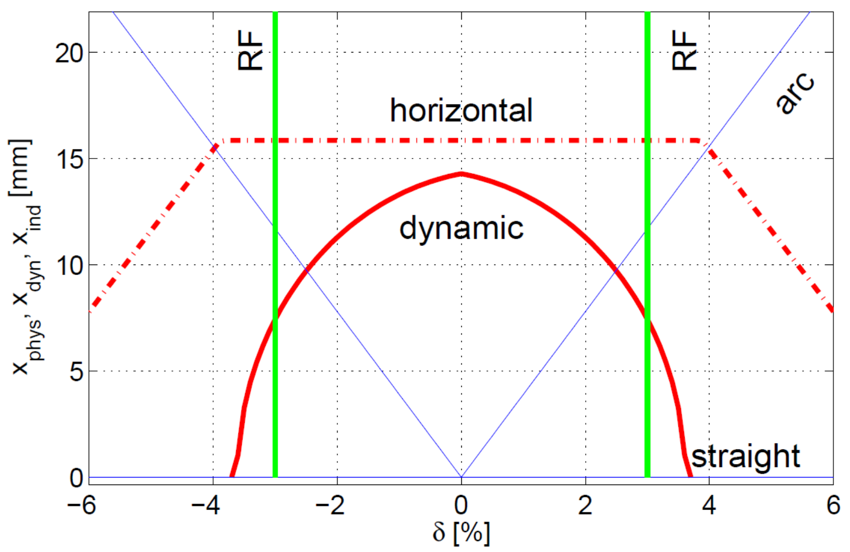
On- and off-momentum (0, ±2%, ±4%) DAs



Off-momentum horizontal DAs

Momentum aperture

- Momentum aperture (MA): maximum momentum deviation that particle can survive
  - Larger MA needed for longer Touschek lifetime
  - Longitudinal MA: the height of RF bucket, also called RF MA
  - Transverse MA
    - Limited by physical aperture (vacuum chamber)
    - Limited by (off-momentum) dynamic aperture: called dynamic MA, usually the main factor limiting MA
  - MA: minimum of longitudinal and transverse MAs

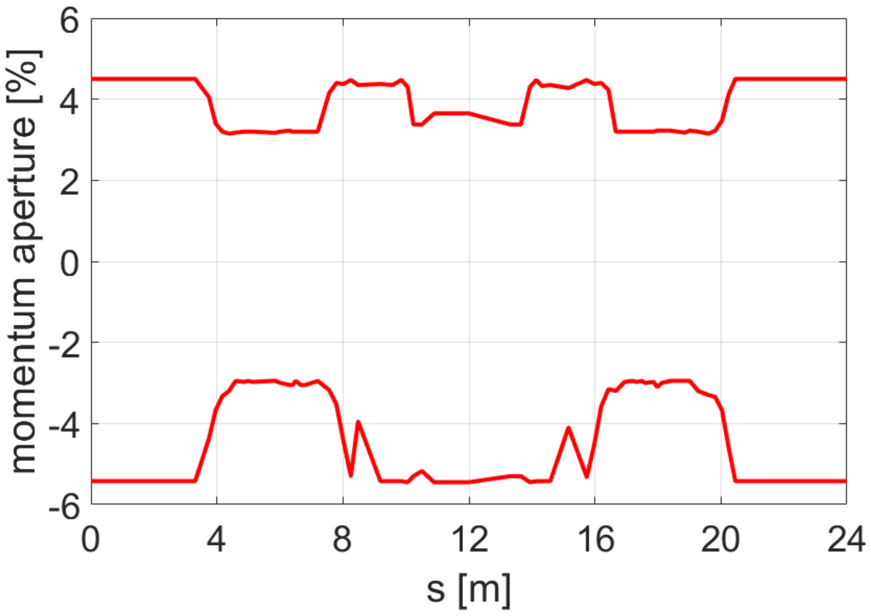


C. Steier et al., PAC2005

- Increasing MA
  - Increasing off-momentum DAs and control of dispersion: lattices with distributed chromatic correction can have large MAs, which are usually adopted by low-energy rings
  - Controlling tune shifts with momentum (mainly higher-order chromaticities): octupoles can be employed to control second-order chromaticities

Momentum aperture

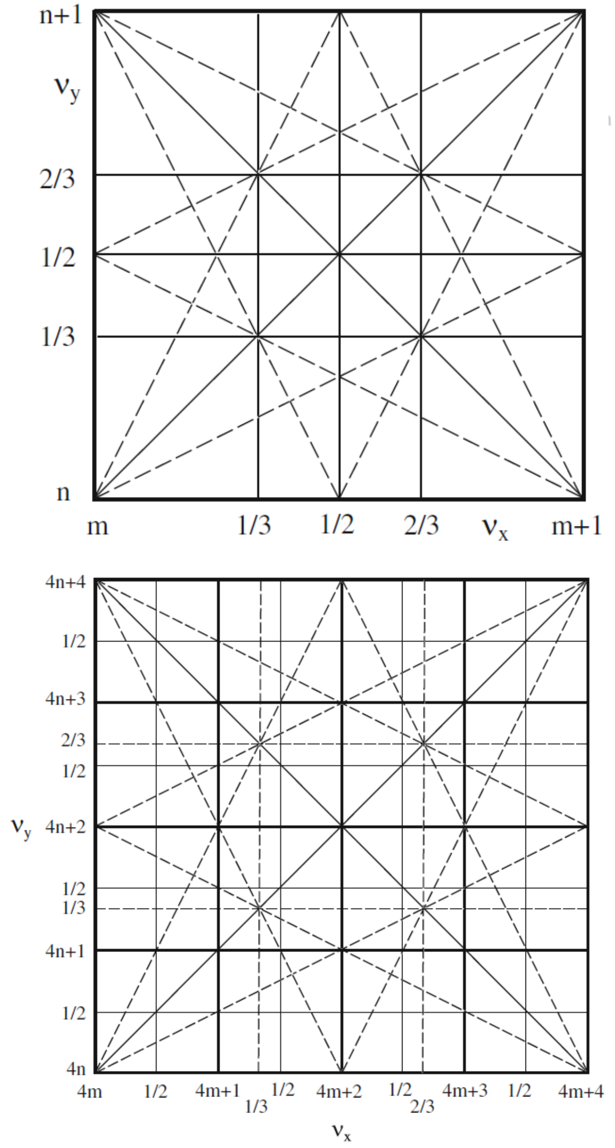
- Tracking MA: at a longitudinal position *s*, starting from initial coordinates (x, x', y, y', z, δ) = (0, 0, 0, 0, 0, δ) to search for maximum ±δ, and then repeating this process at different longitudinal positions to obtain the MA along the longitudinal position
  - MA changes along the longitudinal position: MA also referred to as local momentum aperture (LMA)
  - Physical picture of tracking: when a particle in the beam suddenly gains or loses energy due to scattering, with other coordinates unchanged, and if its momentum deviation exceeds the local MA at the scattering location, the particle will be lost.



LMA along a lattice cell of the HALF storage ring

Resonance lines & periodicity

- The storage ring of a synchrotron light source is usually composed of some identical super-periods. This high periodicity or symmetry significantly reduces the density of resonance lines, which is beneficial for improving nonlinear dynamics performance.
- For a storage ring with long straight sections for high-β_x injection and other specialized applications, maintaining lattice symmetry is crucial for ensuring optimal nonlinear performance. This is achieved by designing the transfer maps between sextupoles across straight sections to be identical, with the same phase advances or additional phase advances of 2π.

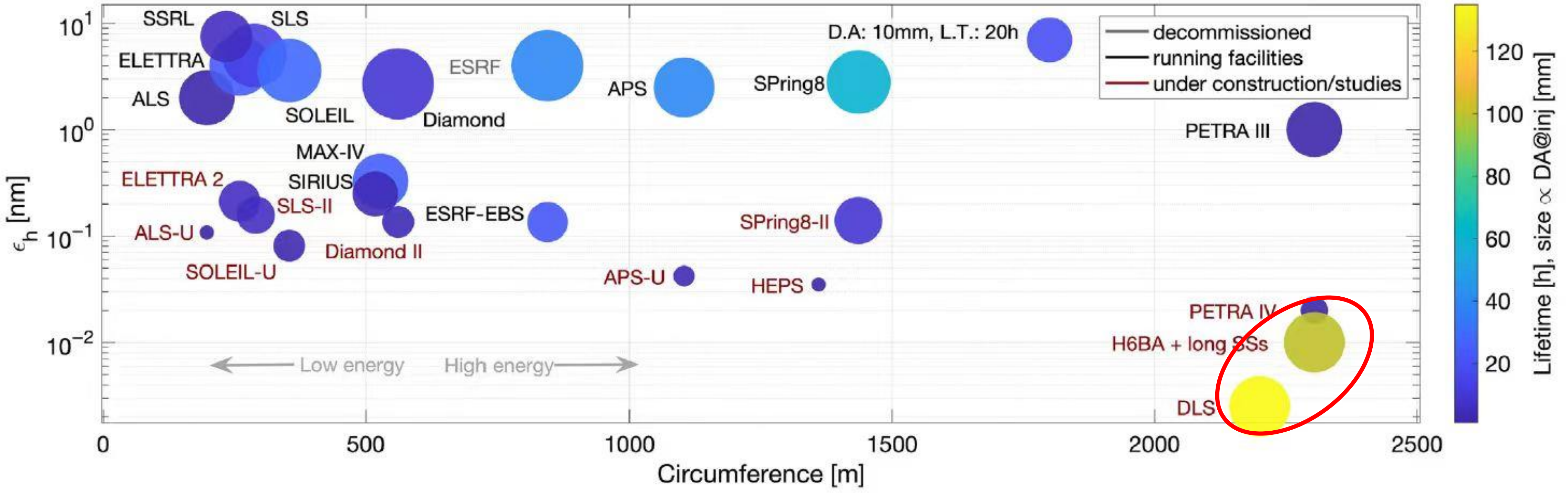


Resonance diagrams for a ring with 1 super-period (upper) 4 super-periods (lower)

H. Wiedemann, Particle Accelerator Physics

Emittance, dynamic aperture & Touschek lifetime

- Lower emittance ➡ lower dispersion ➡ stronger sextupoles ➡ smaller DA & MA
- Generally, DA and Touschek lifetime decrease with reduced emittance. But highly effective nonlinear cancellation can reverse this trend.

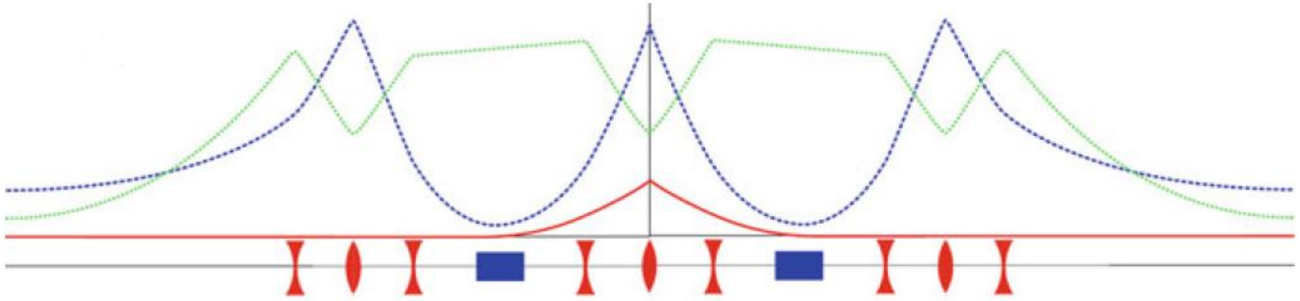


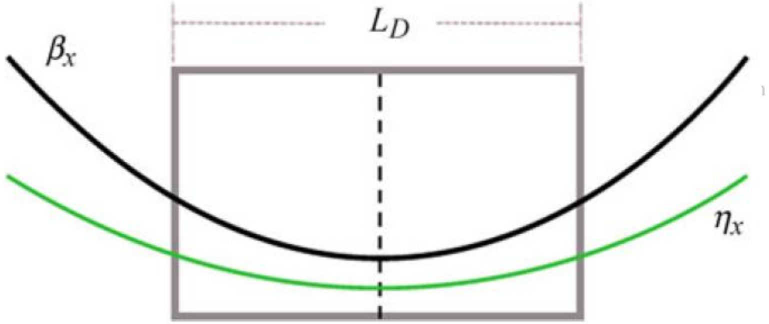
The size of circle is representative of DA and the color is proportional to Touschek lifetime.

P. Raimondi & S. M. Liuzzo, PRAB, 2023

TME & DBA lattices

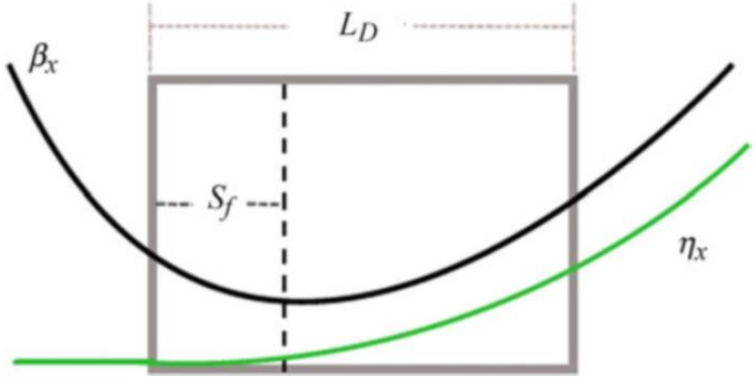
- TME (theoretical minimum emittance) cell with a homogeneous bend
  - Considering practical limitations, TME-like cells are used with a factor of 3~6 larger emittance.
- TME end cell with a homogeneous bend: zero-dispersion on one end
- DBA lattice with two TME end cells
  - The same minimum emittance as the TME end cell, three times larger than that of the TME cell
  - Achromat: zero-dispersion in the straight section preferred for insertion devices, especially wigglers





$$\varepsilon_{TME} = \frac{C_q \gamma^2 \theta^3}{12\sqrt{15} J_x}$$

C_q = 3.832×10^{-13} m, γ is the Lorenz factor of electron, θ is the angle of bend



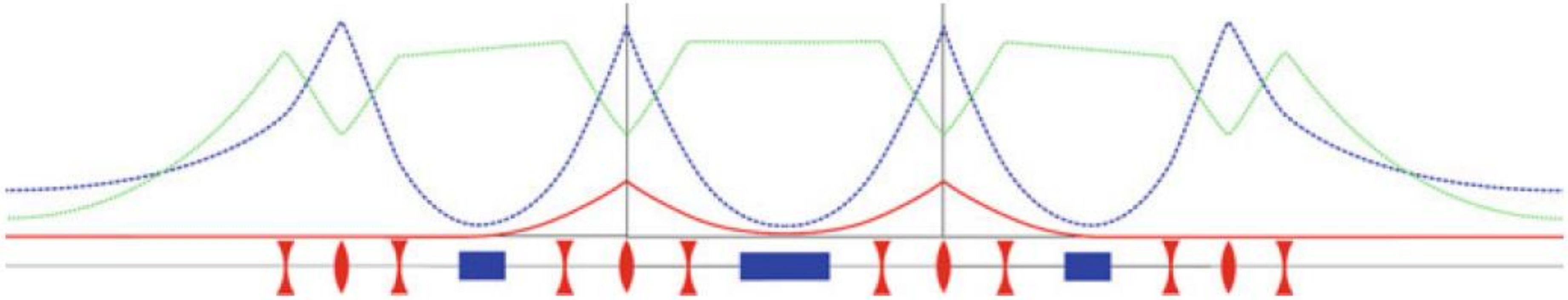
$$\varepsilon_{MEDBA} = \frac{C_q \gamma^2 \theta^3}{4\sqrt{15} J_x}$$

R. Bartolini, 2016

TBA & MBA lattices

- TBA (triple-bend achromat) lattice with one TME cell and two TME end cells
  - Due to the matching condition, the minimum emittance of TBA lattice is not simply the average of one TME cell and two TME end cells, but is given by:

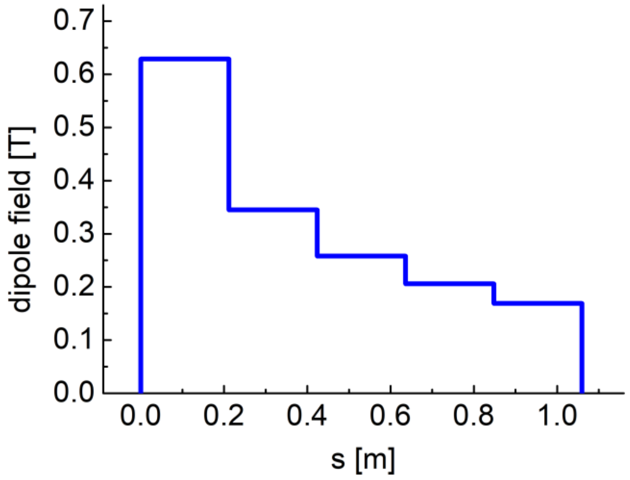
$$\varepsilon_{METBA} \approx 0.66\, \varepsilon_{MEDBA}$$

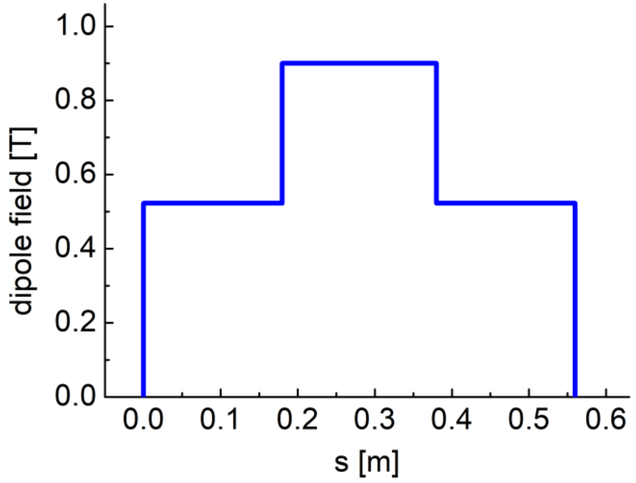


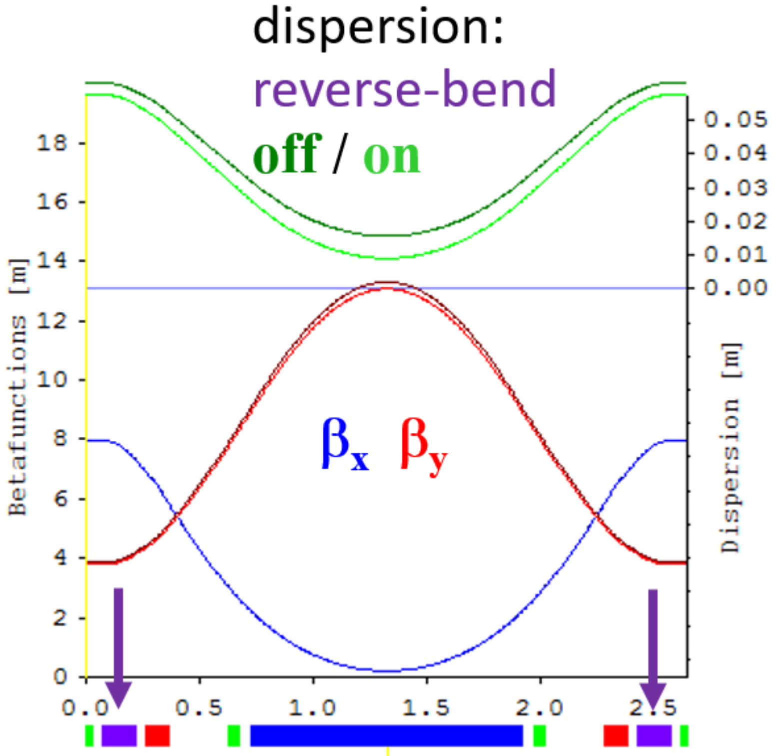
- MBA lattice with ≥ 2 TME cells and two TME end cells
  - Further extending TBA lattice by allowing more than one TME cell
  - Originally proposed in the early 1990s, and used in the design of diffraction-limited storage rings

Longitudinal gradient bend & reverse bend

- Longitudinal gradient bends (LGBs) and reverse bends (RBs) are used in the diffraction-limited storage ring lattice design to reduce the emittance.
- LGB: stronger radiation (higher field) in lower dispersion regions where the quantum excitation is better suppressed







- RB (also called anti-bend): used to disentangle β_x and dispersion so as to reduce the dispersion in the main bend (the focusing of dispersion in a conventional cell is insufficient)

A. Streun, 2014

TME-like cell: LGB/RB unit cell

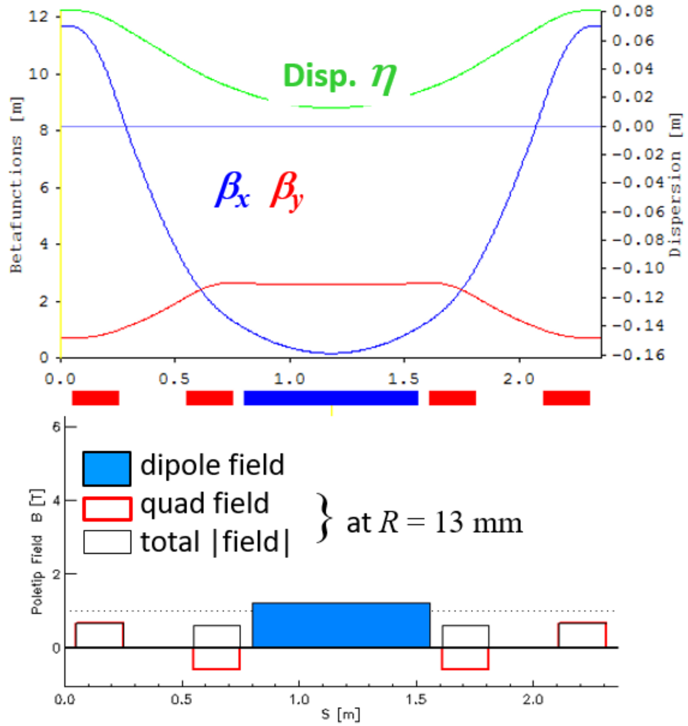
- LGB/RB unit cell: compared to the conventional TME-like cell, LGB/RB cell can achieve even lower emittance than TME.

$$\varepsilon_x = 1.47 \times 10^{-9}(E[\mathrm{GeV}])^2 \frac{I_5}{J_x I_2}$$

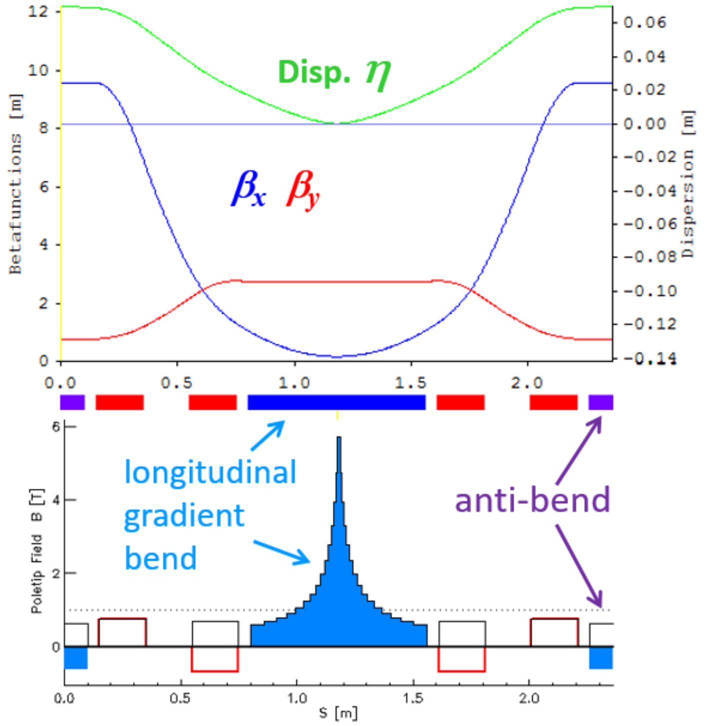
I_5: quantum excitation

I_2: radiation damping

- LGB in the LGB/RB cell
  - Reducing I_5 : higher field at lower dispersion
  - Increasing I_2: more radiation loss than a homogeneous bend
- RB in the LGB/RB cell
  - Reducing I_5: decoupling β_x and dispersion to reduce the dispersion in the main bend
  - Increasing I_2: increasing the total absolute bending angle
  - Increasing J_x : combined with quadrupole



Conventional TME-like cell



LGB/RB unit cell

A. Streun, 2014

Modifying DBA lattice for emittance reduction

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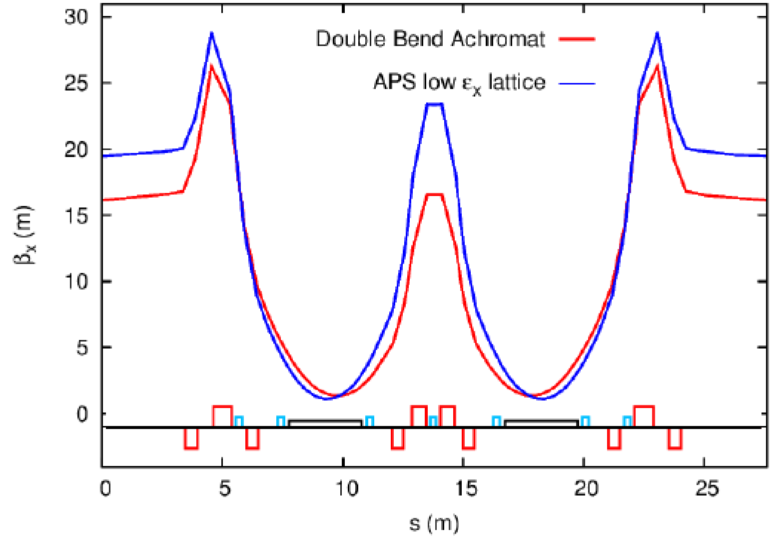
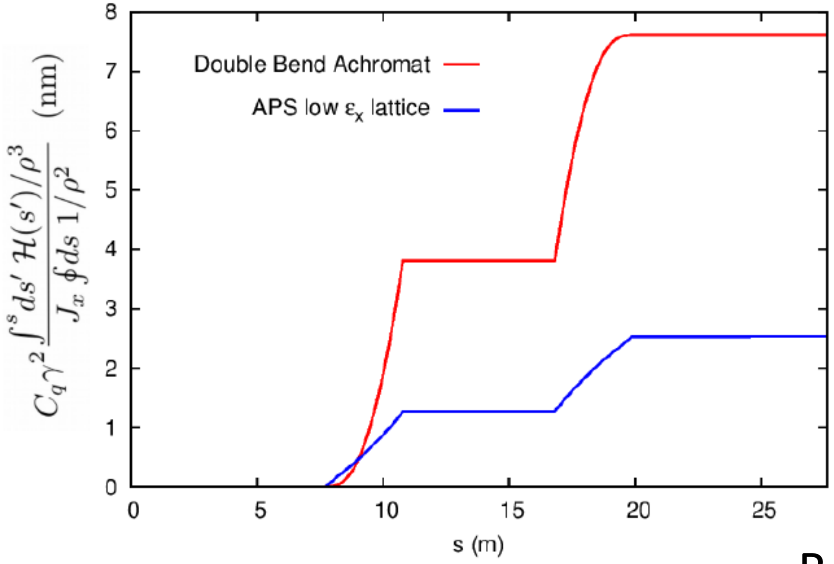
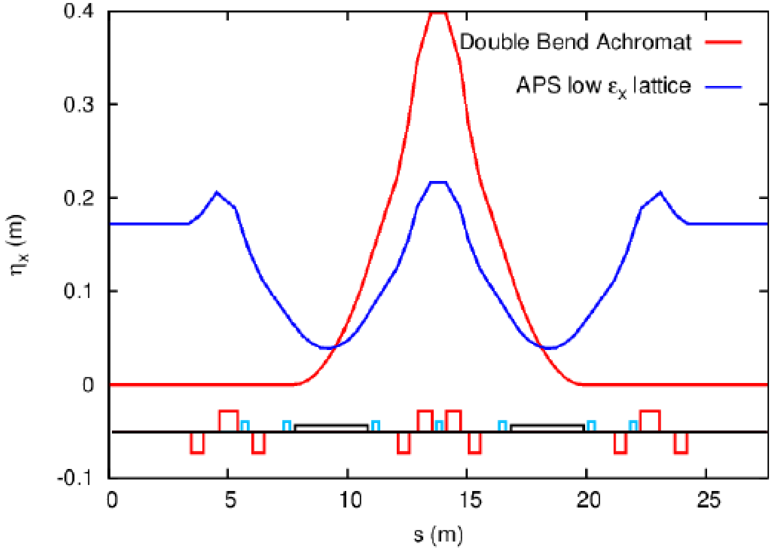
- Allowing some dispersion in the straight section of a DBA lattice to reduce the emittance
  - Zero-dispersion: relatively large emittance
  - High dispersion: the dipole fields of undulators and wigglers can significantly contribute to quantum excitation, thereby increasing the emittance
  - Allowing some dispersion: net benefit for emittance reduction





Equilibrium $\varepsilon_{x,0}$ = 7.6 nm

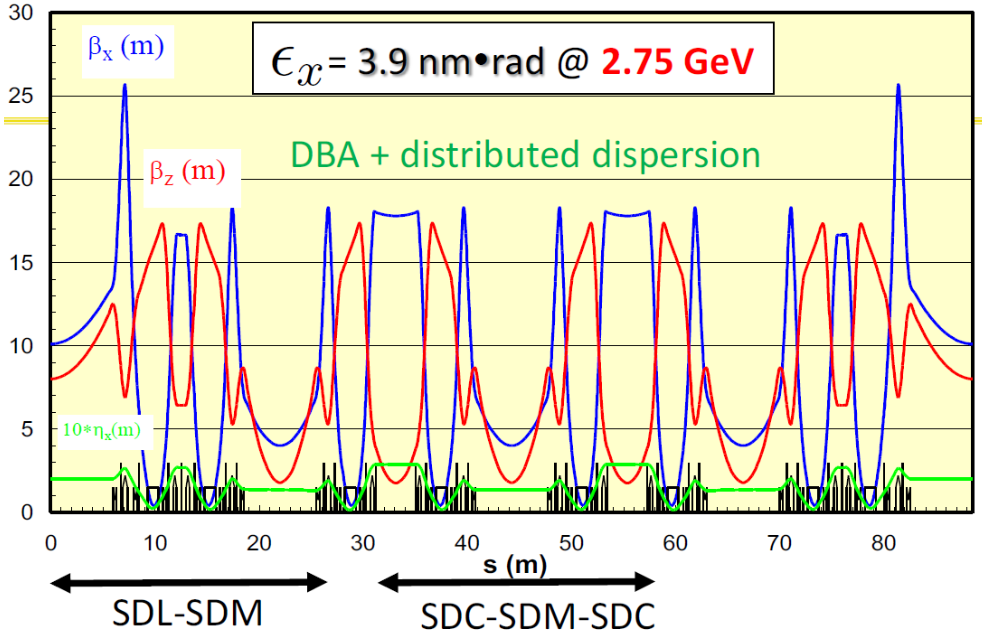
Equilibrium $\varepsilon_{x,0}$ = 2.5 nm
Effective $\varepsilon_{x,\mathrm{eff}}$ = 3.2 nm



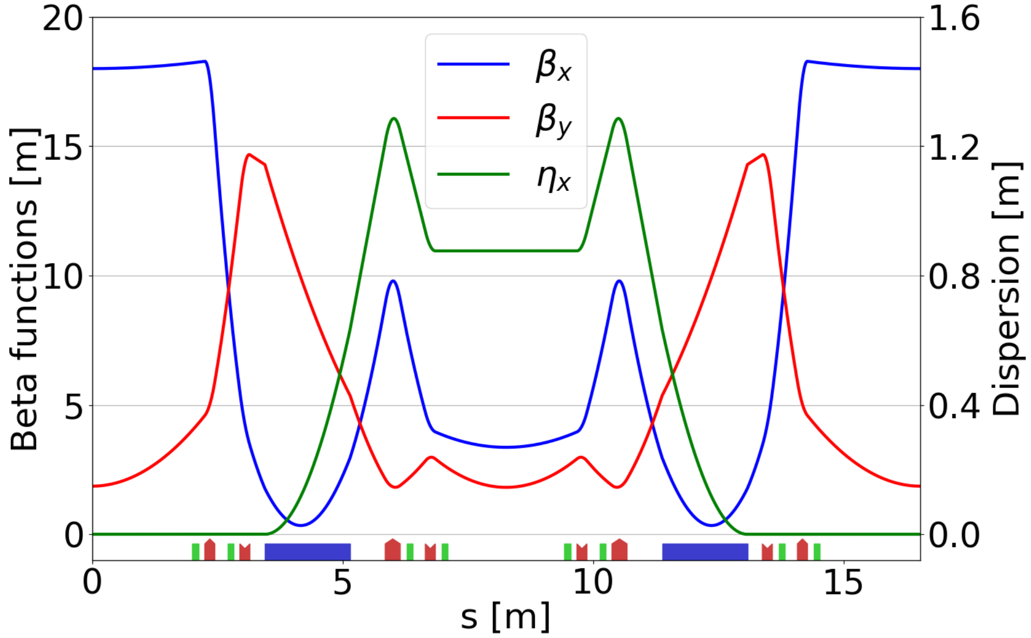
R. Lindberg, NAPAC2019

Modifying DBA lattice for more ID straight sections

- SOLEIL lattice: introducing a (high dispersion) mid-straight inside the DBA
  - 24 straight sections in the ring with a circumference of 354 m
  - The ratio of the total length of straight sections to the ring circumference is ~45%!
- HLS-II lattice: each lattice cell having long and short straight sections
  - 8 straight sections in the 66-m ring, with 6 for insertion devices (IDs)



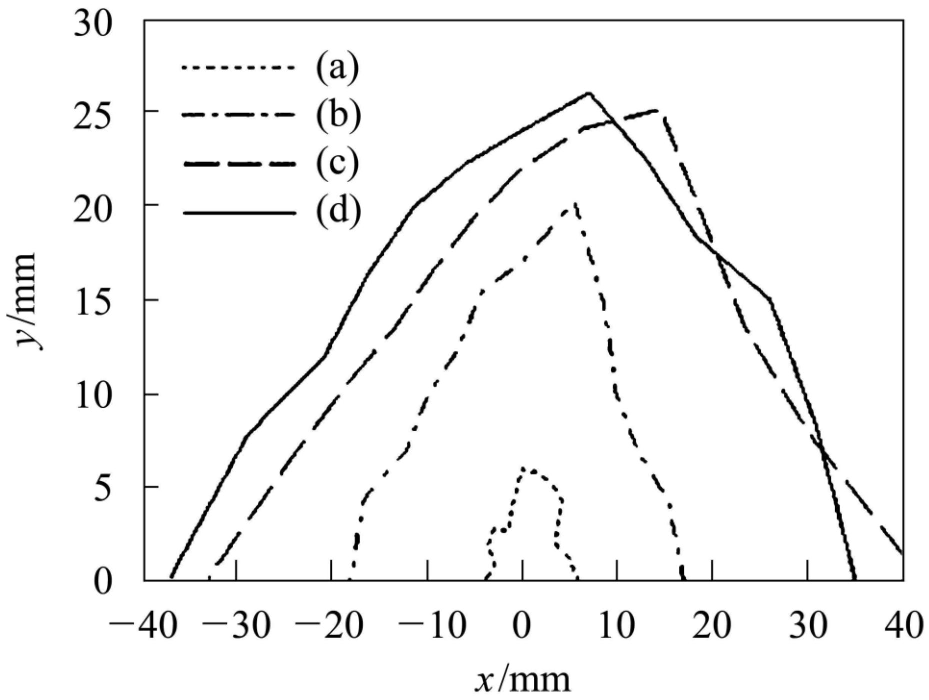
L. S. Nadolski, ESLS2020

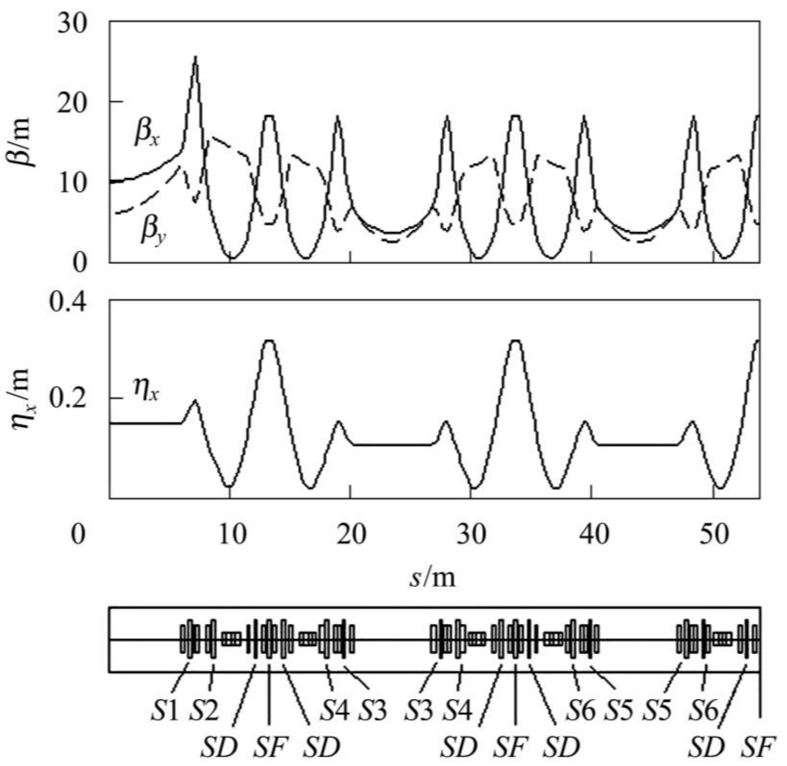


L. Wang et al., IPAC2010

Harmonic sextupoles in the DBA lattice

- Harmonic sextupoles can significantly increase the DA of a DBA lattice: an example of SSRF
  - Due to the dispersion in the straight sections for emittance reduction, the sextupoles in the straight sections are not strictly harmonic sextupoles.





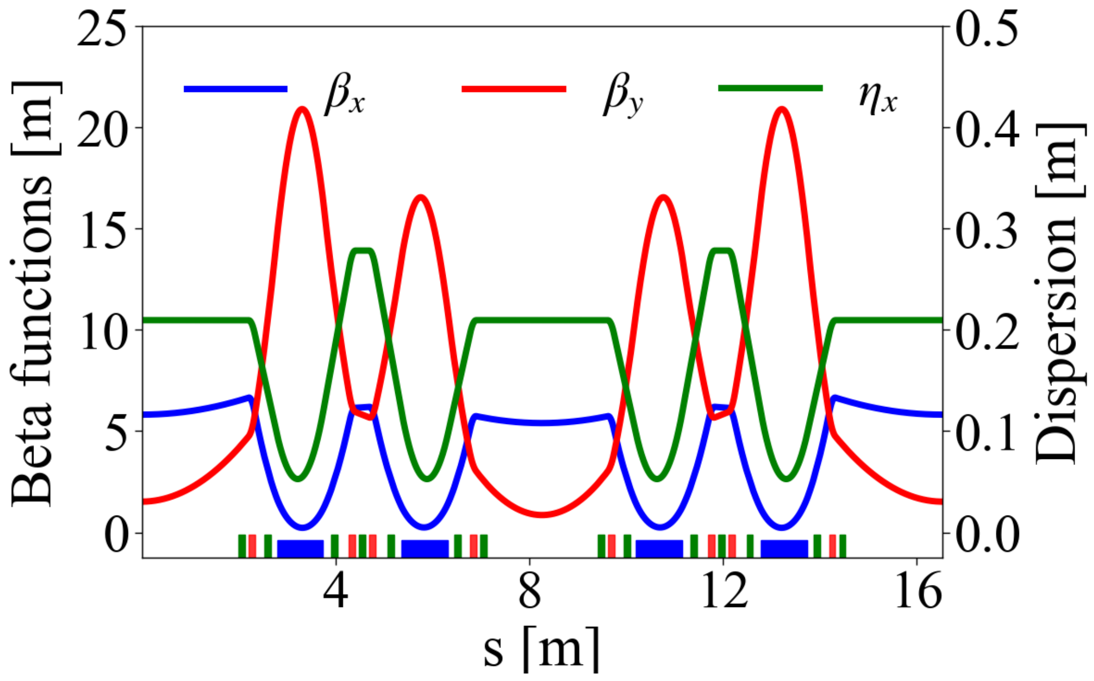
(a) DA without harmonic sextupole
(b) DA with 2 families of harmonic sextupoles
(c) DA with 4 families of harmonic sextupoles
(d) DA with 6 families of harmonic sextupoles

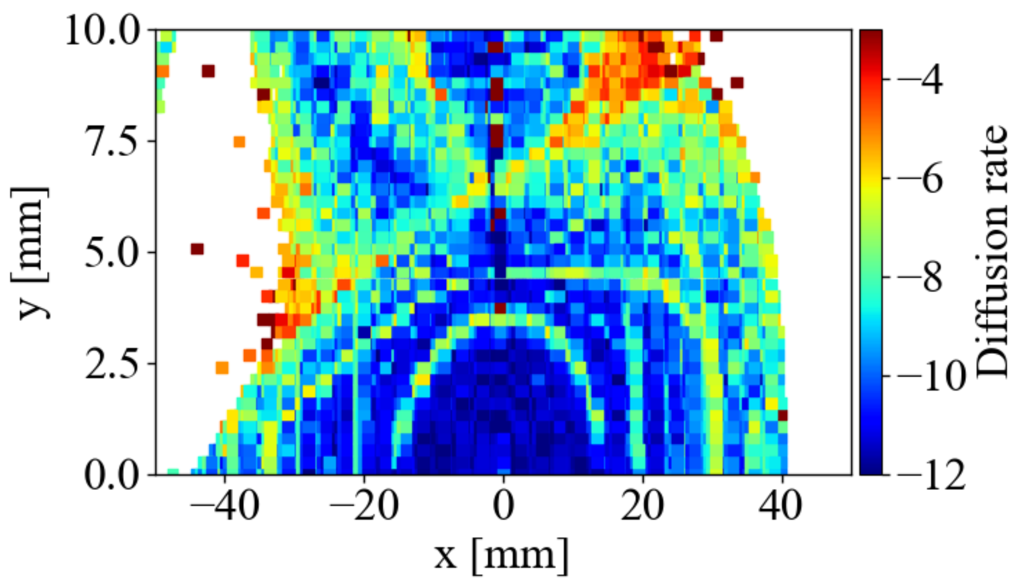
S. Tian et al., CPC, 2009

Harmonic sextupoles in the DDBA lattice

- Sextupoles placed in both long and short straight sections can significantly increase the DA of a DDBA lattice: HLS-III
  - Double-DBA (DDBA) lattice was firstly developed by Diamond

Energy	800 MeV
Circumference	66.1308 m
Number of periods	4
Natural emittance	2.82 nm·rad
Betatron tunes (x/y)	7.420 / 3.157
Natural chromaticities (x/y)	–14.0 / –14.3
Momentum compaction factor	6.74×10^{-3}
Natural energy spread	4.99×10^{-4}
Horizontal damping partition	1.44
Natural damping times (x/y/z)	16.3 / 23.6 / 15.1 ms
Energy loss per turn	14.98 keV



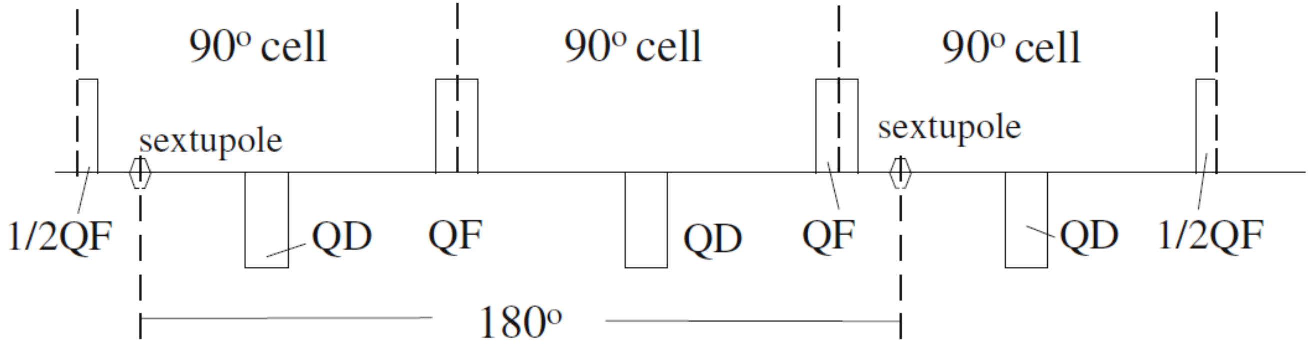


W. Li et al., IPAC2025

Nonlinear cancellation: -I & HOA

- To reduce the nonlinear effects, two basic strategies are used for the cancellation of geometric aberrations caused by sextupoles: -I & HOA.
- -I transformation: place two identical sextupoles (with the same length and strength) separated by a transfer map of –I

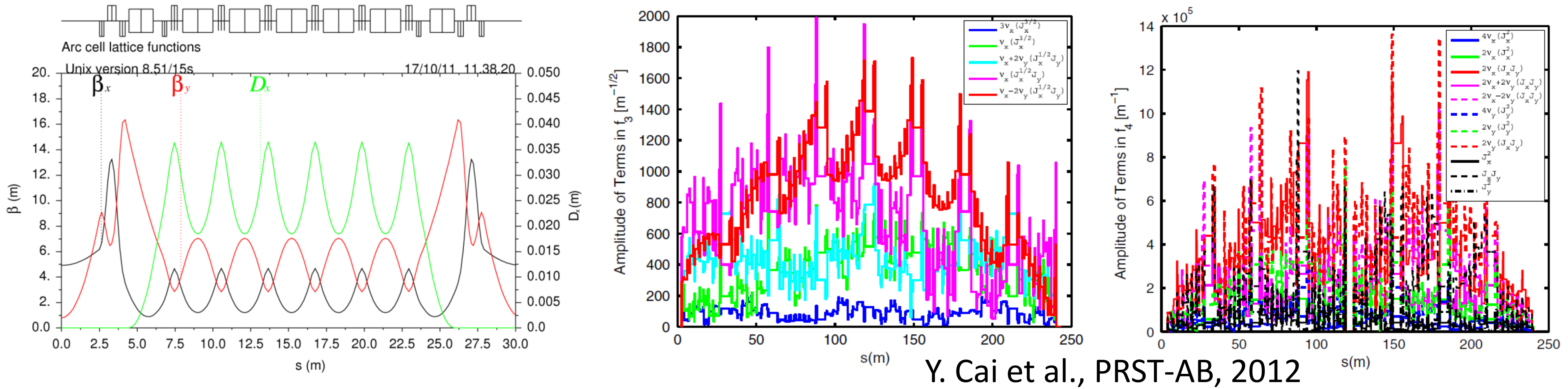
$$-\mathcal{I} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



- Higher-order-achromat (HOA): introduce ≥4 identical cells (lattice cells or unit cells) with the condition of the total horizontal and vertical tunes being integers (i.e., the total phase advances being n·2π), which can cancel the 1st- and 2nd-order sextupole terms (i.e., the 3rd- and 4th-order resonances)
  - Example 1, PEP-X lattice: the horizontal and vertical tunes of a lattice cell are (2+1/8, 1+1/8), with an HOA implemented over 8 lattice cells.

Nonlinear cancellation: -I & HOA

PEP-X: all 3rd- and 4th-order resonances are cancelled over 8 identical lattice cells

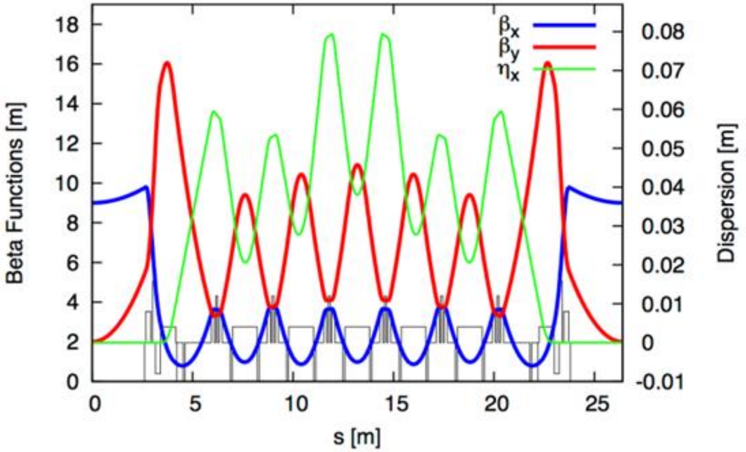


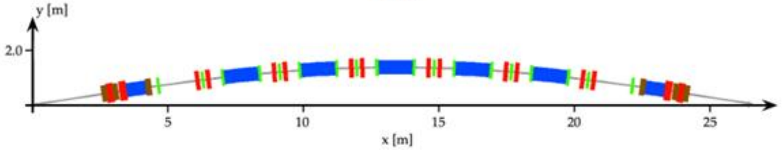
Y. Cai et al., PRST-AB, 2012

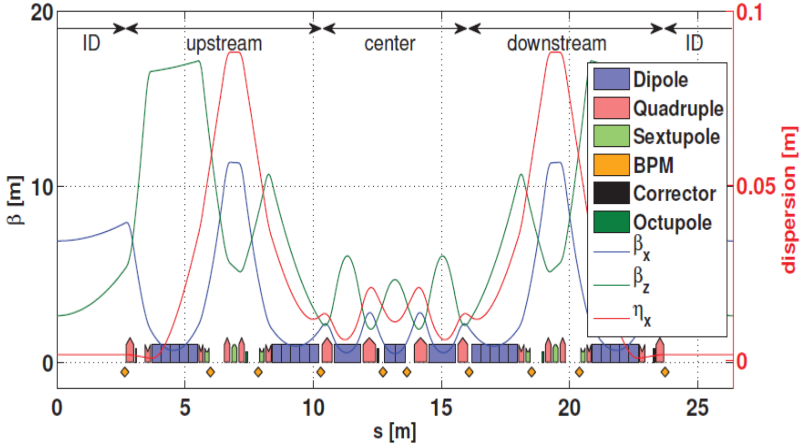
- Example 2, SLS 2.0 lattice: the horizontal and vertical tunes of a unit cell is (3/7, 1/7), with an HOA implemented within a lattice with 7 unit cells.
- Nonlinear cancellation implemented both within a lattice cell and over some lattice cells, used in the lattice design of diffraction-limited storage rings
  - The 7BA lattice of ESRF-EBS has a –I transformation within a lattice cell, and also an approximate HOA is formed over 8 lattice cells with lattice cell tunes of (~2+3/8, ~7/8).

MBA lattices

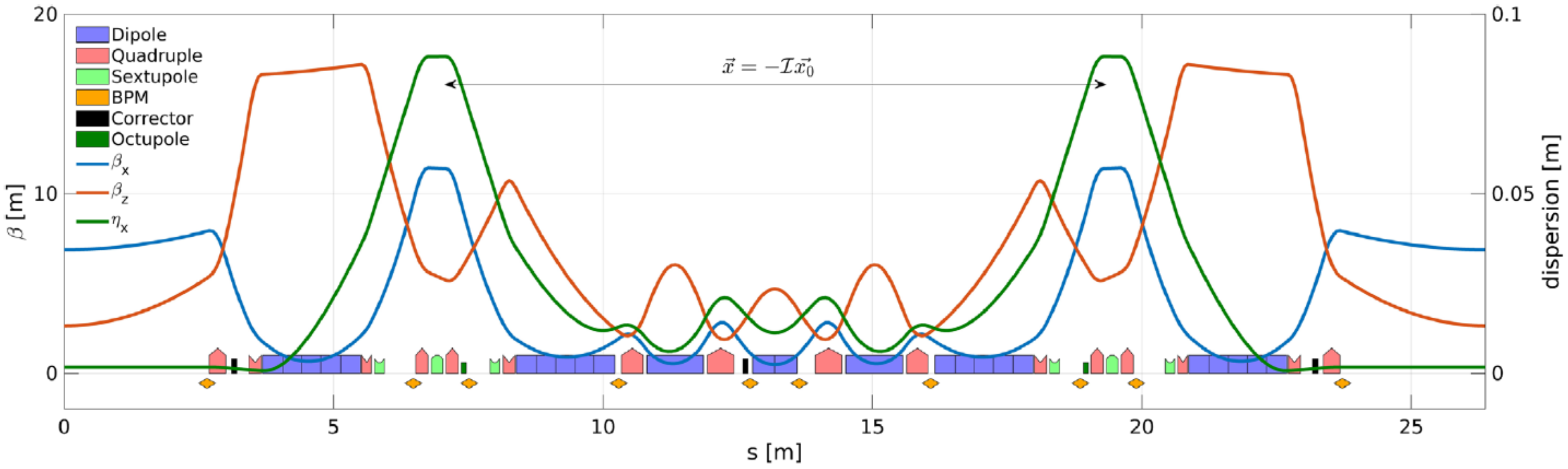
- MBA lattices can be mainly classified into two types: conventional MBA lattices and hybrid MBA lattices
- Conventional MBA lattices
  - Distributed chromaticity correction, used in medium- and low-energy rings
  - Can have lower emittance than hybrid MBA (for the same *E* and N_b) and also larger MA
  - Including MBA lattices based on the HOA cancellation
- Hybrid MBA lattices
  - Dispersion bumps for chromaticity correction, used in many rings with different energies
  - Generally have larger on-momentum DA than conventional MBA







HMBA lattice

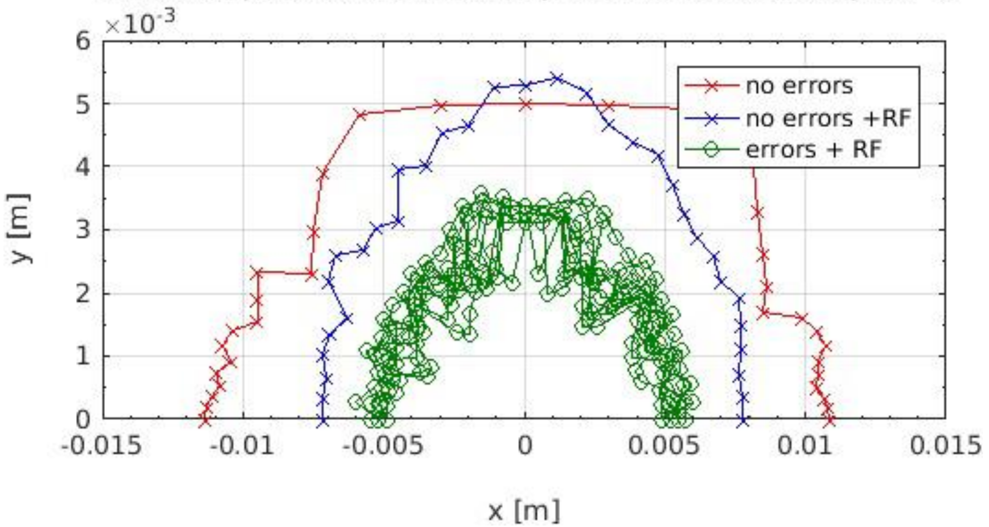


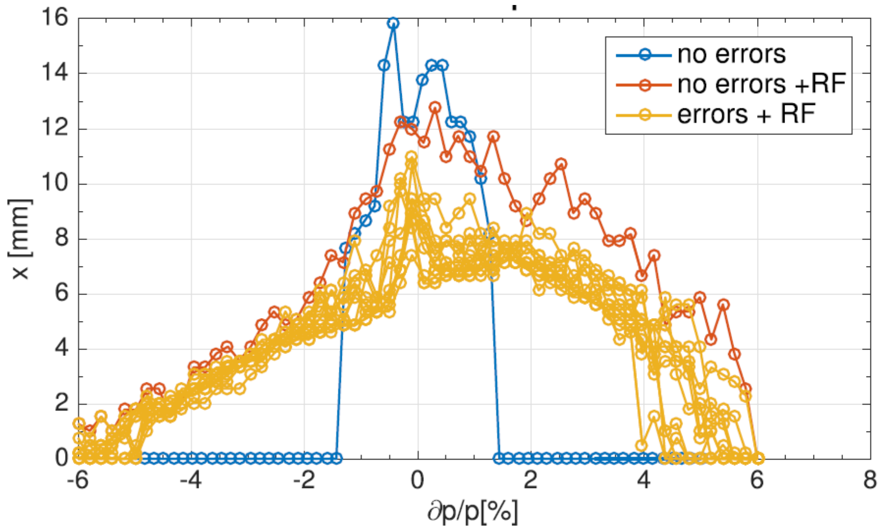
H7BA lattice of ESRF-EBS, P. Raimondi

- Hybrid MBA (HMBA) lattice: proposed by Pantaleo Raimondi (1)
  - Emittance reduction
    - LGBs, firstly used in a light source storage ring: reducing emittance
    - No need of "large" dispersion on the inner bends: giving small dispersion invariant H_x and thus small emittance
  - Nonlinear dynamics improvement
    - Dispersion bump for efficient chromaticity correction: "weak" sextupoles
    - -I transformation between two dispersion bumps: very effective cancellation of geometric aberrations and thus large on-momentum DA
    - Fewer sextupoles than in the DBA lattice

HMBA lattice

- Hybrid MBA (HMBA) lattice: proposed by Pantaleo Raimondi (2)
  - Other nonlinear dynamics features:
    - Reduced 6D DA due to the path lengthening effect (specially designed high-β_x injection section for off-axis injection)
    - Reduced off-momentum DAs





- Widely used in the lattice design of diffraction-limited storage rings (DLSRs)
  - All high-energy DLSRs adopt HMBA due to relatively weak sextupole strengths
  - Some medium- and low-energy DLSRs also adopt HMBA

The core of the HMBA lattice concept

Two non-interleaved dispersion bumps separated by –I

(Horizontal and vertical phase advances of **(1.5, 0.5)×2π for the H7BA lattice)**

Significantly weak strengths of sextupoles

(reducing individual strength)

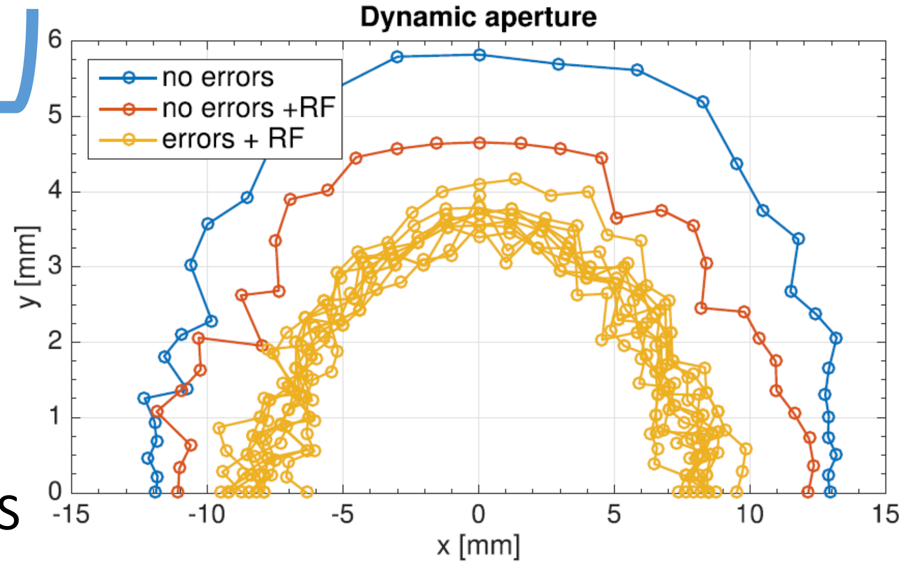
Highly effective cancellation of sextupole effects

(reducing overall strength)

团队强 = 个体强 + 个体间协作强

HMBA的哲学

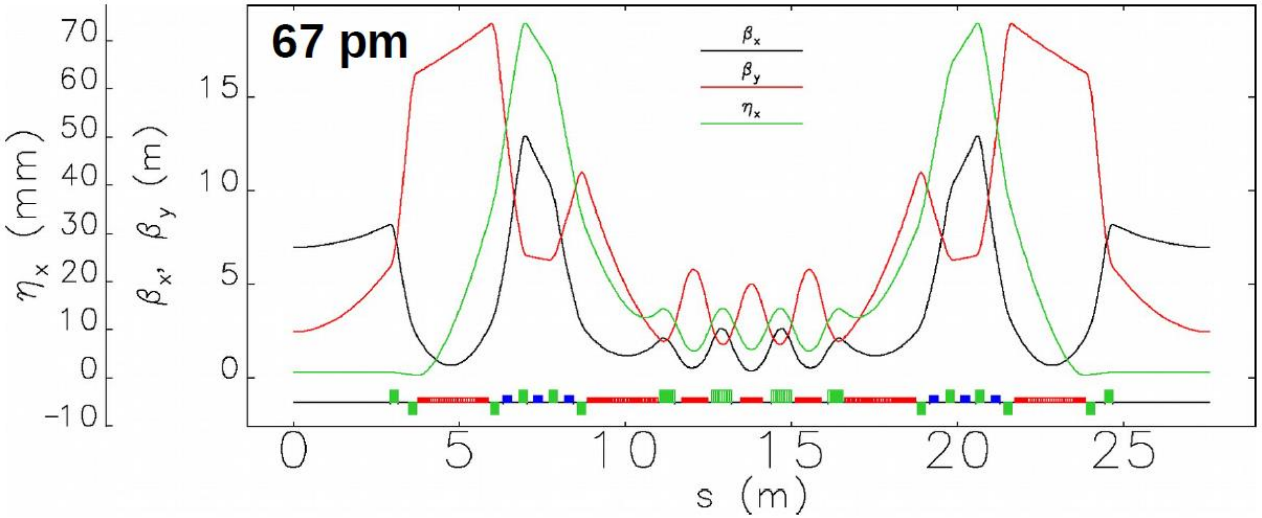
Large on-momentum dynamic aperture

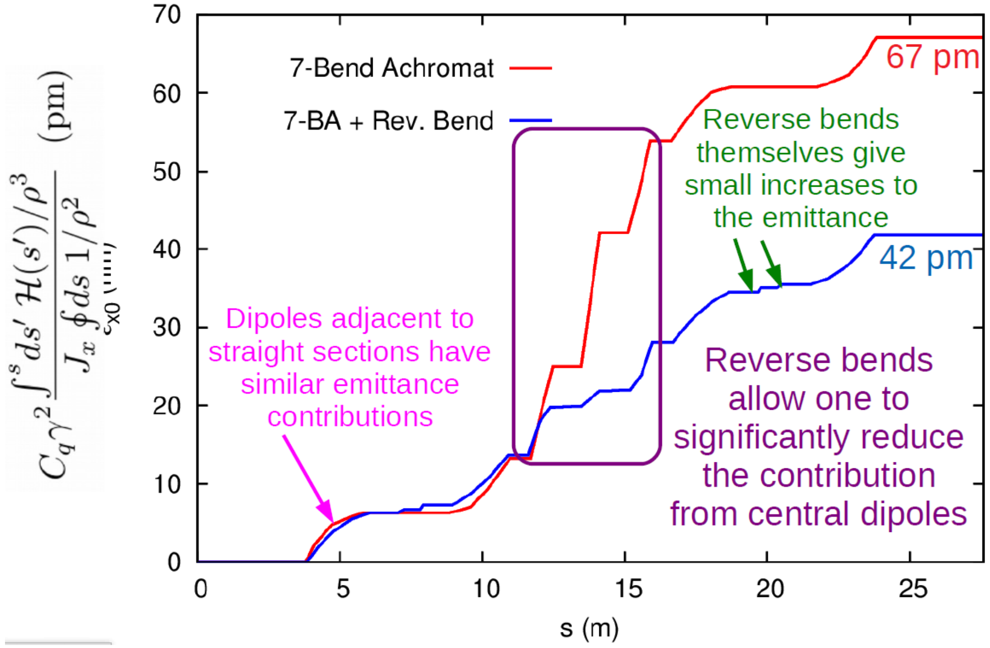


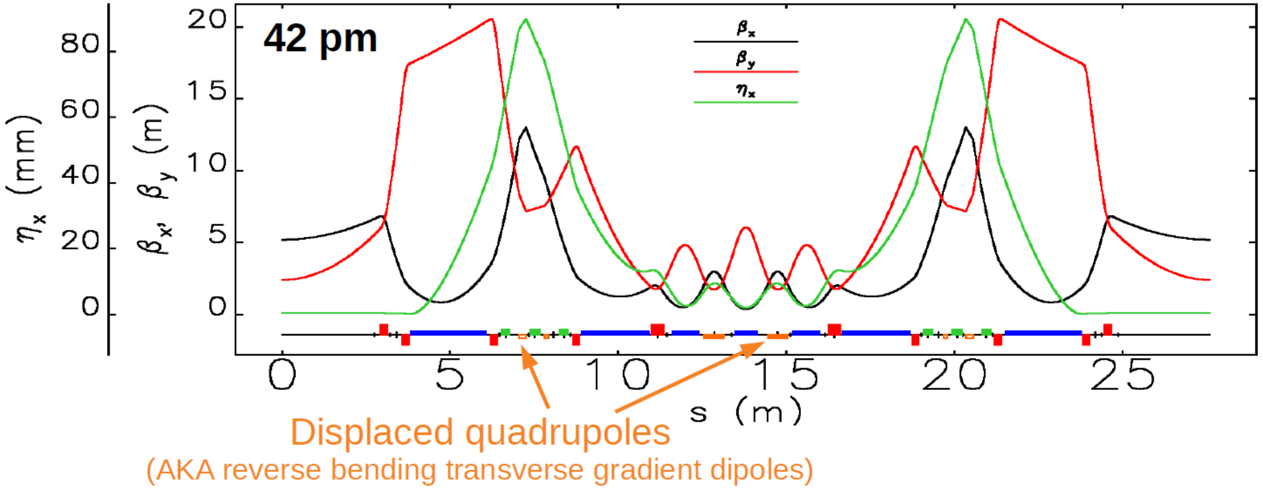
DA of the H7BA lattice of ESRF-EBS

H7BA lattice with RBs

- APS-U proposed an H7BA lattice with RBs (1)
  - Three families of RBs (combined with focusing quadrupoles) used
  - Reducing the natural emittance from 67 pm·rad to 42 pm·rad



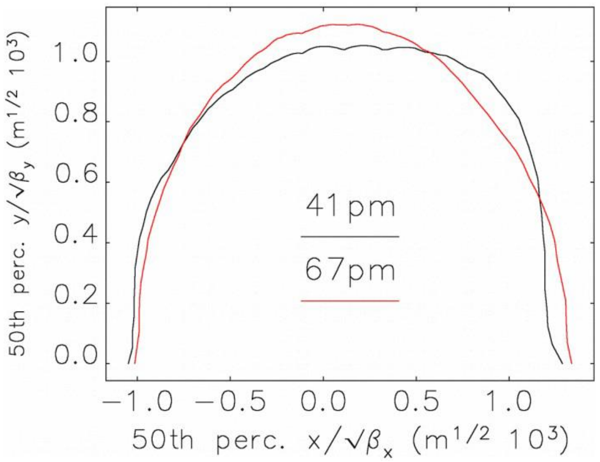


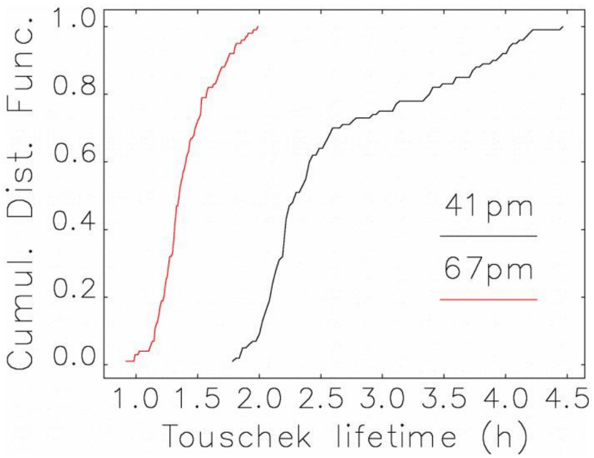


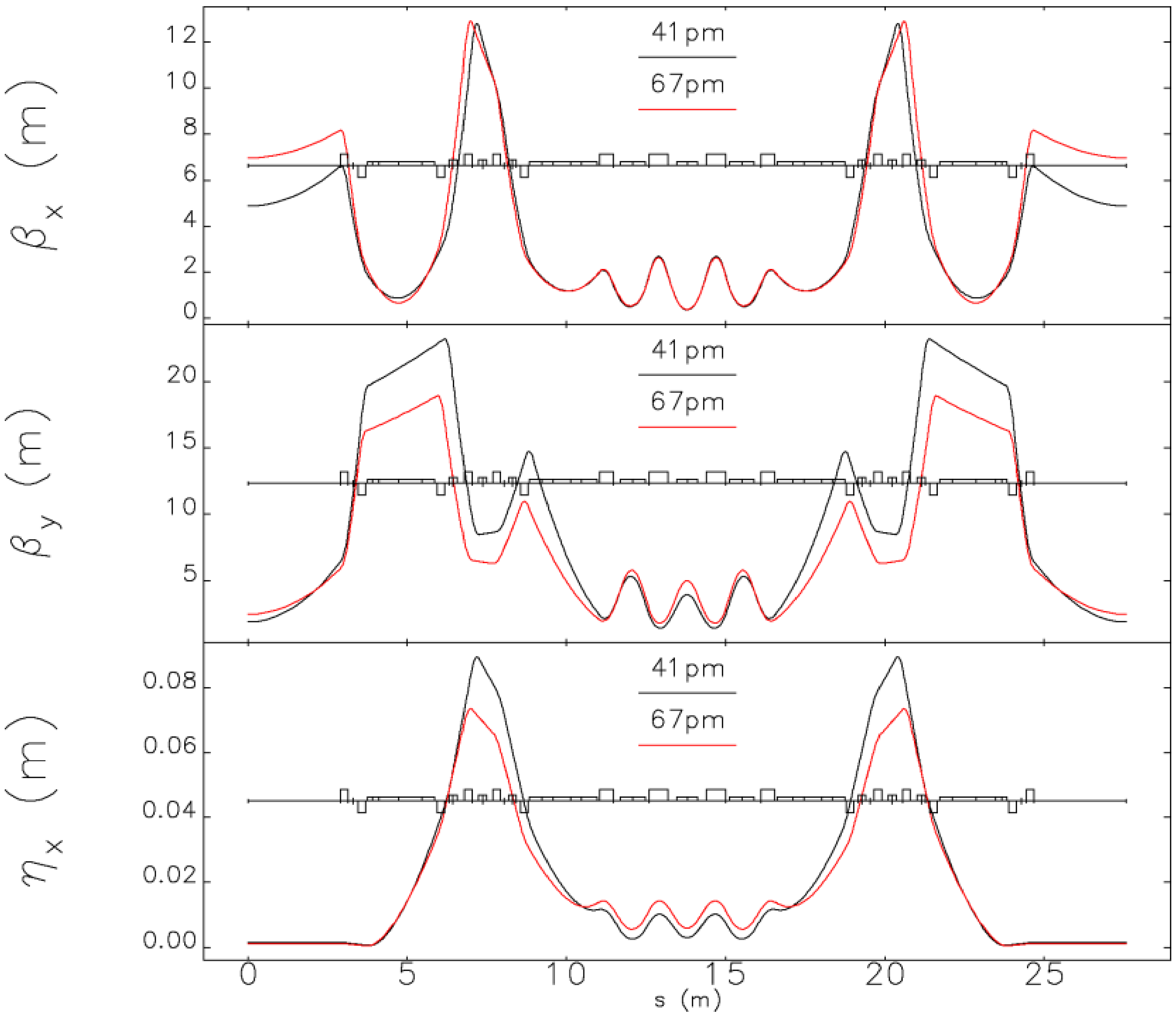
R. Lindberg, NAPAC2019

H7BA lattice with RBs

- APS-U proposed an H7BA lattice with RBs (2)
  - Increasing the peak dispersion from 74 mm to 90 mm: helps to reduce sextupole strengths, thus improving nonlinear dynamics performance
  - Better-optimized beta functions at the ID straight section



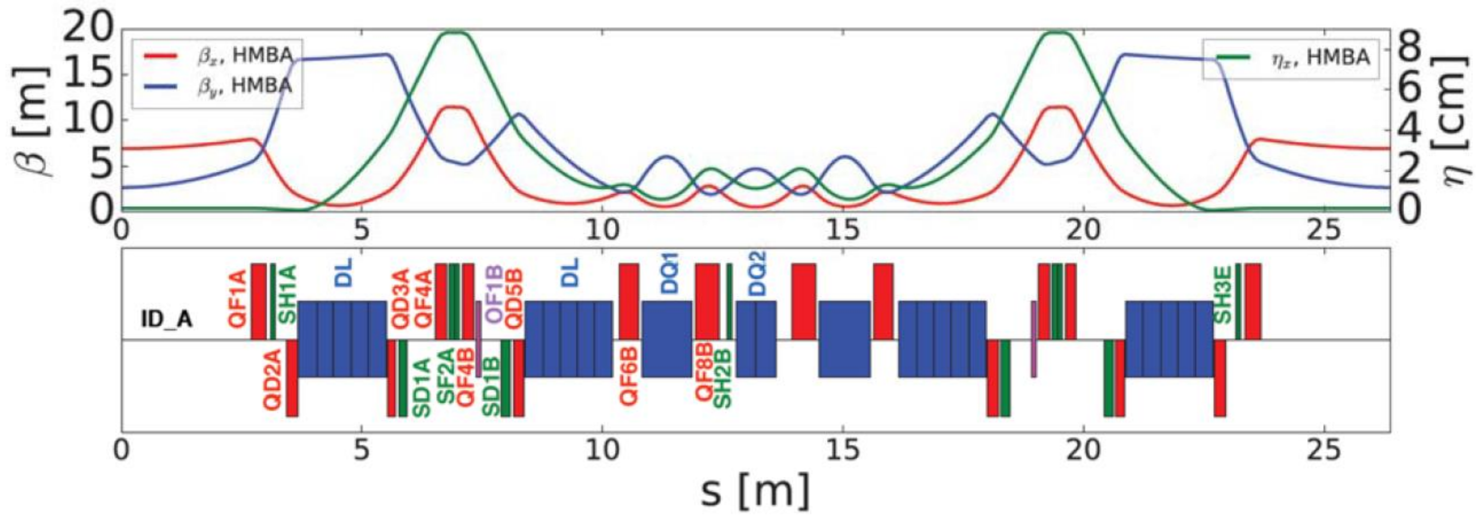


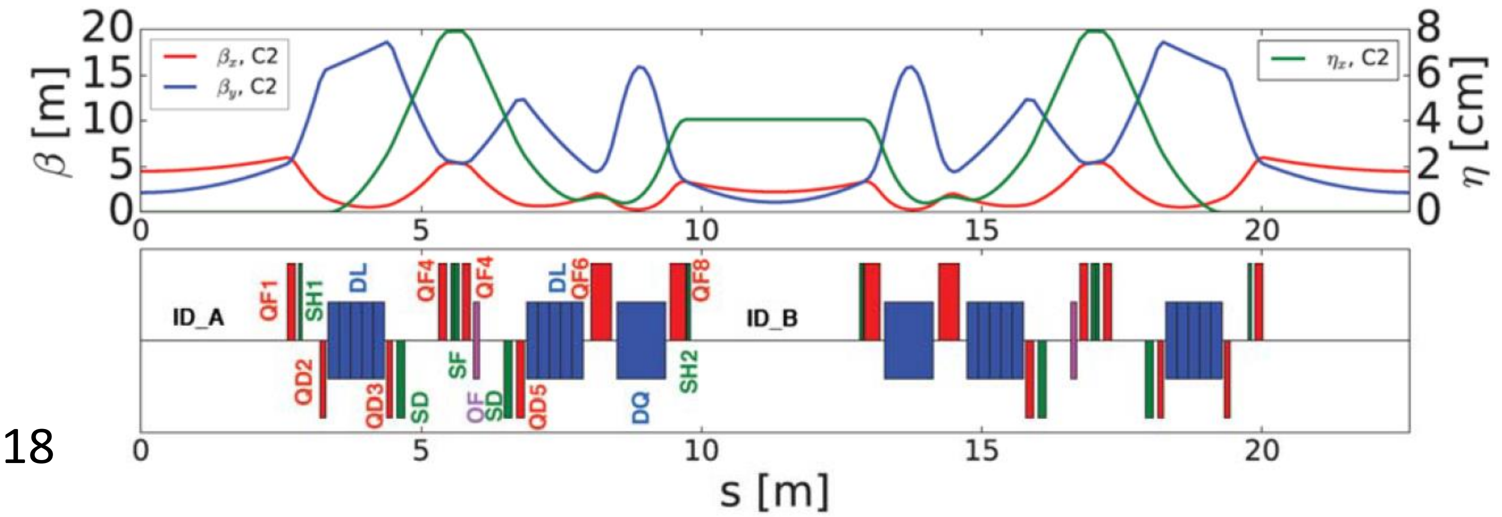


M. Borland, 2017

H6BA lattice with long and short straight sections

- Diamond-II proposed an H6BA lattice with long and short straight sections
  - Inspired by the HMBA and DDBA lattices: replacing the central bend of H7BA by a short straight section
  - Short straight section: non-zero dispersion, low beta functions
  - Doubling the number of ID straight sections at the cost of emittance increase





A. Alekou, 2018

H6BA, H11BA, and H9BA lattices

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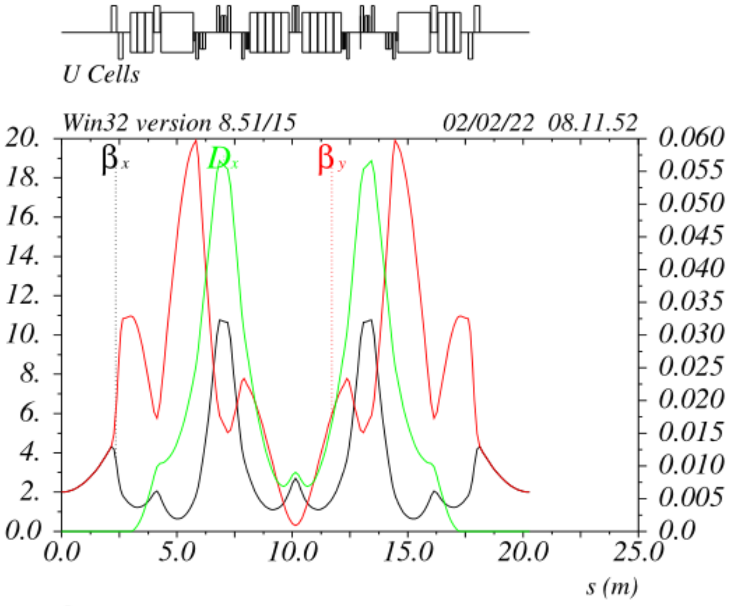
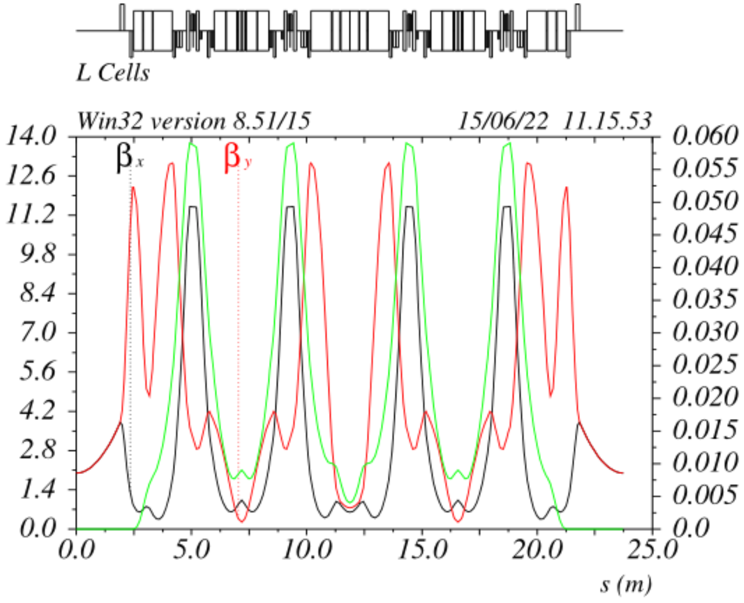
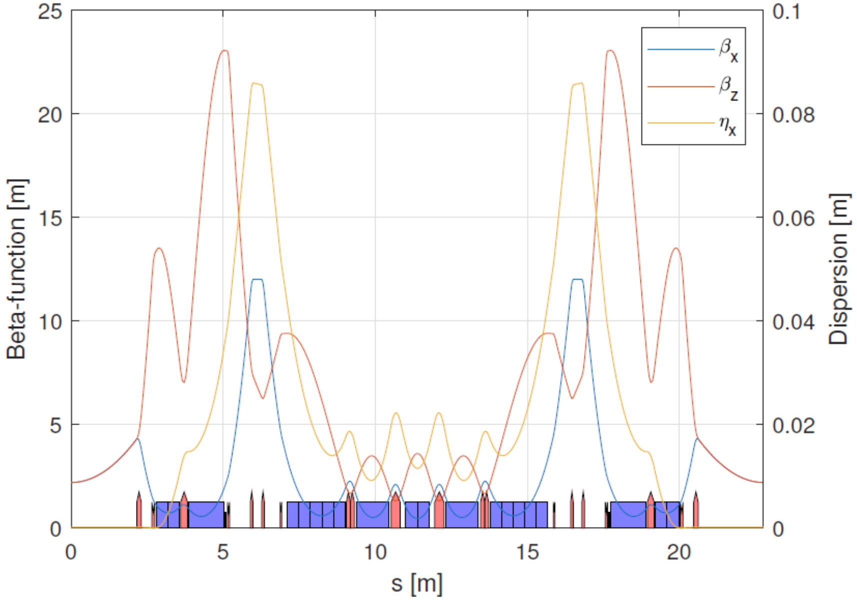
- H6BA lattice:
  - -I with phase advances of (0.5, 0.5)×2π & two central bends
  - Two matching bends on each side & low-beta straight sections
- H11BA lattice: a combination of two H6BA lattices
- H9BA lattice: similar to H6BA, but with phase advances of (1.5, 0.5)×2π



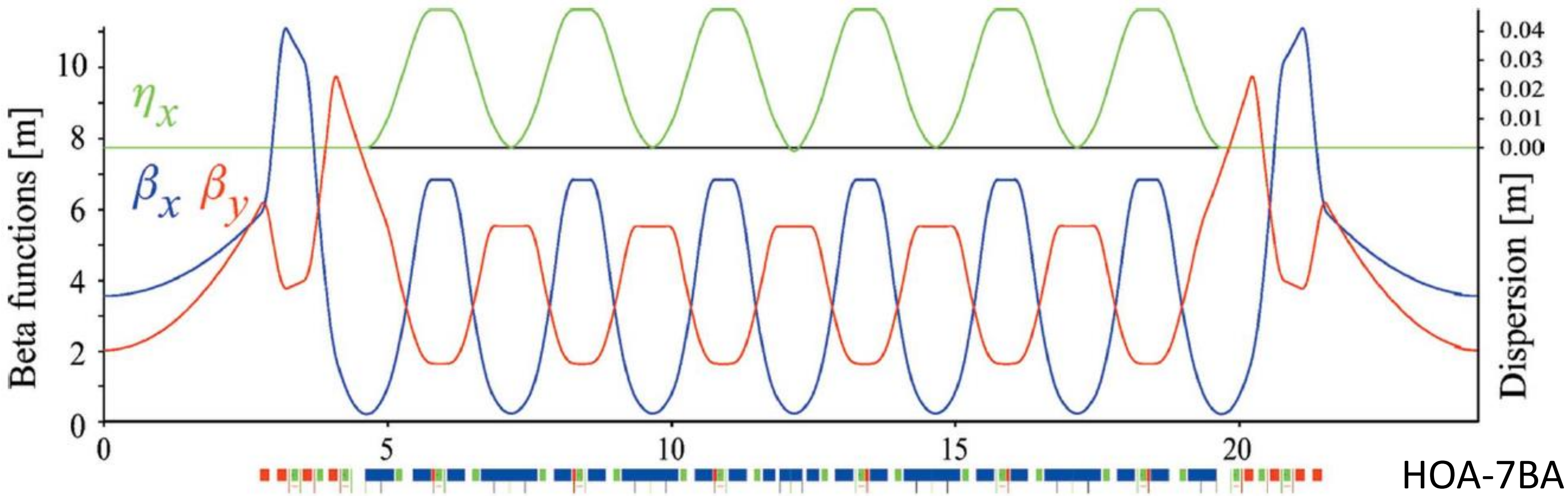




P. Raimondi, LEL2022

HOA-MBA lattice

- Higher-order-achromat MBA (HOA-MBA) lattice
  - Based on identical unit cells, developed by SLS 2.0
  - Introduce a unit cell, repeat it four or more times to generate a super-period, and adjust the total tune to an integer in both planes
  - For 7 identical unit cells, the horizontal and vertical unit cell tunes of (3/7, 1/7) ≈ (0.429, 0.143) provide cancellation of all third-order and fourth-order resonances.



HOA-7BA lattice of SLS 2.0
A. Streun et al., JSR, 2018

HOA-MBA lattice

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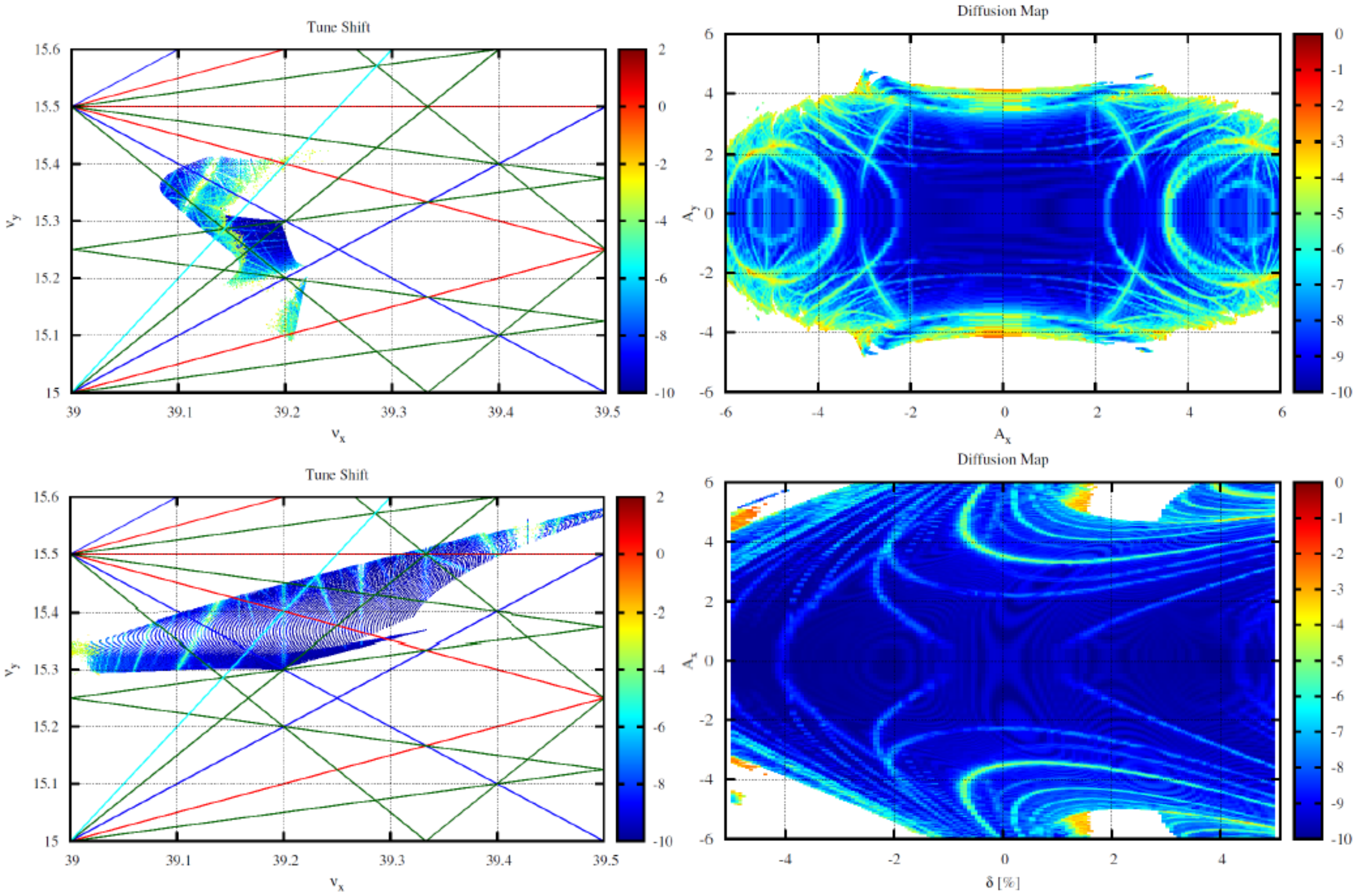
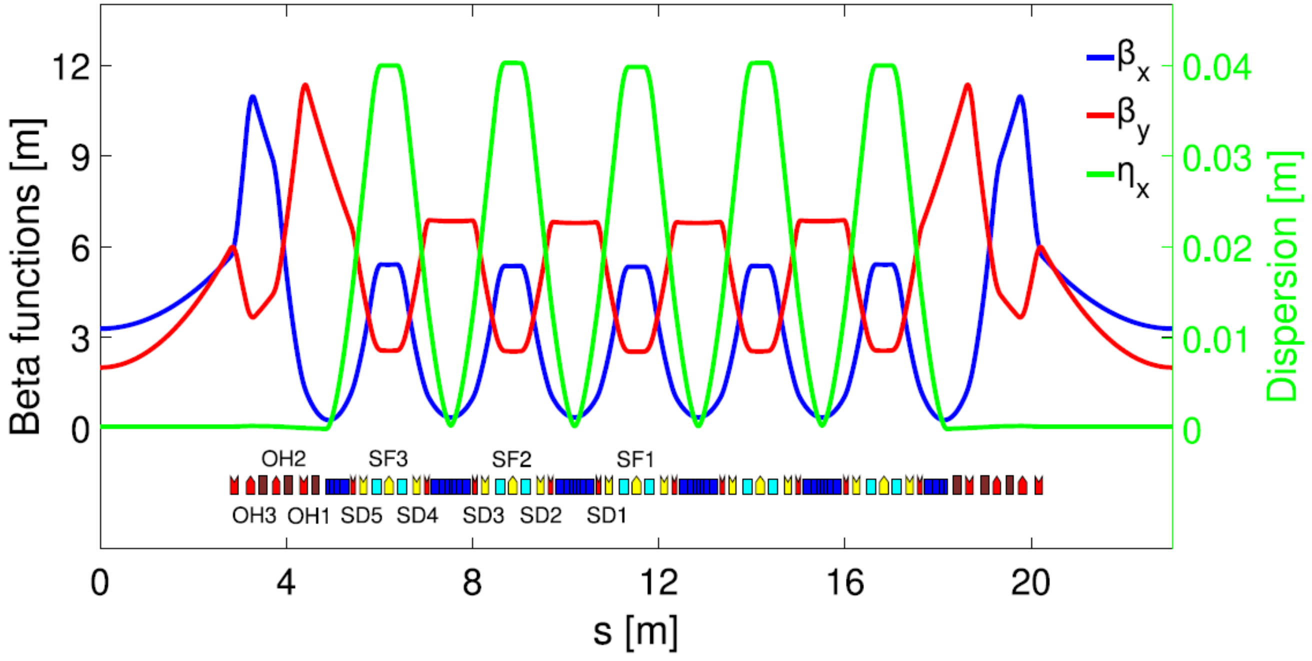
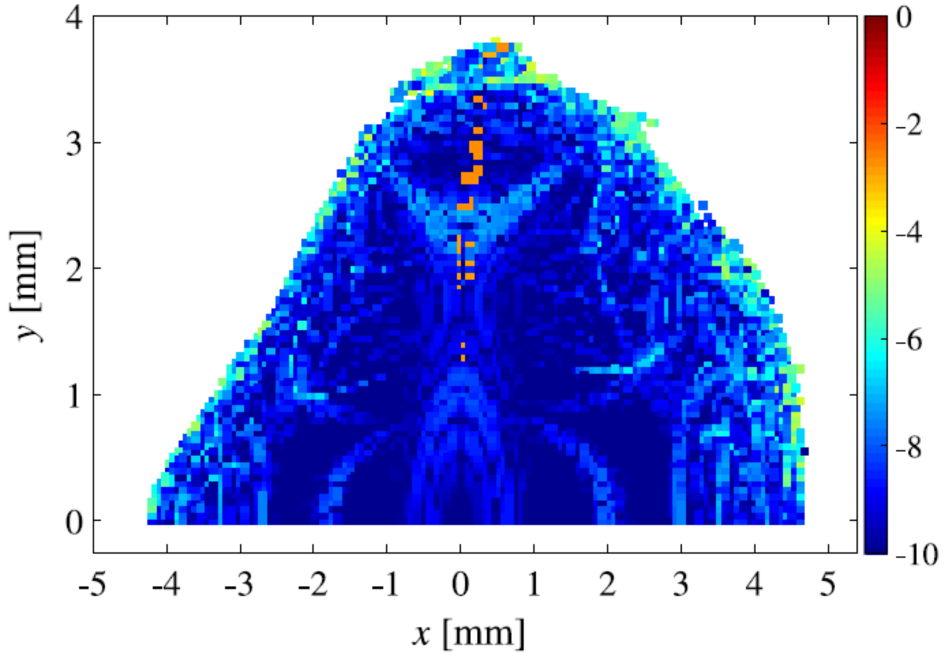
- Nonlinear optimization of the HOA-7BA lattice of SLS 2.0
  - To control non-resonant terms, which are amplitude dependent tune shifts and second-order chromaticities, HOA-based resonance suppression requires to back off the ideal cancellation pattern to some extent.
  - Sextupoles are grouped into some families for better optimization of nonlinear dynamics.



HOA-MBA lattice

- HOA-MBA lattice based on unit cell tunes of (0.4, 0.1)
  - For 5 identical unit cells, the horizontal and vertical unit cell tunes of (0.4, 0.1) provide cancellation of all third-order and fourth-order resonances, except the $2\nu_x + 2\nu_y$ resonance, for normal sextupoles (not skew sextupoles).

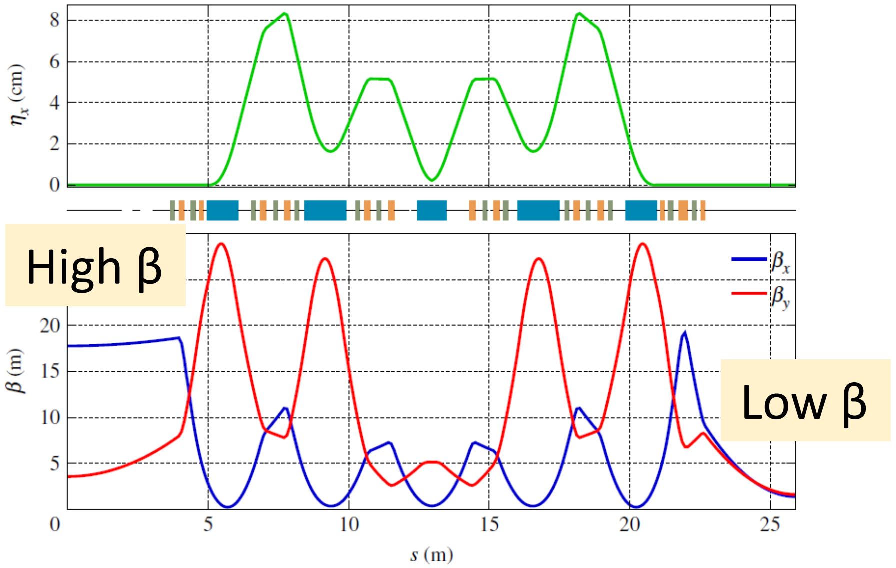




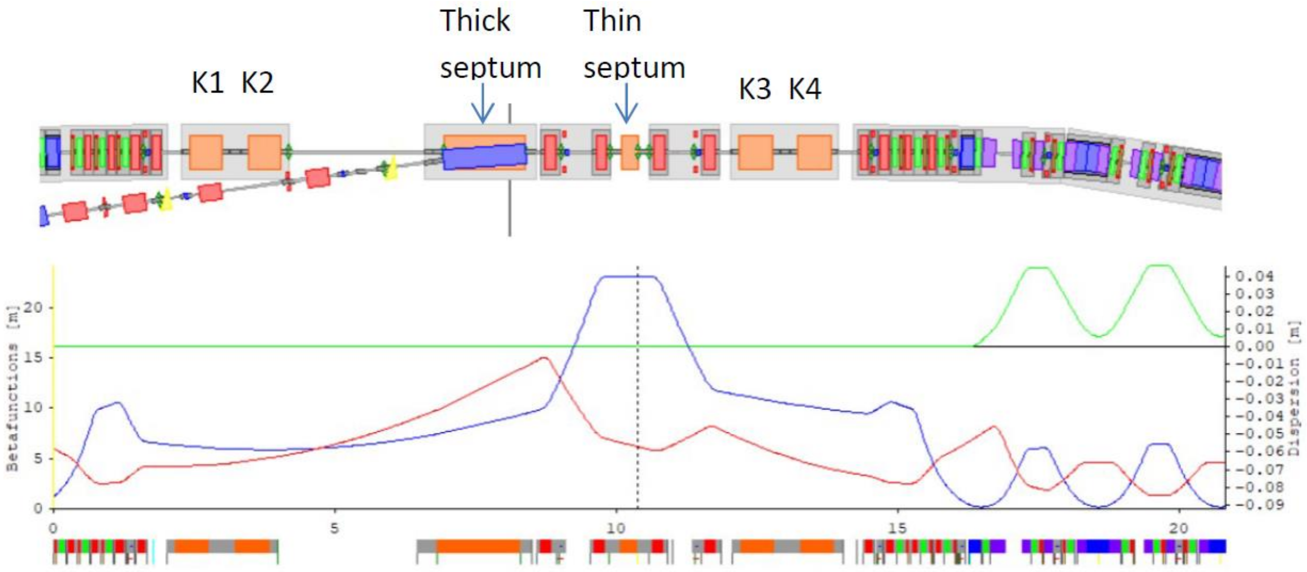
P. Yang et al., NIMA, 2021

High-β_x injection section design of MBA lattices

- High-β_x injection straight section design
  - To obtain a larger DA for off-axis injection: DA is proportional to the square root of beta functions
  - Super-period high- and low-beta design: similar to that of many third-generation light sources, e.g. used by Sirius
  - High-beta design with quasi-periodicity: maintain phases and optics at nonlinear magnets for restoring nonlinear dynamics, e.g. used by ESRF-EBS, SLS 2.0



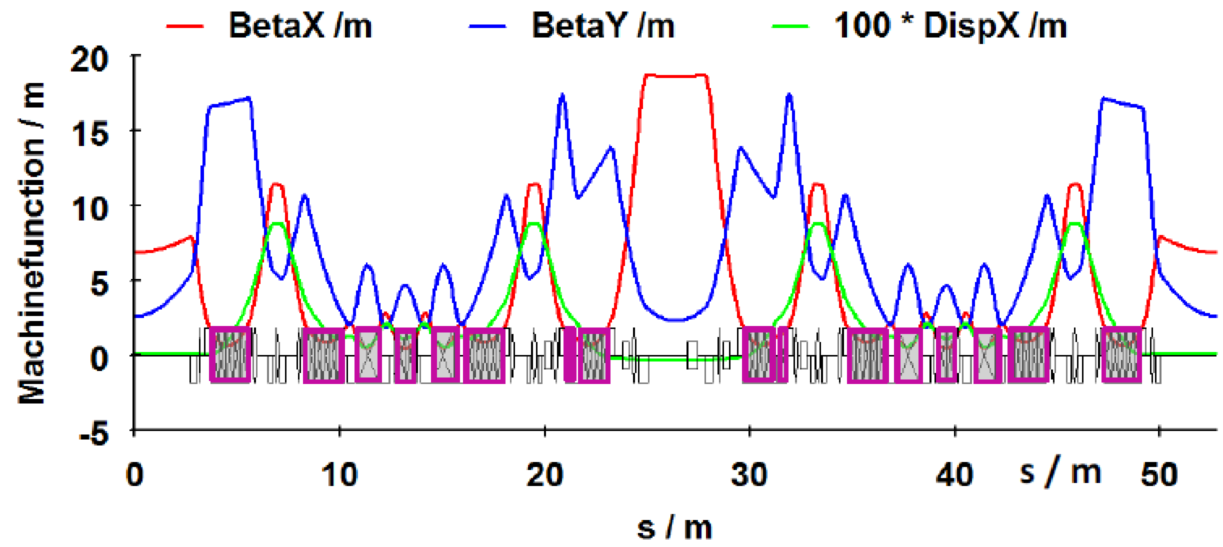
L. Liu et al., IPAC2021

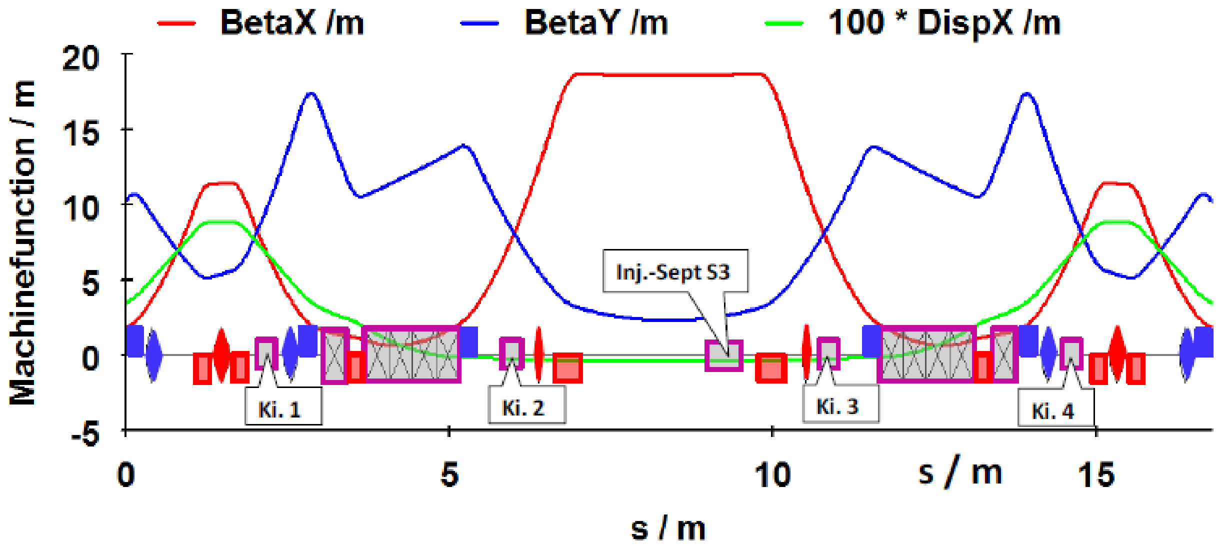


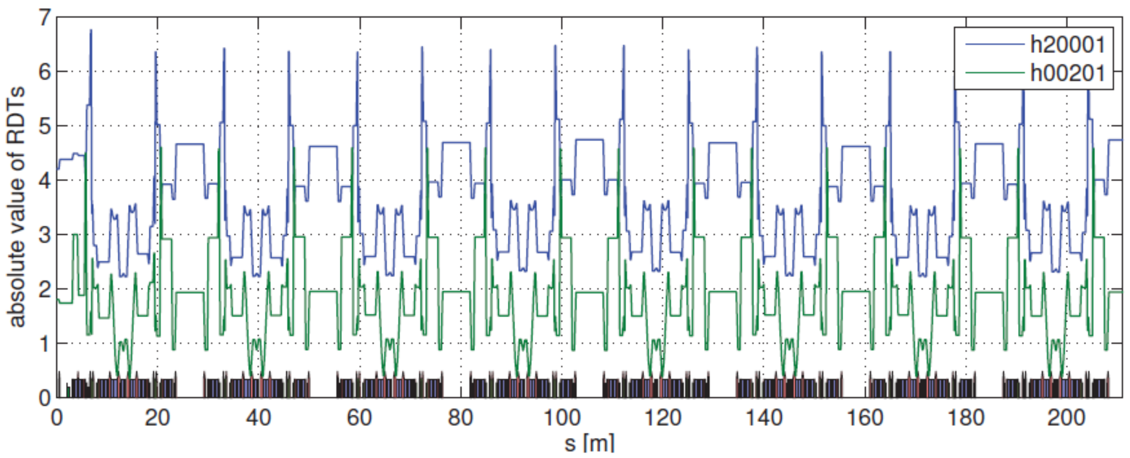
A. Streun et al., 2021

High-β_x injection section design of MBA lattices

- ESRF-EBS high-β_x injection straight section
  - On-momentum symmetry: keeping local phase advances and optics at sextupoles unchanged
  - Off-momentum dynamics: restoring the periodicity of chromatic driving terms (beta function variation with momentum)







N. Carmignani et al., IPAC2015

Interplay of storage ring parameters

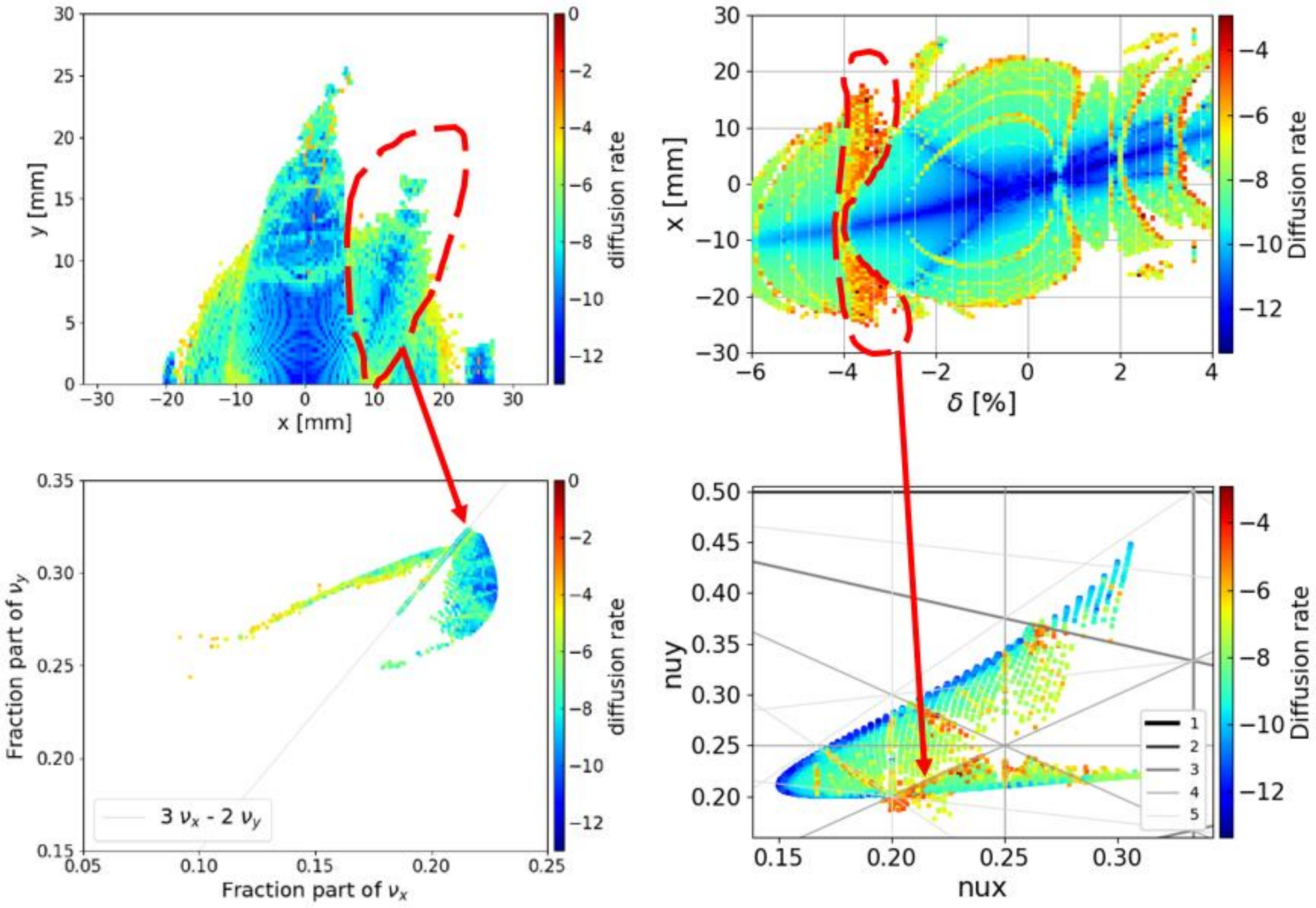
- As the emittance approaches the diffraction-limited emittance, the optimization of trade-offs between emittance, damping times, energy, momentum compaction factor, etc., becomes essential for lattice design.
- An example: HMBA lattice designed with diffraction-limited emittance
  - Relatively small horizontal tunes of unit cells (compared to the unit cells of the SLS 2.0 lattice), longer damping times for the same emittance goal, more severe intra-beam scattering effect
  - If RBs are overused to reduce emittance and damping times, the momentum compaction factor will be very small.
  - For a medium-energy DLSR using HMBA, a beam energy of 3.5~4.0 GeV (rather than 3.0 GeV) with damping wigglers employed provides a better choice for lattice design, including the consideration of achieving lower equilibrium emittance.
- When encountering such issues, the combination of properly increasing beam energy and employing damping wigglers is a more effective solution.

Frequency map analysis

- Frequency map analysis (FMA) constructs the "frequency map" from the space of initial conditions to the tune space
  - Visualizing resonance lines and their strengths, and amplitude dependent tune shifts
  - Quantifying the stability of particle motion through frequency diffusion rates (calculated from the tune differences between the first *N* turns and the following *N* turns)

$$d_r = \log_{10}\sqrt{(\nu_{x,1}-\nu_{x,2})^2+(\nu_{y,1}-\nu_{y,2})^2/N}$$

$(x, y) \rightarrow (\nu_x, \nu_y)$ $(\delta, x) \rightarrow (\nu_x, \nu_y)$



Resonance driving terms

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- The one-turn map of a storage ring has the Lie series representation:

  $\mathcal{M} = e^{:h_2:}e^{:h_3:}e^{:h_4:}\ldots$ where h_2 is the linear part, and h_3 & h_4 are nonlinear

- The nonlinear part can be expanded using resonance basis. The leading order h_3 caused by sextupoles is written as:

$$h_3 = \sum_{j+k+l+m+p=3} h_{jklmp}\,(2J_x)^{\frac{j+k}{2}}(2J_y)^{\frac{l+m}{2}}e^{i[(j-k)\phi_x+(l-m)\phi_y]}\delta^p$$

where $h_{jklmp} \sim \sum_{n=1}^{N}(b_3L)_n\,\beta_{x,n}^{\frac{j+k}{2}}\beta_{y,n}^{\frac{l+m}{2}}\,\eta_{x,n}^{p}e^{i[(j-k)\mu_{x,n}+(l-m)\mu_{y,n}]}$

- The importance of linear optics design in nonlinear dynamics optimization
  - Resonance driving terms (RDTs) h_{jklmp} are functions of linear optics (beta functions, dispersion, phase advances) and sextupole strengths (also calculated from linear optics).
  - Linear optics offers extensive degrees of freedom for nonlinear optimization.

Resonance driving terms

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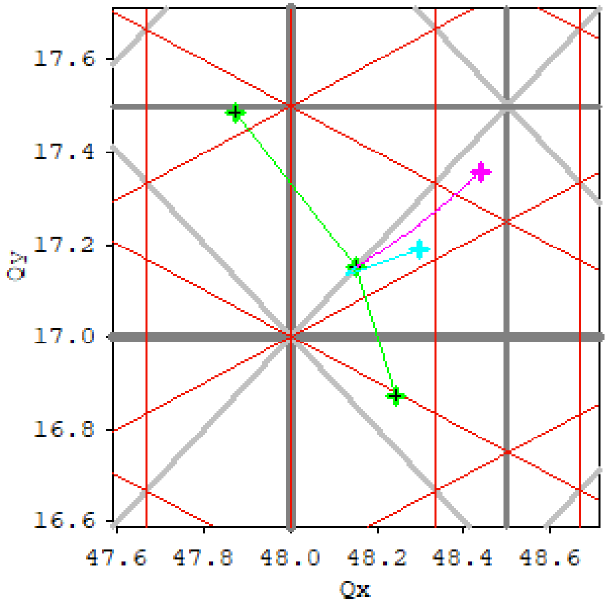
- The effects of nonlinear driving terms

chromatic terms (p≠0)			Geometric terms (p=0)					
			Amplitude-dependent tune shift driving terms		Resonance driving terms			
h_{11001}	$\partial\nu_x/\partial\delta$	Chromaticity	h_{22000}	$\partial\nu_x/\partial J_x$	h_{21000}	ν_x	h_{31000}	$2\nu_x$
h_{00111}	$\partial\nu_y/\partial\delta$	Chromaticity	h_{11110}	$\partial\nu_{x,y}/\partial J_{y,x}$	h_{30000}	$3\nu_x$	h_{40000}	$4\nu_x$
h_{20001}	$\partial\beta_x/\partial\delta$	Chromatic beta-beat	h_{00220}	$\partial\nu_y/\partial J_y$	h_{10110}	ν_x	h_{20110}	$2\nu_x$
h_{00201}	$\partial\beta_y/\partial\delta$	Chromatic beta-beat			h_{10020}	$\nu_x - 2\nu_y$	h_{11200}	$2\nu_y$
h_{10002}	$\partial\eta_x/\partial\delta$	Closed-orbit distortion			h_{10200}	$\nu_x + 2\nu_y$	h_{20020}	$2\nu_x - 2\nu_y$
							h_{20200}	$2\nu_x + 2\nu_y$
							h_{00310}	$2\nu_y$
							h_{00400}	$4\nu_y$

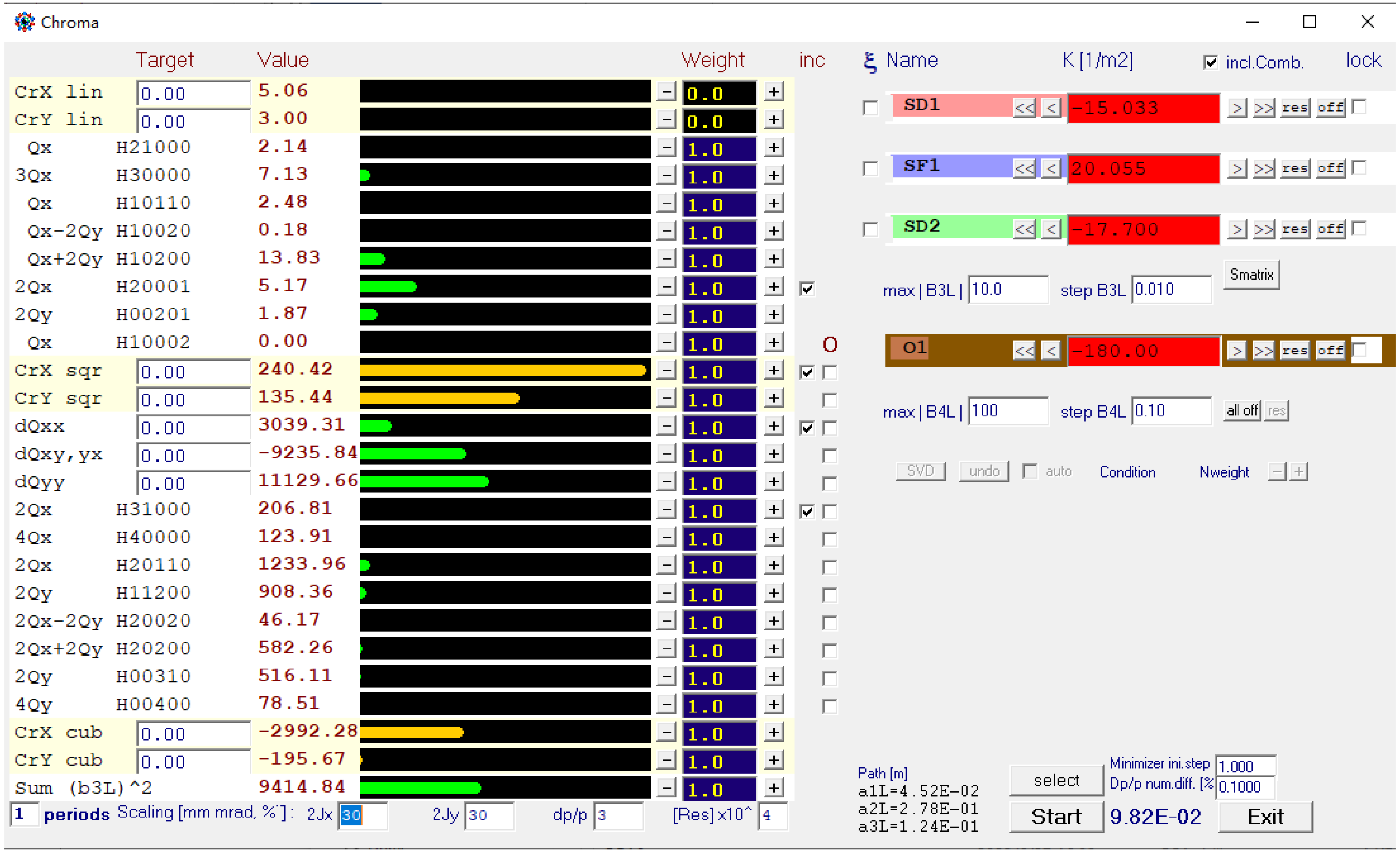
- Minimizing nonlinear driving terms in nonlinear optimization
  - Enlarging on-momentum DA: Reducing 3rd- and 4th-order RDTs & amplitude dependent tune shifts
  - Improving off-momentum nonlinear dynamics: control of 2nd- and 3rd-order chromaticities & chromatic driving terms

Optimization of nonlinear driving terms with OPA

- The OPA program offers powerful functionality and a user-friendly interface for optimizing the nonlinear driving terms.



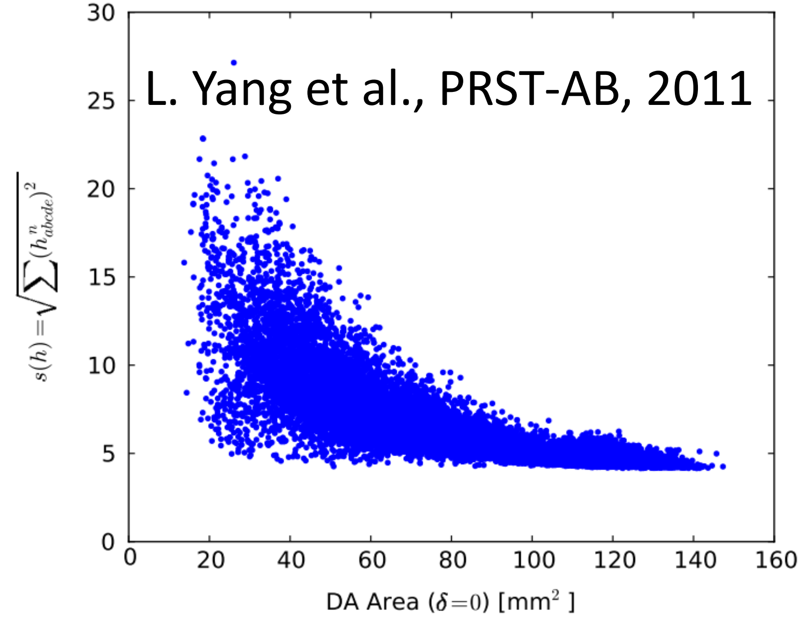
Tune shifts with amplitude (green) and momentum (magenta/cyan)

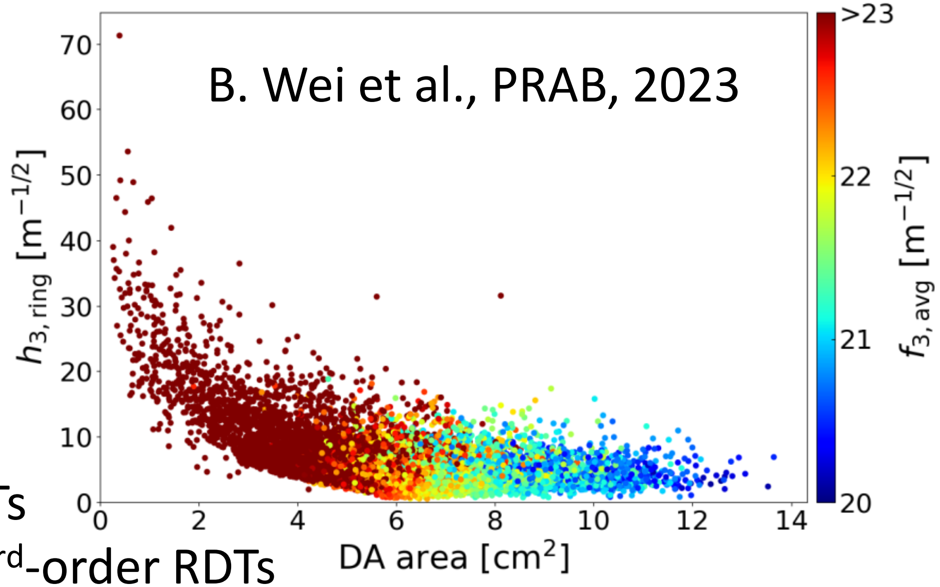


https://ados.web.psi.ch/opa/

Minimizing the fluctuation of resonance driving terms

- Minimizing the commonly-used one-turn RDTs is a necessary but not sufficient condition for large DA.
    - The optimization results are dependent on the lattice designers' experience.
- Recently it was found that minimizing the variation or fluctuation of RDTs along the longitudinal position of a ring is much more effective than minimizing one-turn RDTs in enlarging DA.
    - Qualitative analysis via the BCH formula: reducing the fluctuation of lower-order RDTs can control higher-order nonlinear terms.



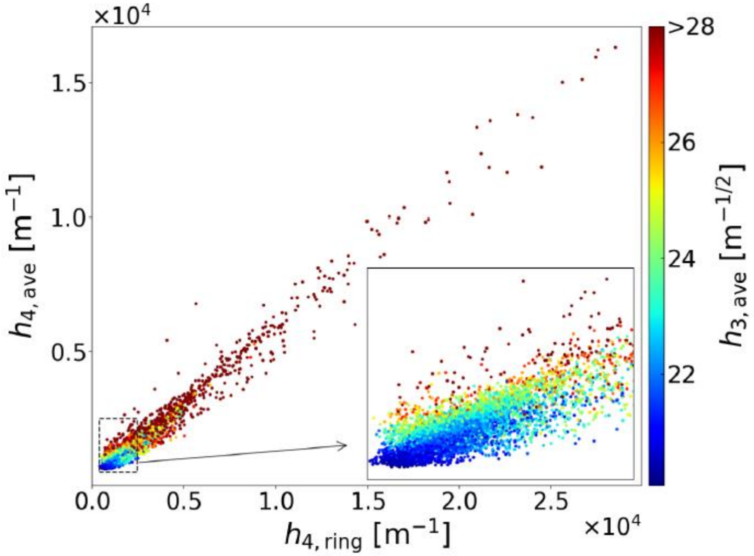


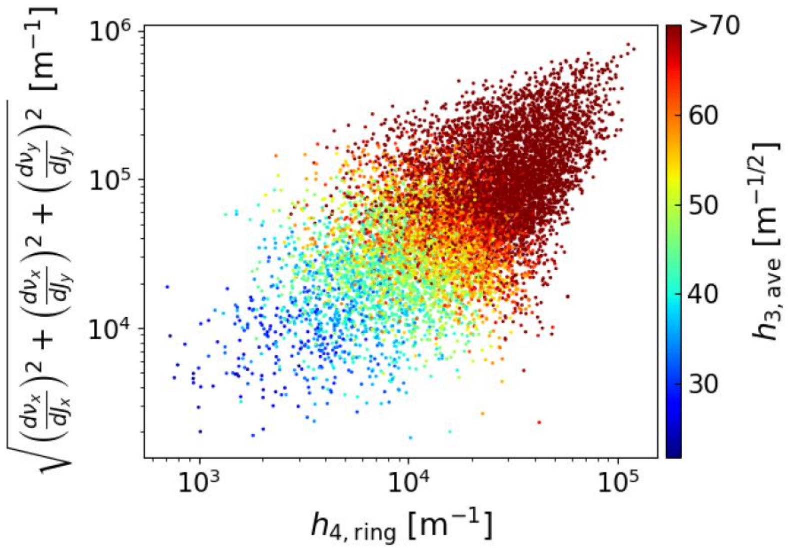
Vertical axis: one-turn RDTs

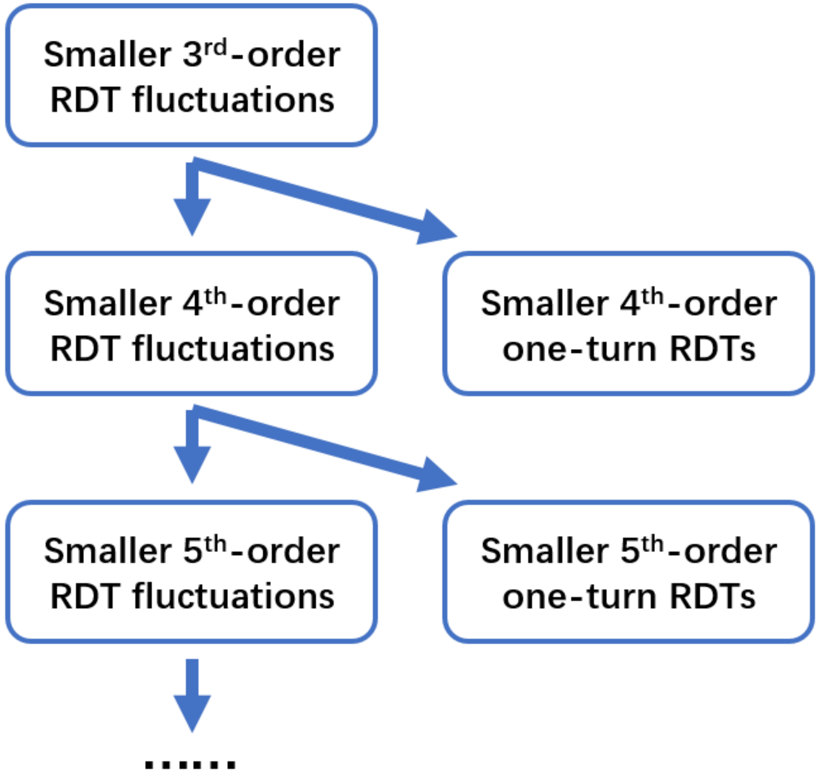
Color bar: fluctuation of 3rd-order RDTs

Minimizing the fluctuation of resonance driving terms

- Reducing lower-order RDT fluctuations controls higher-order nonlinear terms
  - Minimizing 3rd-order RDT fluctuations reduces 4th-order RDT fluctuations, 4th-order one-turn RDTs, ADTS, and even higher-order RDTs.
  - Controlling lower-order RDT fluctuations in the DA optimization can avoid the need for calculating higher-order RDTs to a large extent, which are not only more computationally complicated but also more numerous.



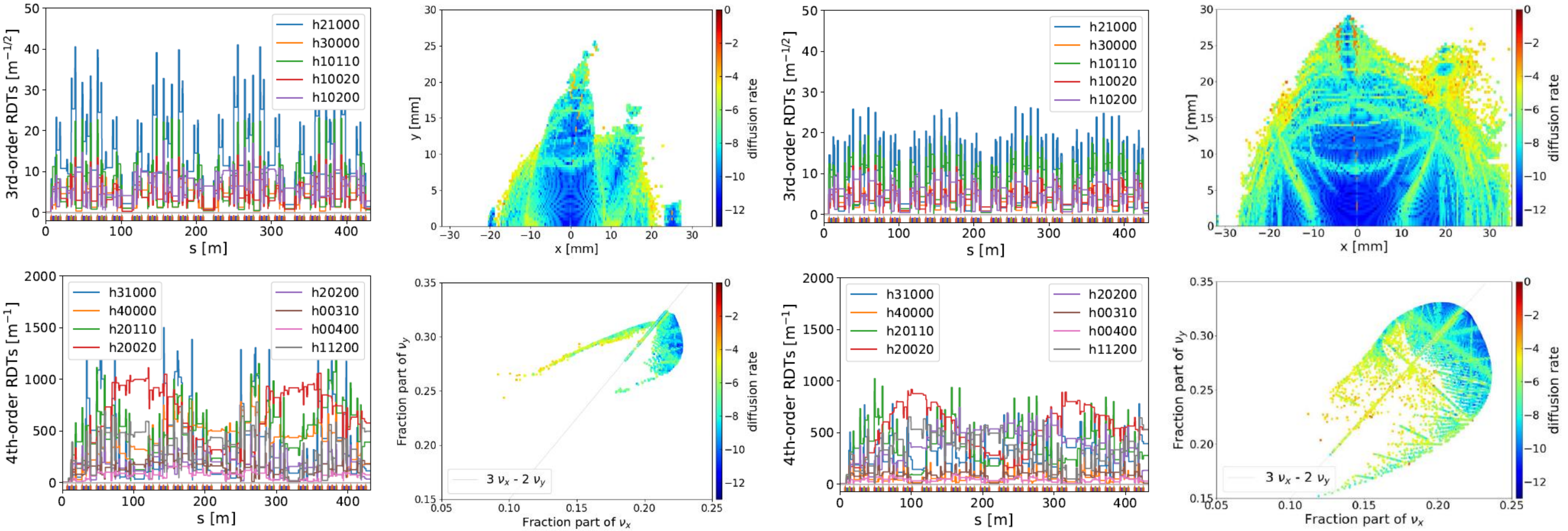




Physics of reducing RDT fluctuations

Minimizing the fluctuation of resonance driving terms

- An example of the SSRF lattice: the 5th-order structure resonance $3\nu_x - 2\nu_y$ is effectively suppressed by reducing 3rd- and 4th-order RDT fluctuations, and a larger DA is obtained with smaller RDT fluctuations.



Optimization algorithms for lattice design

- Powerful optimization algorithms for linear optics design and nonlinear dynamics optimization
  - Scanning method: e.g. GLASS
    - Giving a global and comprehensive understanding, but suitable for optimizations with ≤4 variables
  - Computational intelligence techniques: evolutionary algorithms & machine learning
    - Efficiently finding global optima & handling optimizations with large numbers of variables
    - Evolutionary algorithms: multi-objective genetic algorithms (MOGA), particle swarm optimization (PSO), differential evolution (DE)
- GLASS (GLobal scan of All Stable Settings) for linear optics design
  - (1) Scan ALL possible quadrupole settings; (2) find ALL stable settings; (3) compute properties of ALL stable settings; (4) filter by property those settings that may be of interest.
  - A database with all possible solutions and associated properties is obtained.

Optimization algorithms for lattice design

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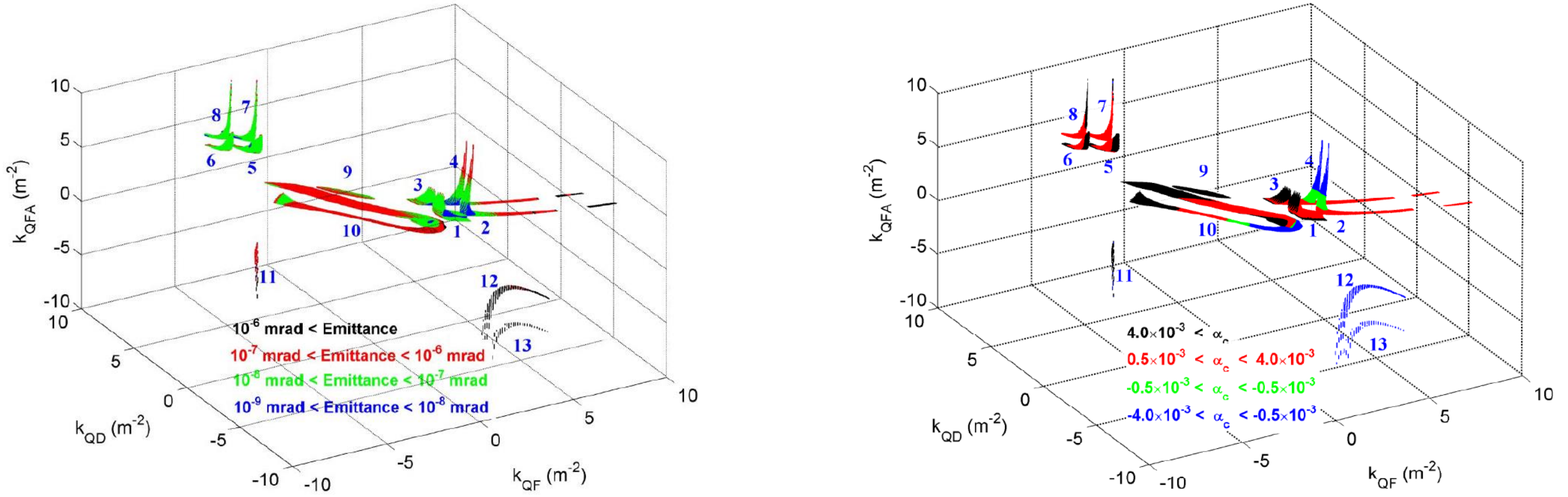
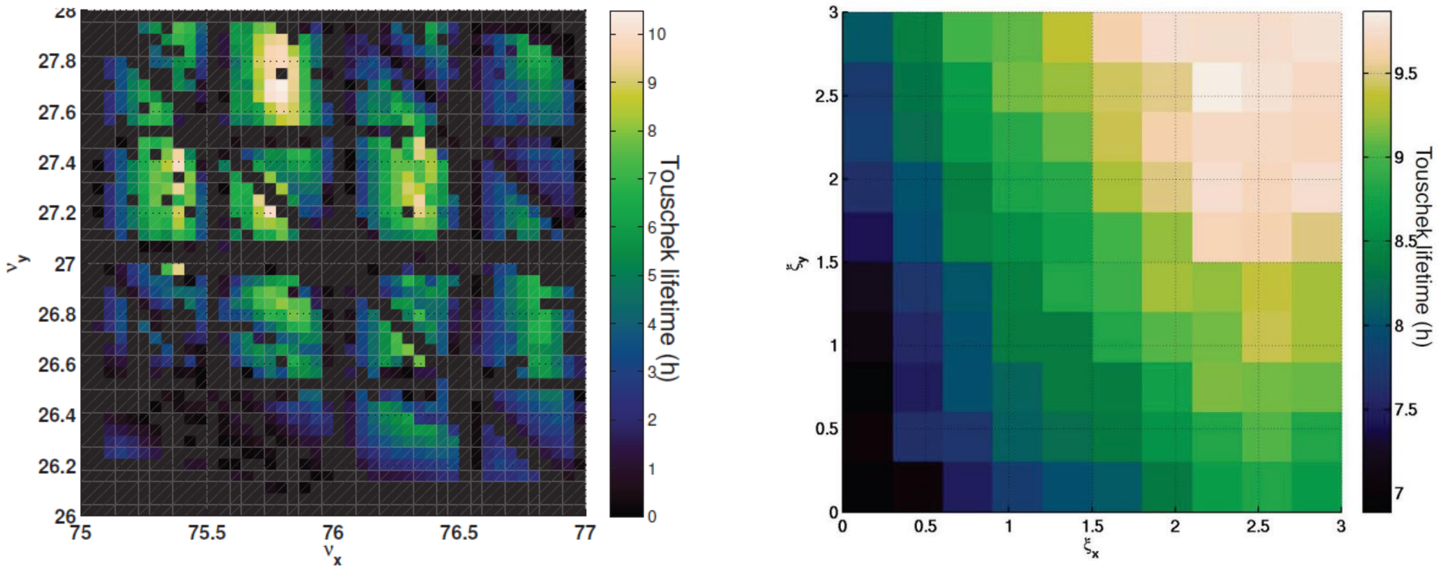


D. S Robin et al., PRST-AB, 2008

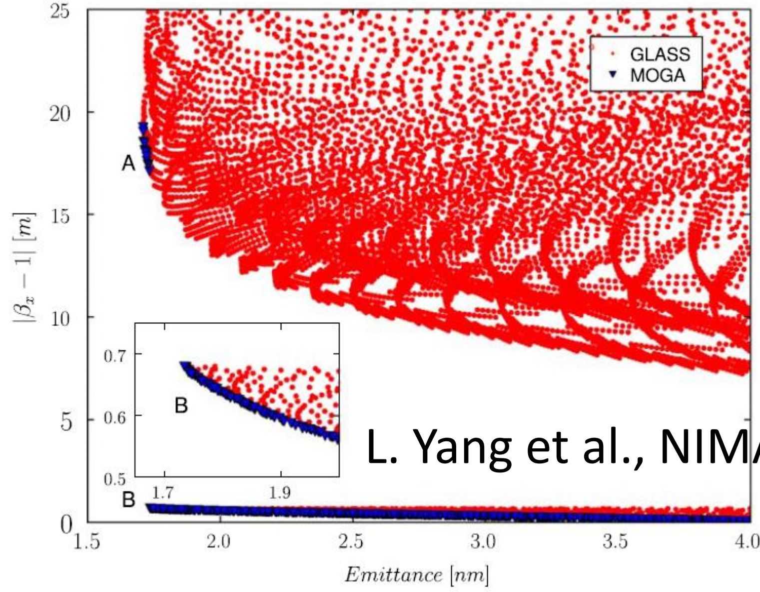
- Scanning method for nonlinear optimization
  - Nonlinear magnet strength scan
  - Working point and chromaticity scan



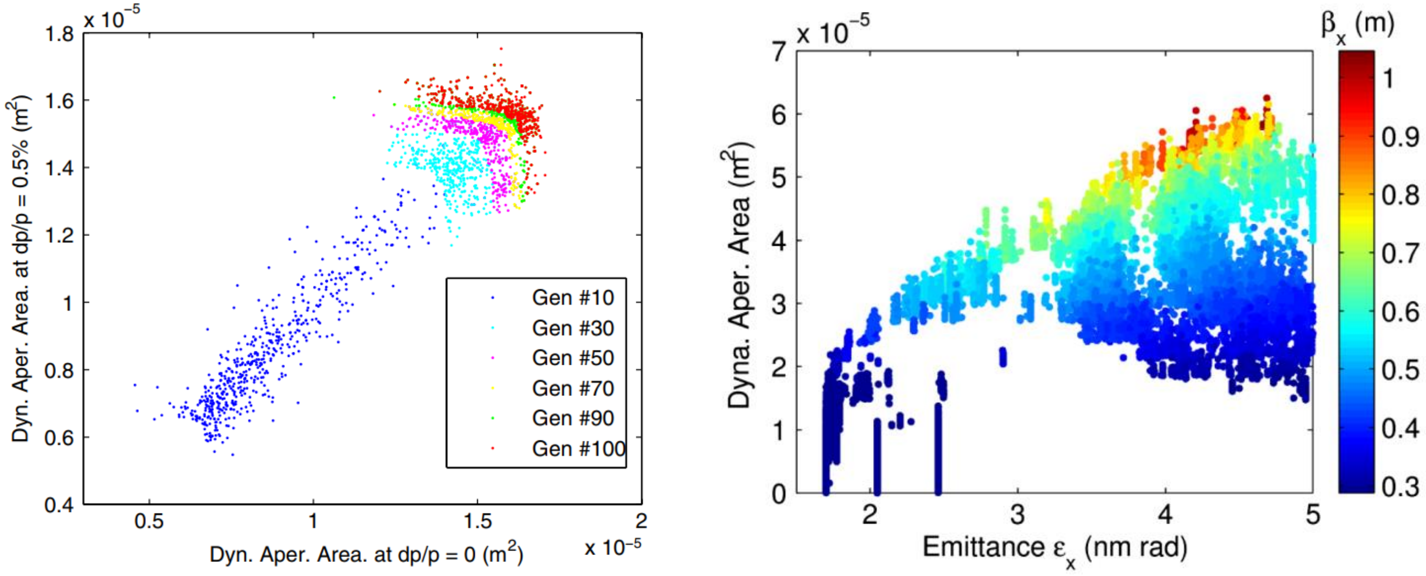
N. Carmignani et al., IPAC2015

Optimization algorithms for lattice design

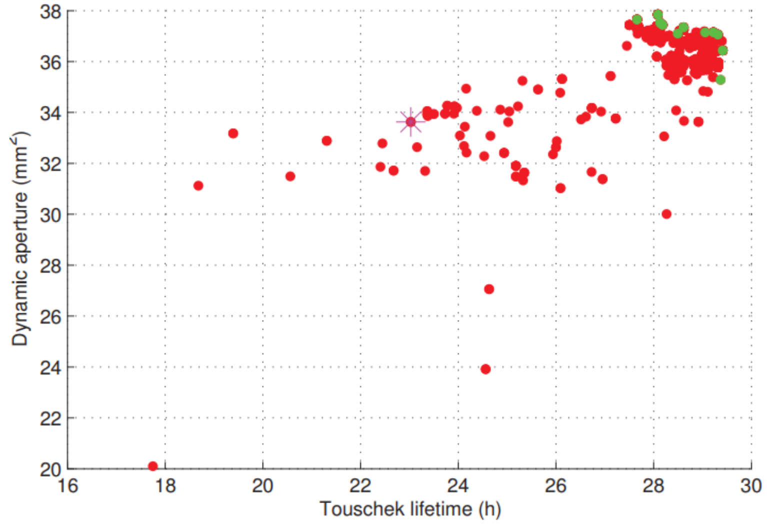
- Applying evolutionary algorithms in linear optics design and nonlinear optimization
  - Multi-objectives: natural emittance & beta function, on- and off-momentum DAs, DA & Touschek lifetime, natural emittance & DA
  - Pareto front: the set of all optimal trade-off solutions (non-dominated solutions)



L. Yang et al., NIMA, 2009



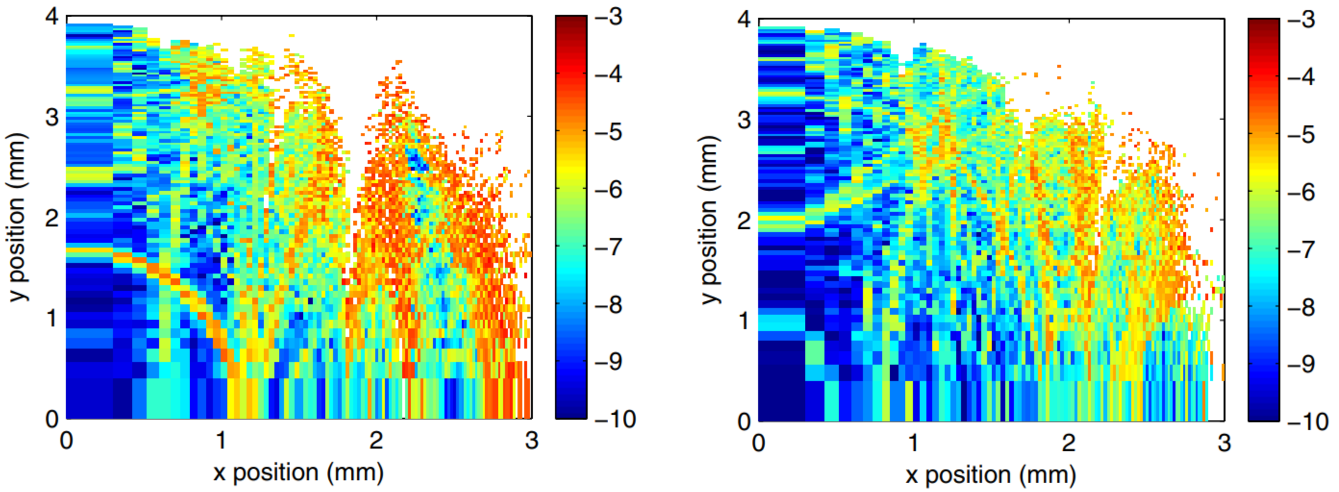
C. Sun et al., PRST-AB, 2012



N. Carmignani et al., IPAC2015

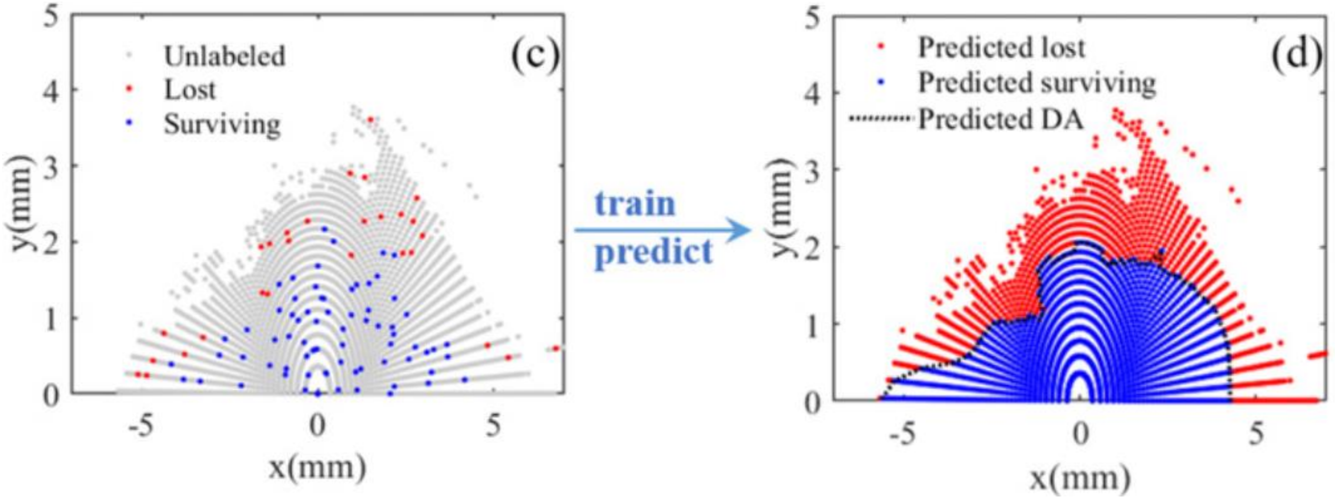
Optimization algorithms for lattice design

- DA optimization: using the total diffusion rate as objective yields higher-quality DA than using the DA area.



C. Sun et al., PRST-AB, 2012

- Application of machine learning
  - Enhancing the performance of evolutionary algorithms
  - Modeling complex physical processes, such as fast evaluation of DA
  - Discovering the underlying correlations among parameters

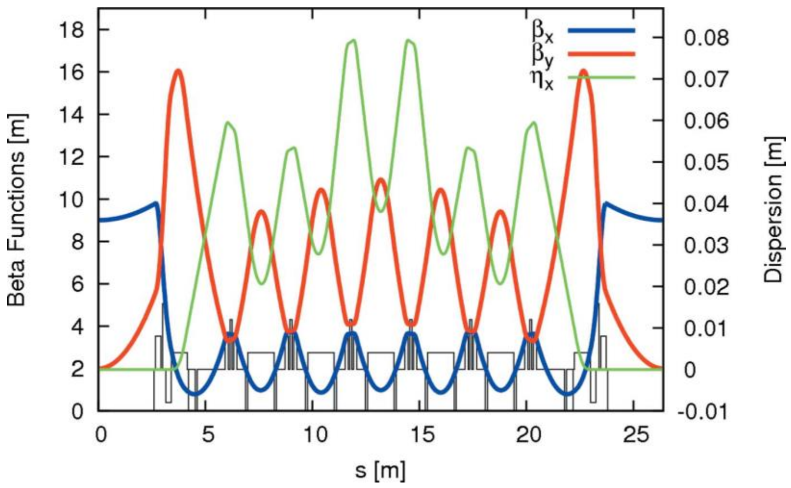


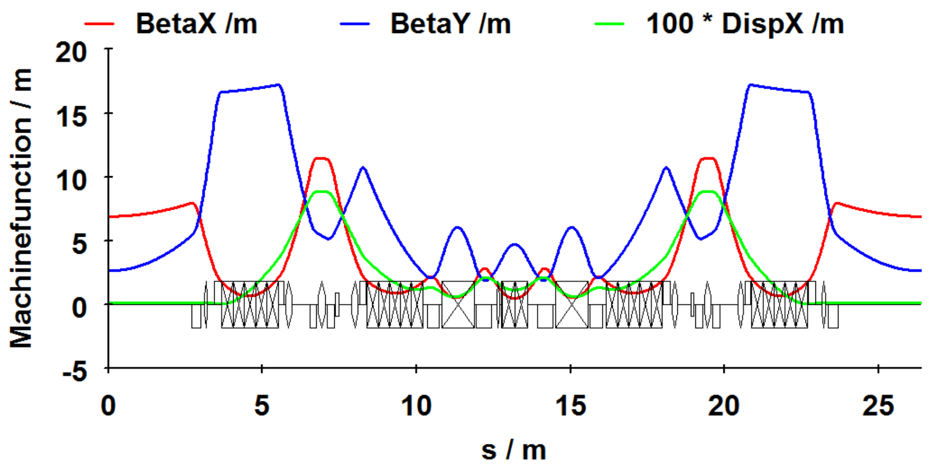
J. Wan, Y. Jiao, NJP, 2022

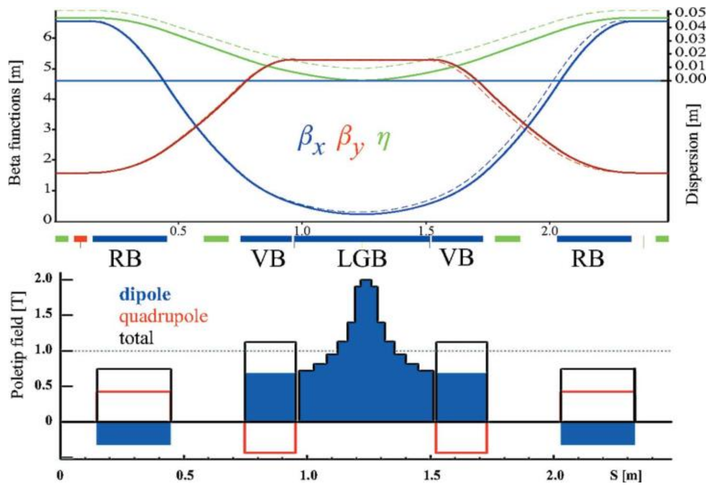
MBA lattice design

NSRL
National Synchrotron Radiation Laboratory

- Low-emittance lattice concepts
  - MAX IV: the first light source based on the MBA lattice concept
  - ESRF-EBS: developed the HMBA lattice concept
  - SLS 2.0: developed the LGB/RB unit lattice concept





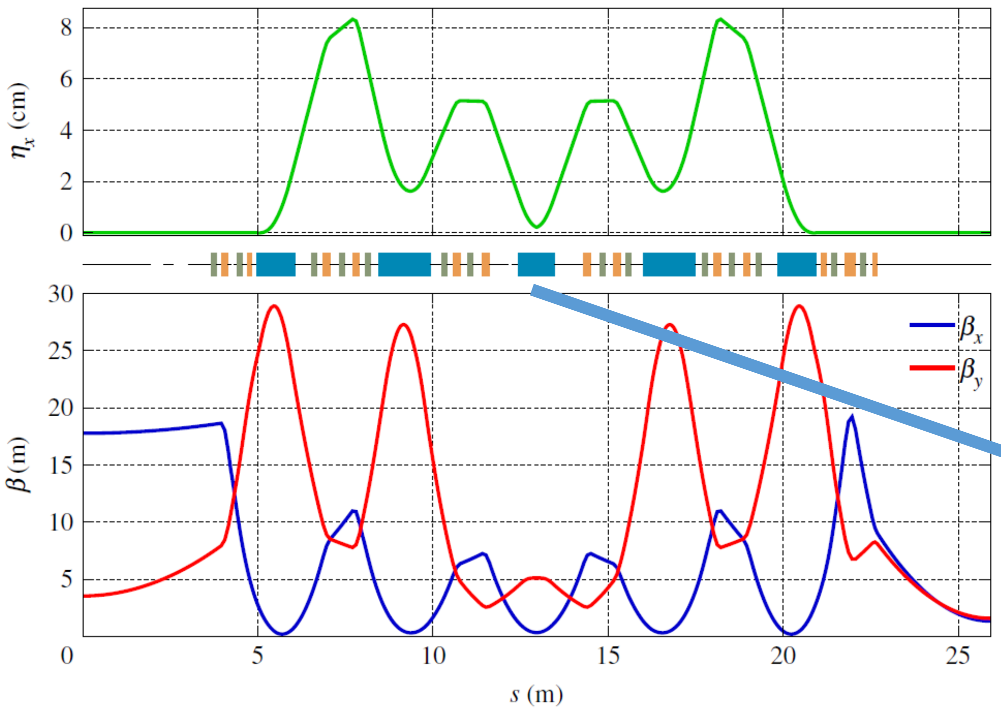


- These concepts have promoted the development of ultra-low-emittance synchrotron light sources:
  - Sirius, APS-U, HEPS, Korea-4GSR, SPring-8-II, PETRA IV, ALS-U, SKIF, SOLEIL II, NanoTerasu, Diamond-II, SPS-II, HALF, Elettra 2.0, etc.

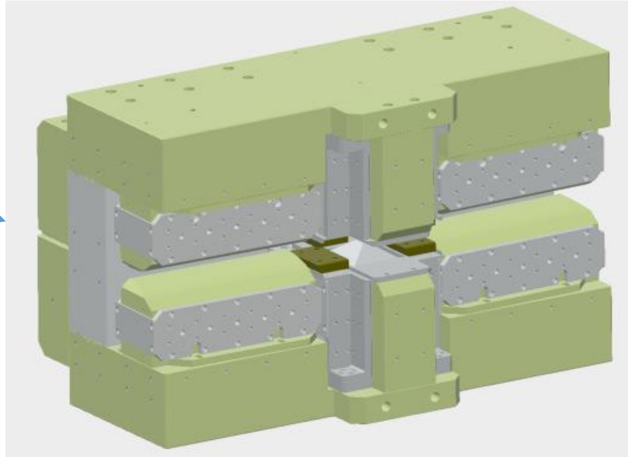
Sirius: 5BA lattice

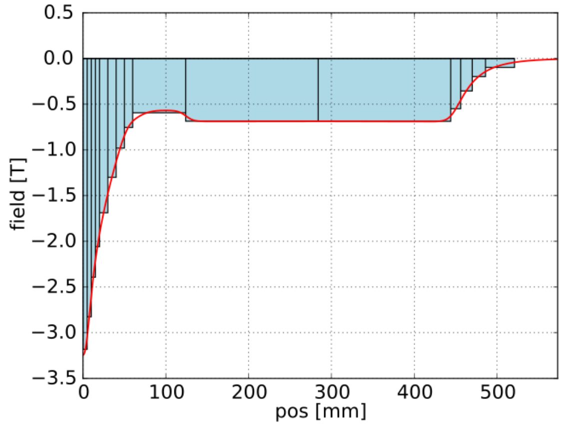
- 20 × 5BA with low-beta straights and superbends
  - 5 high-beta straight sections + 15 low-beta ones with βx ≈ βy ≈ 1.5 m
  - Low-beta straight sections: enhancing ID brightness, allowing for Delta type IDs, reducing ID effect
  - Central bend: superbend with 3.2 T for bend beamline

e^--Beam energy	3.0 GeV
Circumference	518.4 m
Lattice	20 x 5BA
Hor. emittance (bare lattice)	0.25 nm.rad
Hor. emittance (with undulators)	0.15 nm.rad
Betatron tunes (H/V)	49.11 / 14.17
Natural chrom. (H/V)	-119.0 / -81.2
Energy spread (rms)	0.85×10^{-3}
Energy loss/turn (dipoles)	473 keV
Damping times (H/V/L) [ms]	16.9 / 22.0 / 12.9
Nominal beam current (top up)	350 mA



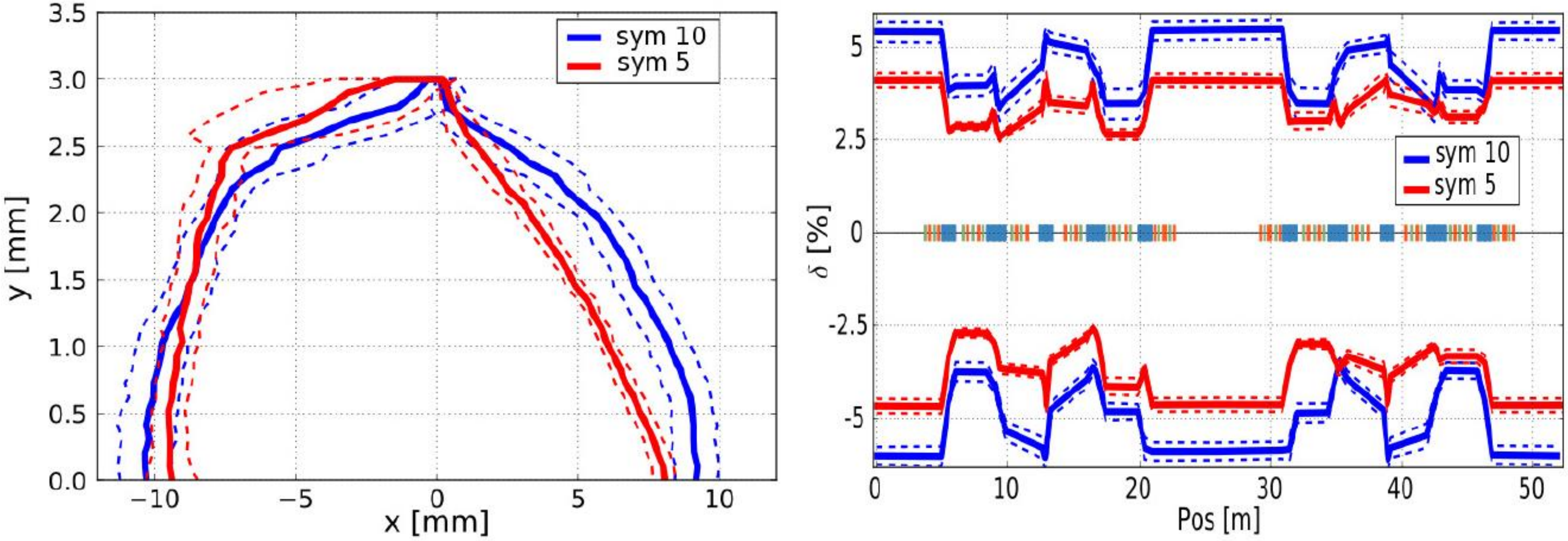
L. Liu et al., IPAC2016, IPAC2021





Sirius: 5BA lattice

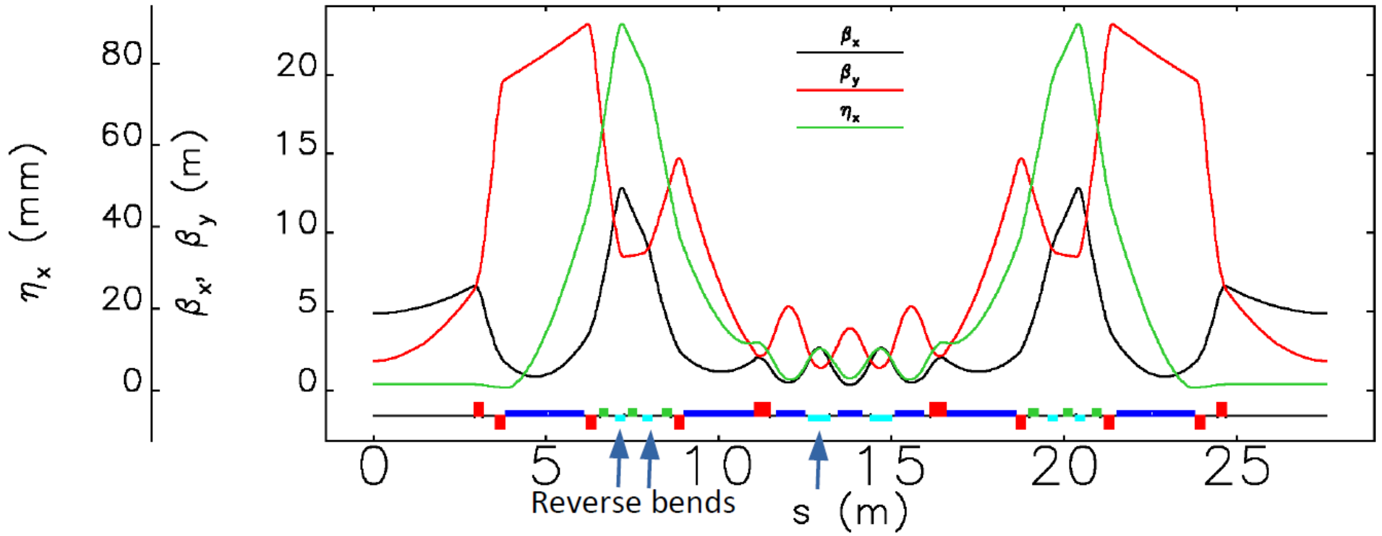
- Nonlinear dynamics performance (red curves in the figures)
  - To optimize the nonlinear dynamics, sextupoles were grouped into 21 families.
  - The optimization was performed using MOGA with chromaticities set to 2.5 in both transverse planes.



Simulations include all perturbations and the solid curve indicates the average over 20 random seeds.

APS-U: H7BA lattice

- 40×H7BA with RBs
  - Three families of RBs (horizontally offset quadrupoles) used for emittance reduction: one family close to the central bend, two in dispersion bumps
  - Natural emittance: 41.7 pm·rad @ 6 GeV
  - Full-coupling beam: working point on the linear difference resonance line

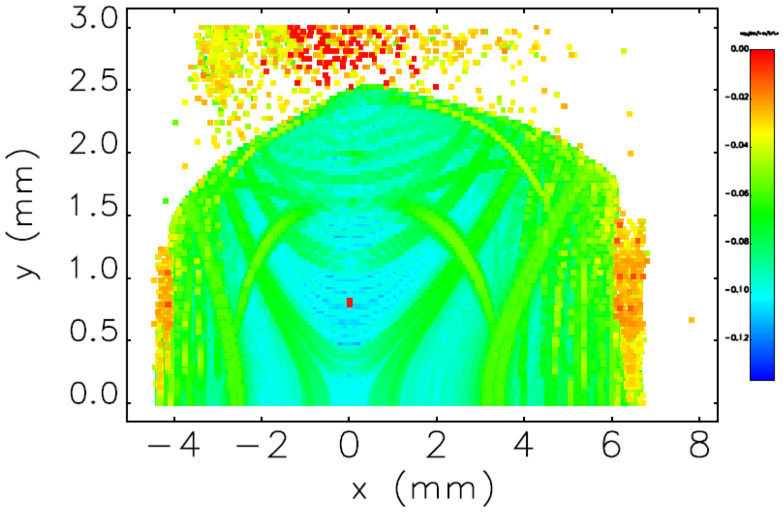


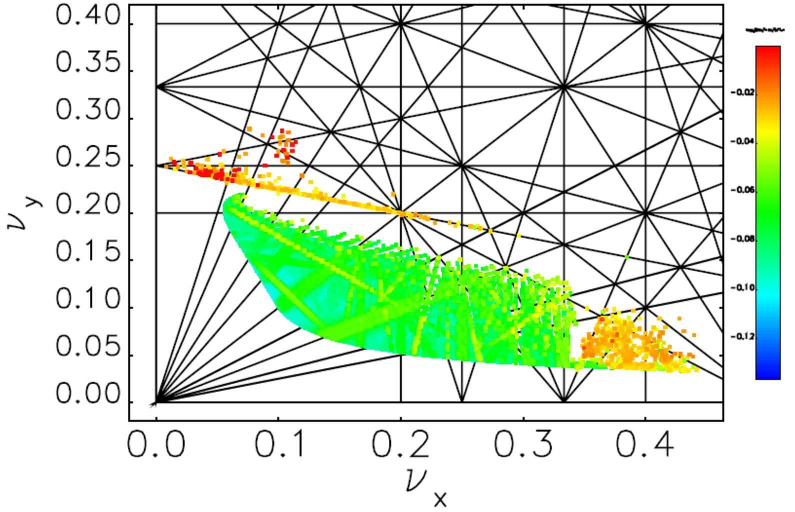
Tunes and Chromaticities		
ν_x	95.100	
ν_y	36.100	
ν_x/N_s	2.3775	per sector
ν_y/N_s	0.9025	per sector
ξ_x	8.02	
ξ_y	5.48	
Natural ξ_x	-133.50	
Natural ξ_y	-111.50	
Lattice functions		
Maximum β_x	13.01	m
Maximum β_y	20.59	m
Maximum η_x	0.090	m
Average β_x	3.76	m
Average β_y	8.42	m
Average η_x	0.031	m
Radiation-integral-related quantities at 6 GeV		
Natural emittance	41.68	pm
Energy spread	0.135	%
Horizontal damping time	6.85	ms
Vertical damping time	15.40	ms
Longitudinal damping time	20.51	ms
Energy loss per turn	2.87	MeV
ID Straight Sections		
β_x	5.19	m
β_y	2.40	m
η_x	0.39	mm
$\epsilon_{x,eff}$	41.7	pm
Miscellaneous parameters		
Momentum compaction	4.04×10^{-5}	
Circumference	1103.6083	
$\Delta f_{rf}/f_{rf}$	3.42×10^{-4}	
Damping partition J_x	2.25	
Damping partition J_y	1.00	
Damping partition J_δ	0.75	

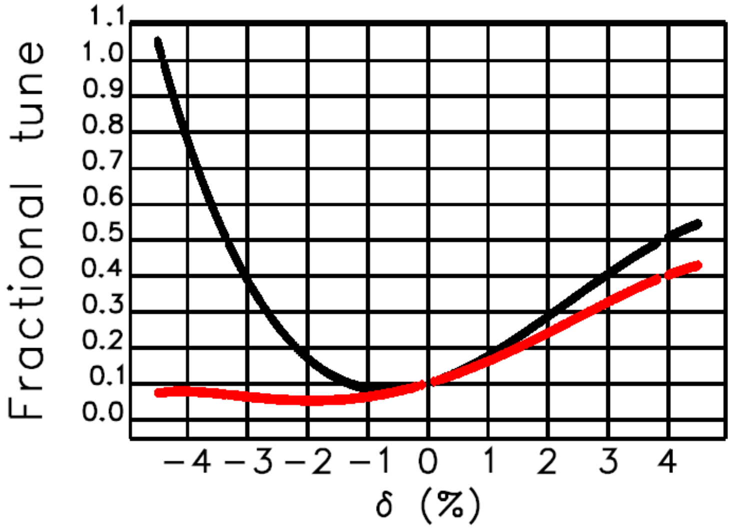
APS-U Final Design Report, 2019

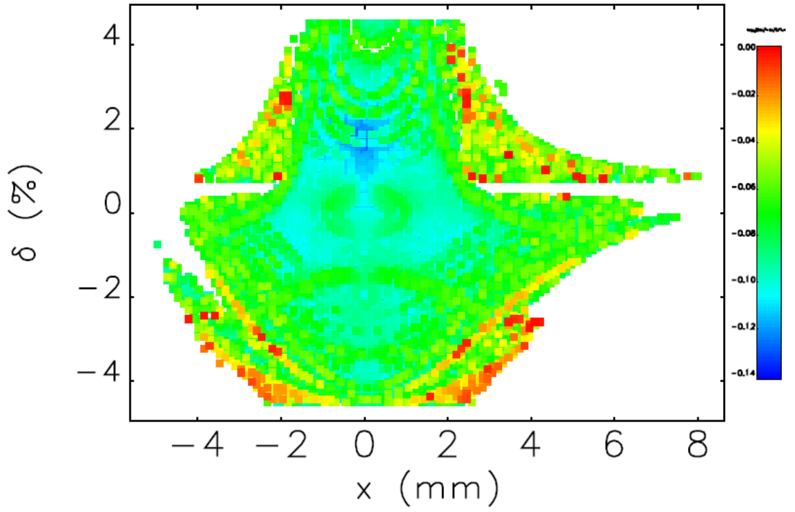
APS-U: H7BA lattice

- Nonlinear dynamics performance
  - Nonlinear cancellation: -I within a lattice cell + HOA over 8 lattice cells, note that cell tunes ≈ (2+3/8, 7/8)
  - Sextupoles grouped into 12 families: a two-sector translational symmetry





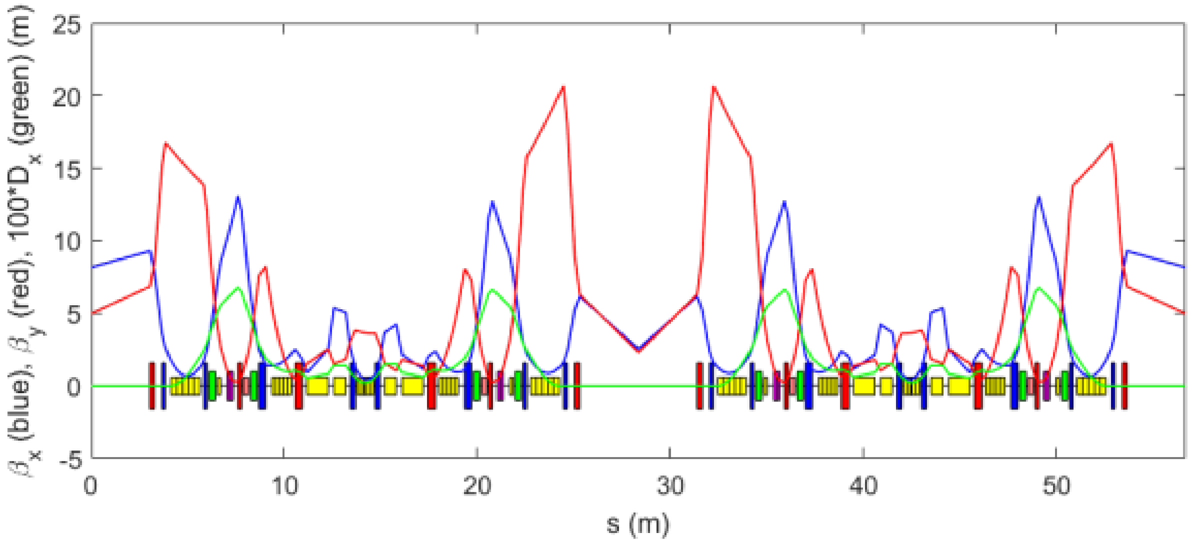


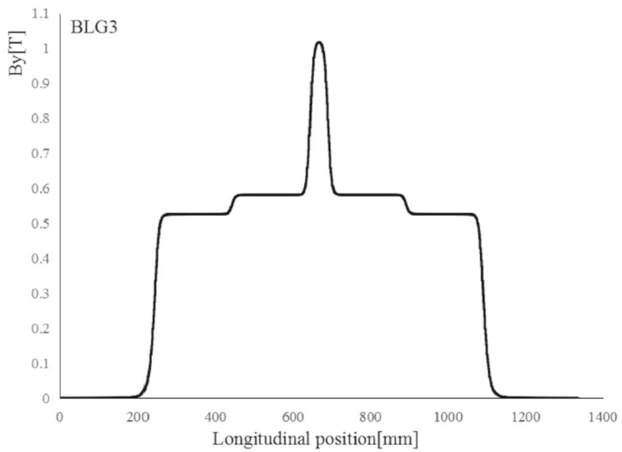


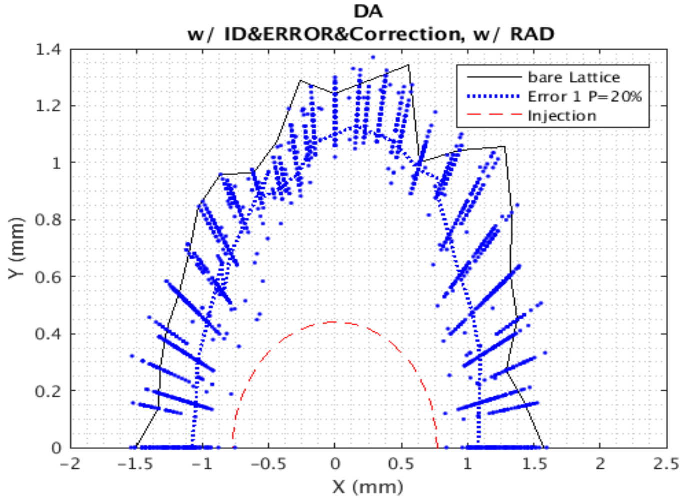
HEPS: H7BA lattice

- 48×H7BA
  - 24 super-periods with high- and low-beta straight sections (higher brightness of IDs in low-beta straights)
  - LGBs + CBs + RBs, central LGB with 1 T for bend beamline
  - Natural emittance: 34.8 pm·rad @ 6 GeV

Parameters	Units	Bare lattice	14-ID lattice
Circumference	m	1360.4	
Energy	GeV	6	
Natural emittance	pm	34.82	27.75
Coupling factor	–	10%	
Betatron tune (ν_x/ν_y)	–	115.15/104.29	
Corrected chromaticity (ξ_x/ξ_y)	–	4.95/5.01	4.84/5.08
Damping partition numbers ($J_x/J_y/J_E$)	–	1.90/1/1.10	1.60/1/1.40
Radiation damping time ($\tau_x/\tau_y/\tau_\delta$)	ms	10.86/20.62/18.71	8.59/13.72/9.79
Radiation loss per turn	MeV	2.64	3.97
Equilibrium energy spread	–	1.02e-3	1.06e-3
Momentum compaction factor	–	1.83e-5	
Main RF frequency (f_{RF})	MHz	166.6	
Harmonic number of main RF	–	756	
Harmonic cavity frequency (f_{HC})	MHz	499.8	
Harmonic ratio (f_{HC}/f_{RF})	–	3	
Bucket height	–	4%	





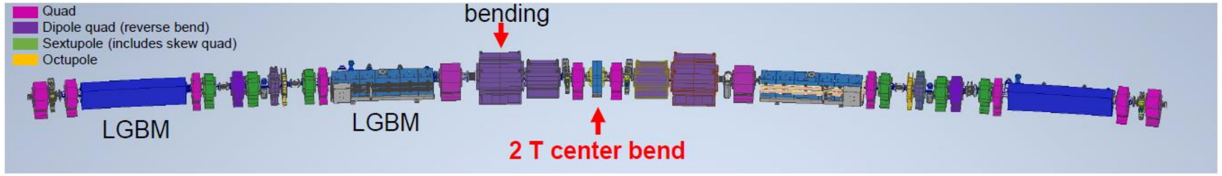


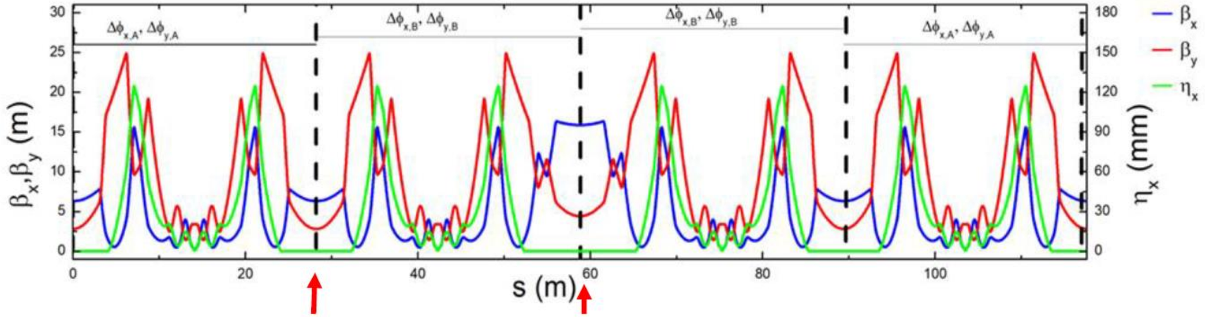
Y. Jiao et al., IPAC2021

Korea-4GSR: H7BA lattice

- 28×H7BA
  - LGBs + CBs + RBs + central bend with 2 T for bend beamline
  - Nonlinear cancellation: -I within a lattice cell + HOA over 7 lattice cells, note that cell tunes ≈ (2+3/7, 6/7)
  - High-beta straight section for off-axis injection

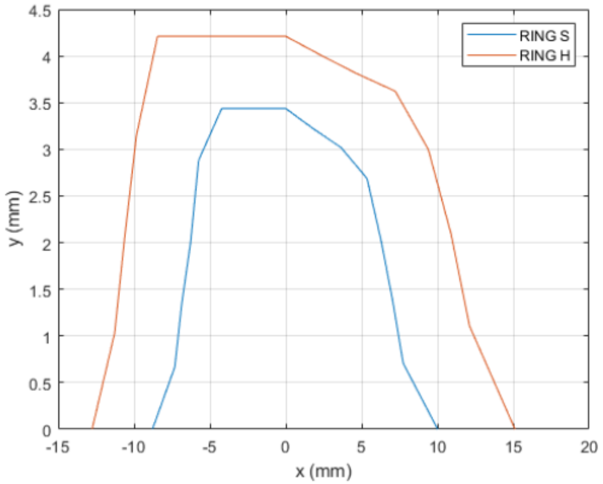
Parameters	Value
Energy (GeV)	**4.0**
Circumference (m)	**799.297**
Emittance (pm)	**62**
Tunes (H,V)	**68.18, 23.26**
Natural chromaticity (H,V)	**-112.1, -85.3**
Chromaticity (corrected) (H,V)	**5.8 , 3.5**
Hor. Damping partition	**1.84**
Momentum compaction	**0.78×10^{-4}**
Energy spread (σ_δ)	**1.26×10^{-3}**
Energy loss per turn (MeV)	**1.097**
Main RF voltage (MV)	**3.5**
Beam current (mA)	**400**
Bunch length (σ_z) (mm) (w/o HC, w/ HC)	**3.66 / 14.66**

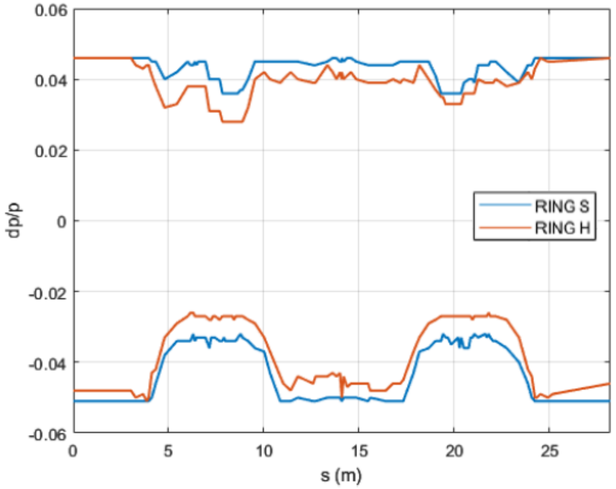




Beta functions at the center of ID SS:
(β_x, β_y) = (6.33 m, 2.84 m)

Beta functions at the center of High-beta SS:
(β_x, β_y) = (15.90 m, 4.45 m)



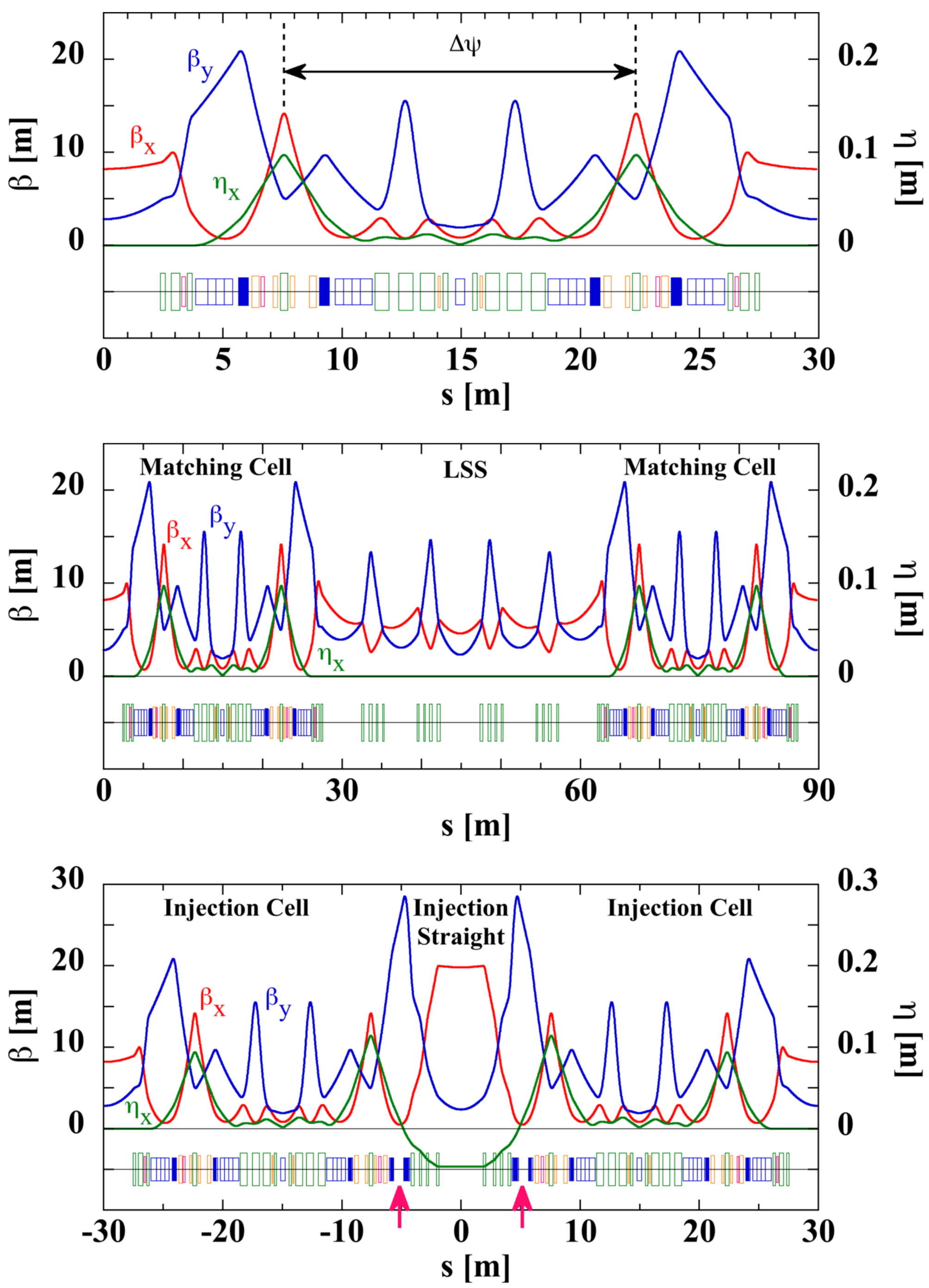


Jimin Seok, LER2024

SPring-8-II: 5BA lattice

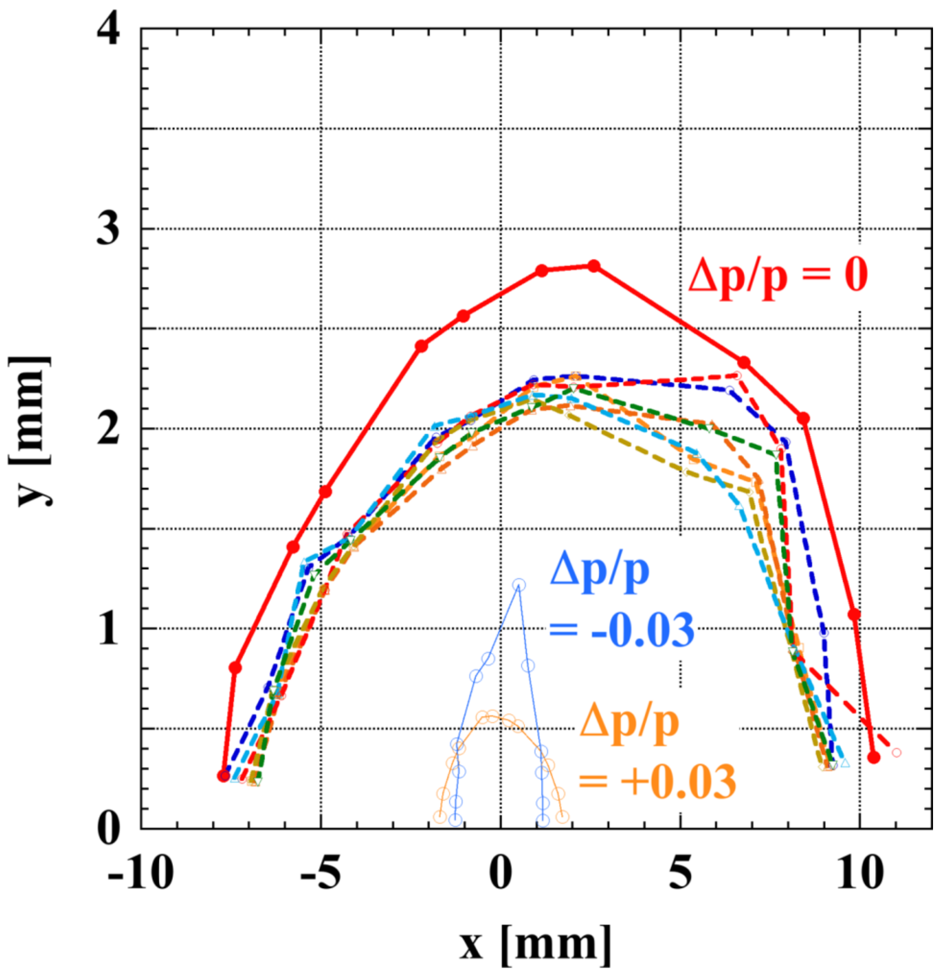
- 44×5BA with damping wigglers
  - Normal lattice cell: two dispersion bumps following HMBA, with phase advances of (1.488, 0.495)×2π (small deviations from –I for better nonlinear dynamics)
  - Four long straight sections for damping wigglers
    - Optics designed to avoid the generation of large natural chromaticities
    - Adjusting phase advances to 2π over the long straight section to keep the relative phase relation among arcs unchanged
  - Damping wigglers reduce the emittance from 111 pm·rad to 50 pm·rad @ 6 GeV
  - High-beta straight for off-axis injection

H. Tanaka et al., JSR, 2024



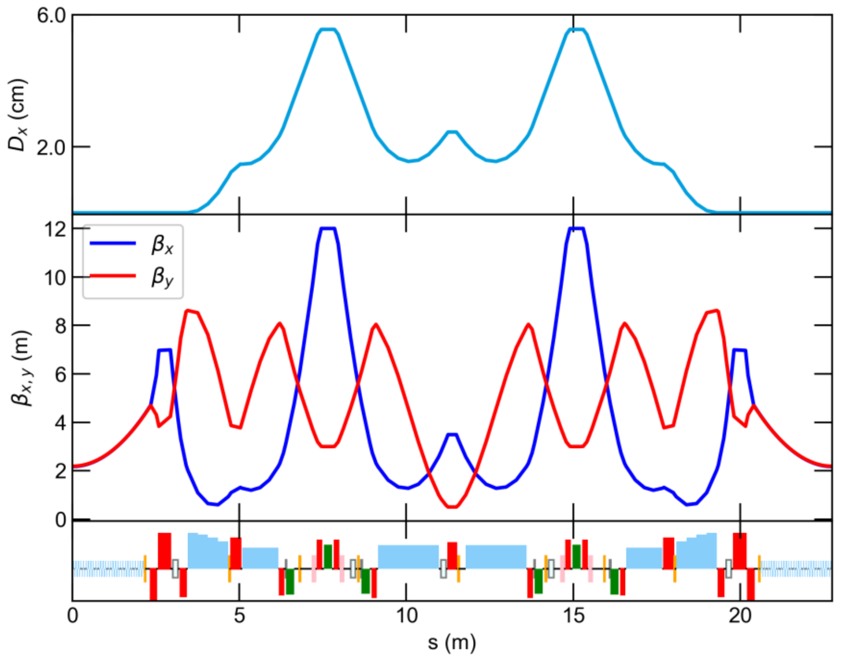
SPring-8-II: 5BA lattice

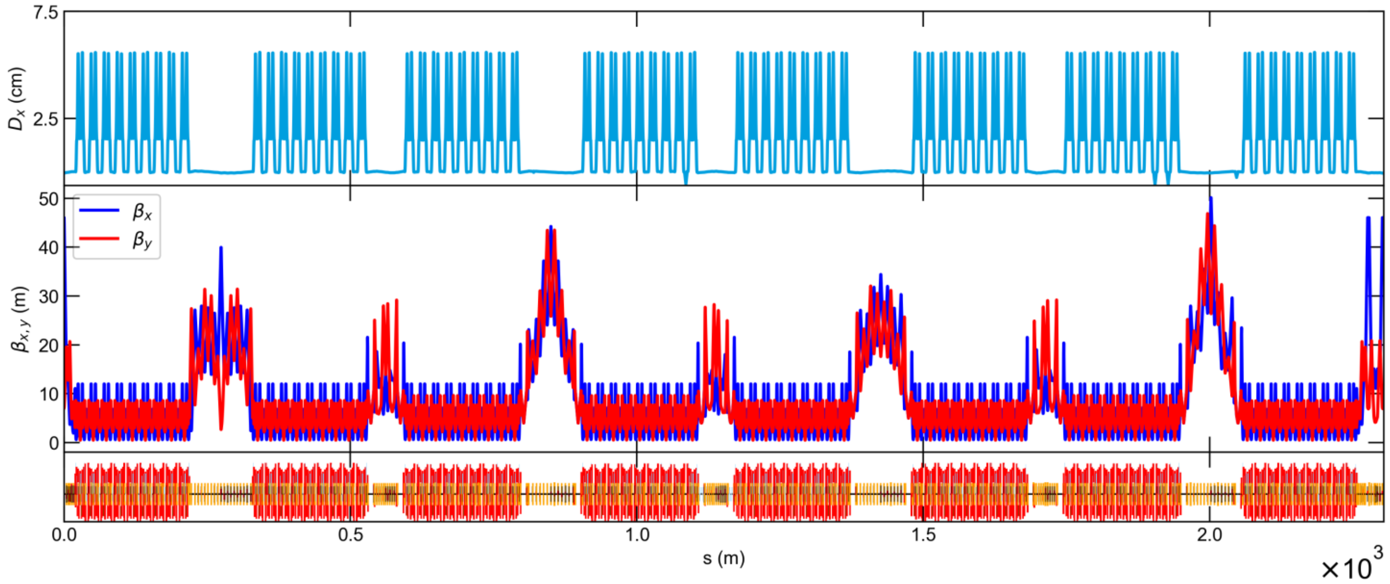
Parameter	SPring-8-II (new)	SPring-8 (present)
Lattice type	Five-bend	Double-bend
Energy (GeV)	6	8
Circumference (m)	1435.428	1435.949
Stored current (mA)	200	100
Emittance (pm rad)	50 with DWs (111 for bare lattice)	2400
Betatron function (m) at ID straight	8.2 / 2.8	31.2 / 5.0
Dispersion function (m) at ID straight	0.0	0.146
Tune ν_x / ν_y	108.10 / 42.58	41.14 / 19.35
Natural chromaticity ξ_x / ξ_y	−153 / −151	−117 / −47
Momentum compaction factor	4.13×10^{-5}	1.60×10^{-4}
Relative energy spread (%)	0.098	0.109
Damping partition number J_x / J_y / J_s	1.38 / 1.0 / 1.62	1.0 / 1.0 / 2.0
Radiation loss by dipoles (MeV per turn)	2.6	8.9
Radiation loss by damping wigglers (MeV per turn per long-straight)	0.5	



PETRA IV: H6BA lattice

- 72 × H6BA with damping wigglers (1)
  - Feature: two bumps with -I of (0.5, 0.5) × 2π, matching section with two bends, low beta straight sections
  - Damping wigglers reduce the emittance from 43 pm·rad to 20 pm·rad @ 6 GeV
  - Ultra-large ring circumference + less aggressive optics + more damping wigglers: better nonlinear dynamics and larger momentum compaction factor



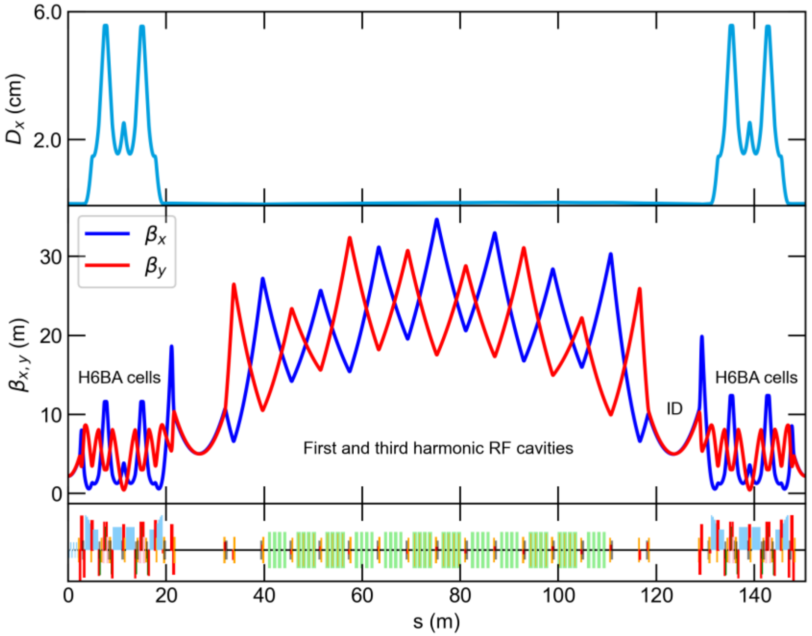


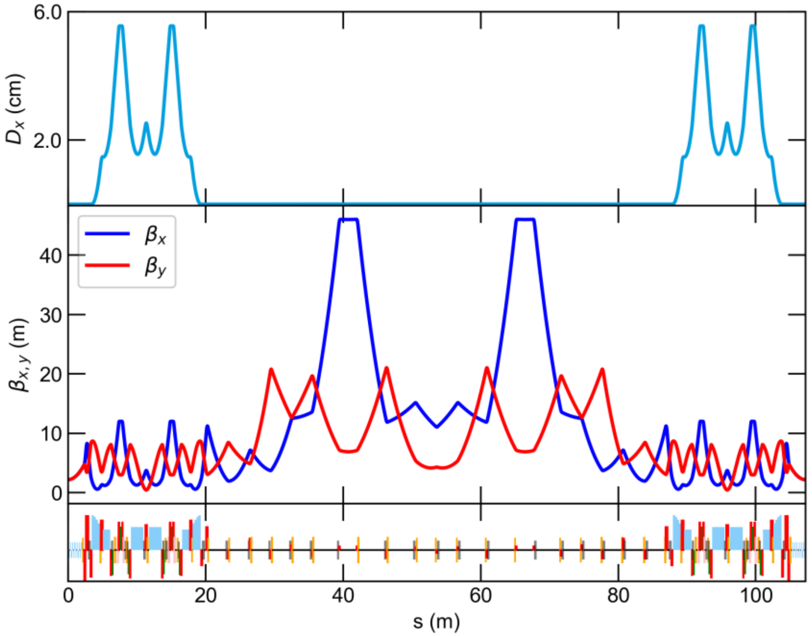
Parameter	Value
Tunes Q_x,Q_y	164.18, 68.27
Natural chromaticity ξ_x,ξ_y	-230, -196
Corrected chromaticity ξ_x,ξ_y	6, 6
Momentum compaction factor α_C	3.3×10^{-5}
Standard ID space	4.9 m
$\beta_{x,y}$ at ID, standard cell	2.2 m, 2.2 m
$\beta_{x,y}$ at ID, flagship IDs	4 m, 4 m
Nat. hor. emittance ε_x no DW, zero current	43 pm rad
Nat. hor. emittance ε_x with DW, zero current	20 pm rad
Rel. energy spread δ_E no DW, zero current	0.7×10^{-3}
Rel. energy spread δ_E with DW, zero current	0.9×10^{-3}
Energy loss per turn, lattice no DW	1.3 MeV
Energy loss per turn, lattice with DW	4.0 MeV

I. Agapov et al., arXiv:2408.07995v1, 2024

PETRA IV: H6BA lattice

- 72×H6BA with damping wigglers (2)
  - Eight very long straight sections for flagship IDs (10-m long, high flux), injection, RF cavities, collimation, etc.
  - On-momentum periodicity: setting the phase advances of long straight sections to 2π in both planes to have the ring with a periodicity of 72 for on-momentum particles
  - Reducing off-momentum effect: control of chromatic functions and chromaticities of straight sections





PETRA IV: H6BA lattice

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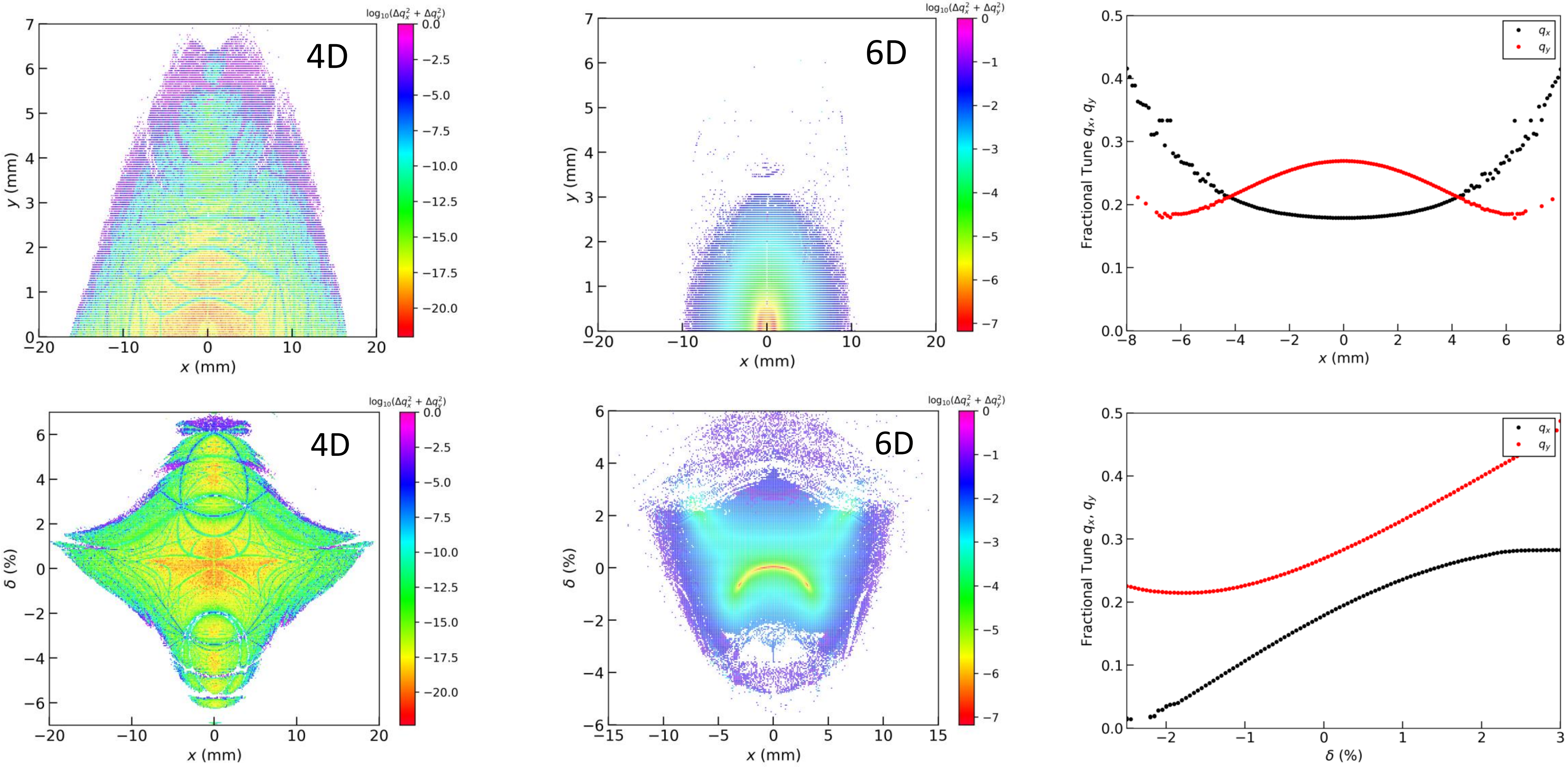
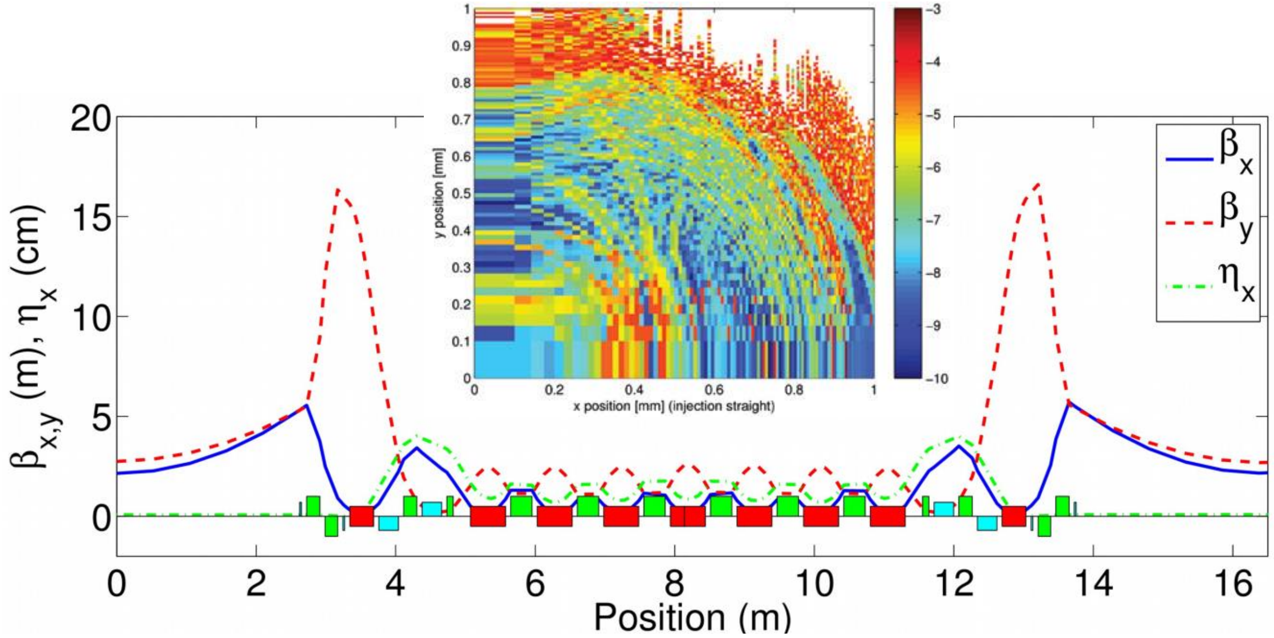
- Nonlinear dynamics performance

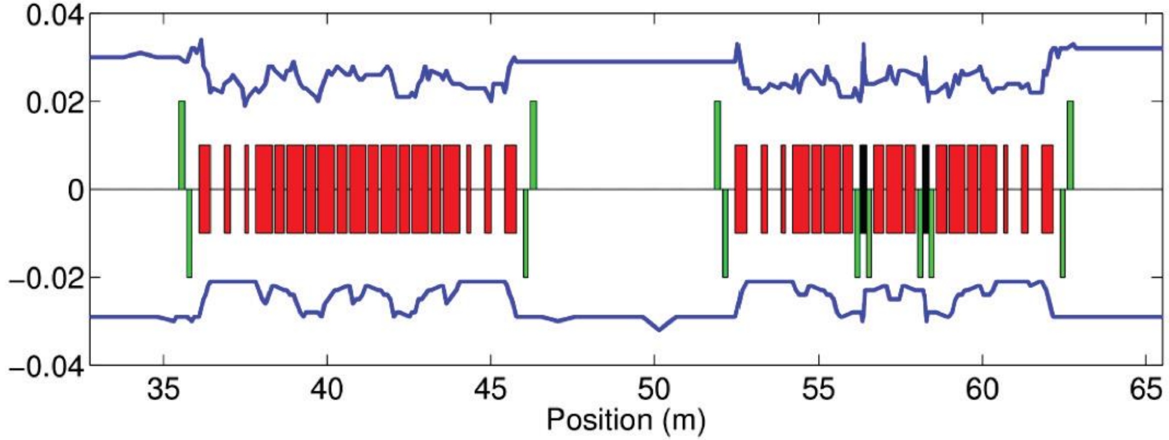


ALS-U: 9BA lattice

- 12×9BA
  - Two dispersion bumps following HMBA
  - CBs + RBs + superbends with 5 T field
  - Lower natural emittance: 108 pm·rad @ 2 GeV
  - Horizontal tune of lattice cell: near -I
  - DA of ~1 mm & lifetime of ~1 h (full-coupling beam + harmonic cavity)

Electron energy, E	2.0 eV
Circumference, C	196.51 m
Tune, ν_x/ν_y	41.358/20.353
Natural chromaticity, ξ_{0x}/ξ_{0y}	−64.3/ − 64.8
Chromaticity during operation, ξ_x/ξ_y	2/1
Momentum compaction, α_c	2.025×10^{-4}
Bunch charge, Q	1.15 nC
Natural rms emittance, ϵ_{x0}	108 pm rad
Natural rms energy spread, σ_δ	1.02×10^{-3}
Radiation energy loss/turn (no IDs), U_0	245 keV
Damping times, $\tau_x/\tau_y/\tau_z$	5.56/10.7/9.97 ms
Harmonic number, h	328
Main rf cavity frequency	500.390 MHz
Main rf cavity voltage	0.6 MV
Synchrotron tune (w/o 3HC, no IDs), ν_s	1.6×10^{-3}

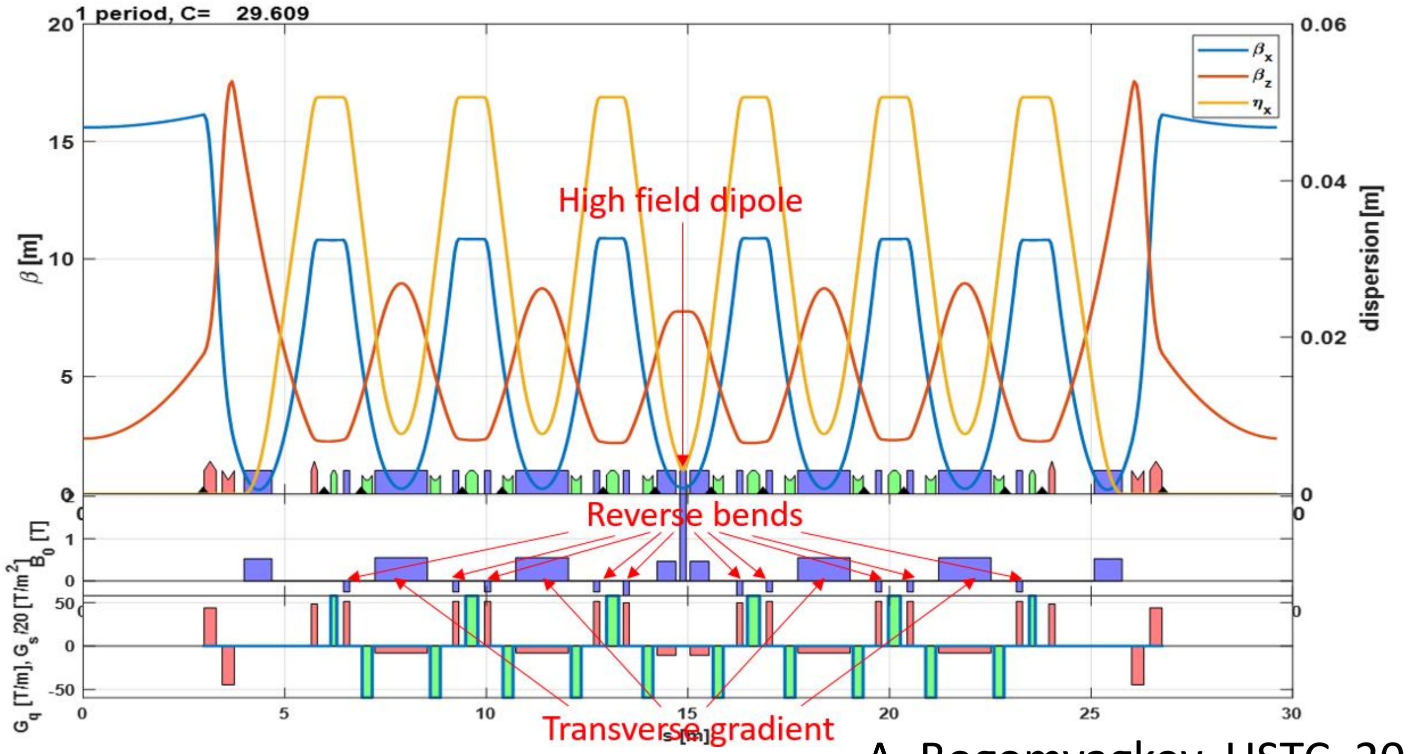




C. Steier et al., IPAC2018

SKIF: 7BA lattice

- 16×7BA : similar to the 7BA lattice of SLS 2.0, but very different in the matching section
  - CB+RB, quadrupole doublet in the matching section
  - Only two families of chromatic sextupoles used, no sextupole in the matching section



A. Bogomyagkov, USTC, 2024

Energy, GeV	3
Symmetry	16
Circumference, m	476.14
Revolution period, μs	1.588
Hor. Emittance, pm	73.2
Energy spread	1×10^{-3}
Energy loss per turn, keV	536
Betatron tunes, x/y	50.806/18.84
Momentum compaction factor	7.64×10^{-5}
Natural chromaticity, x/y	-149/-55
RF harmonic number	567
RF frequency, MHz	357
RF amplitude, MV	0.77
Energy acceptance	$\pm 3\%$
Synchrotron tune	1.13×10^{-3}
Bunch length, mm	5.3
Partitions, x/δ	1.94/1.06
Damping times, x/δ, ms	9.2/16.7

SKIF: 7BA lattice

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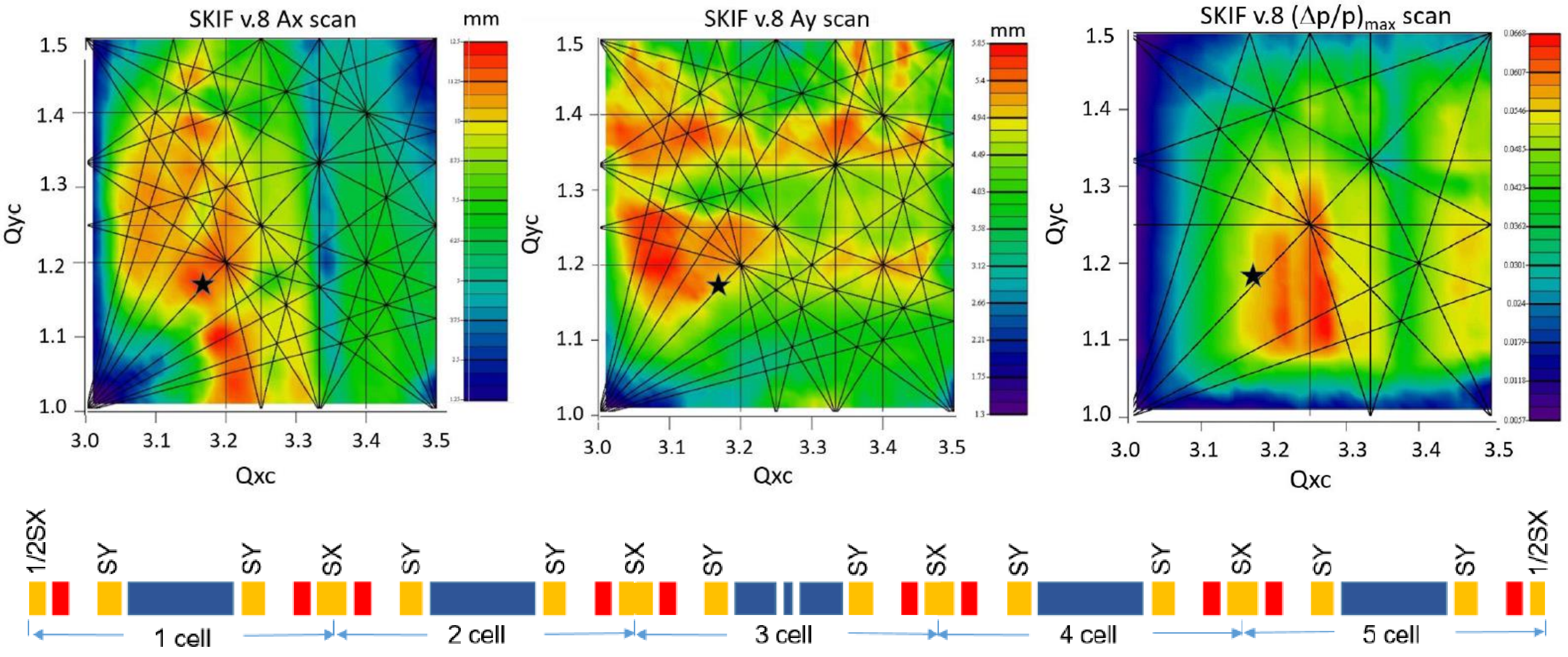
- Betatron tunes, phase advances between chromatic sextupoles, and their strengths were carefully adjusted to maximize DA and MA by scanning.



SKIF: 7BA lattice

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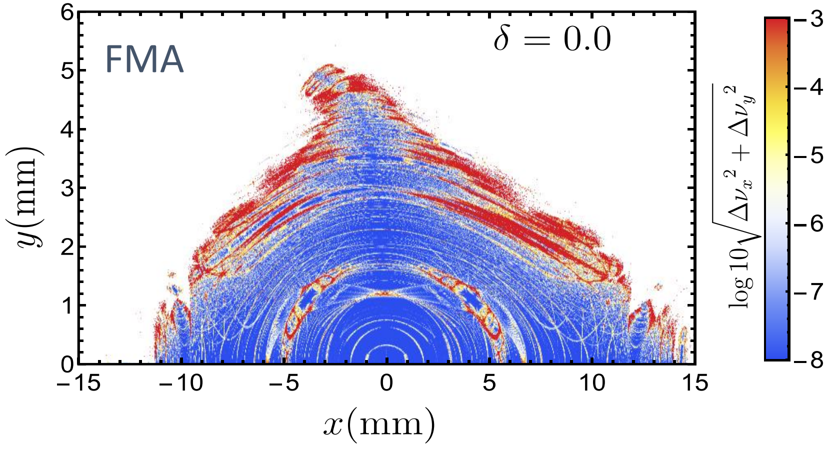
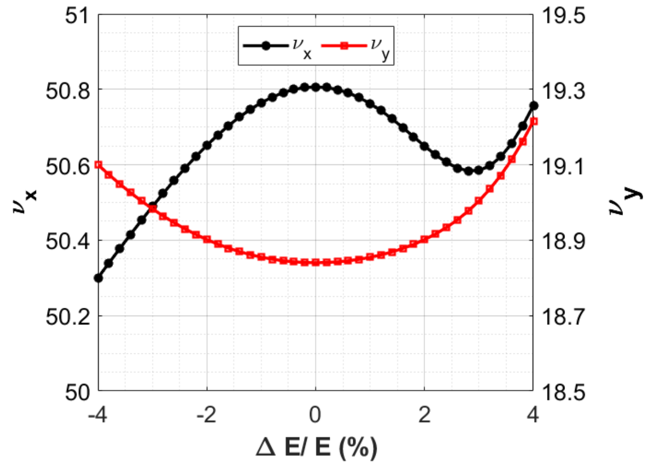
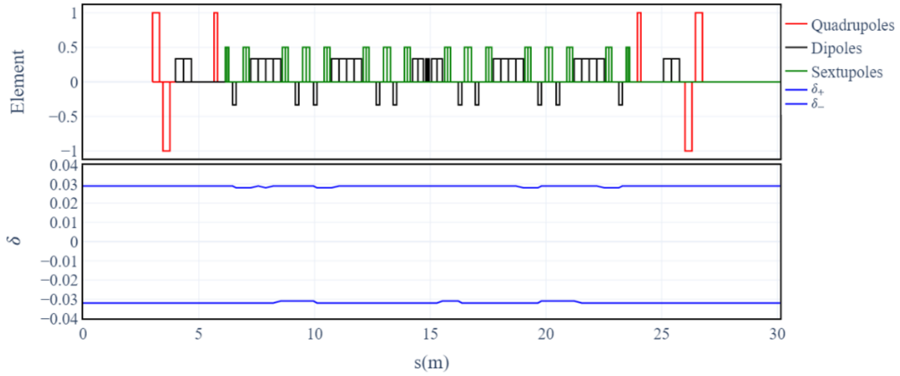
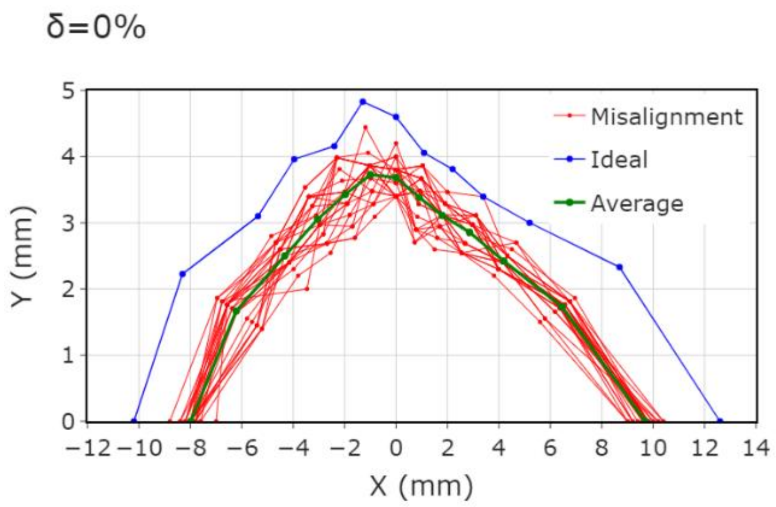
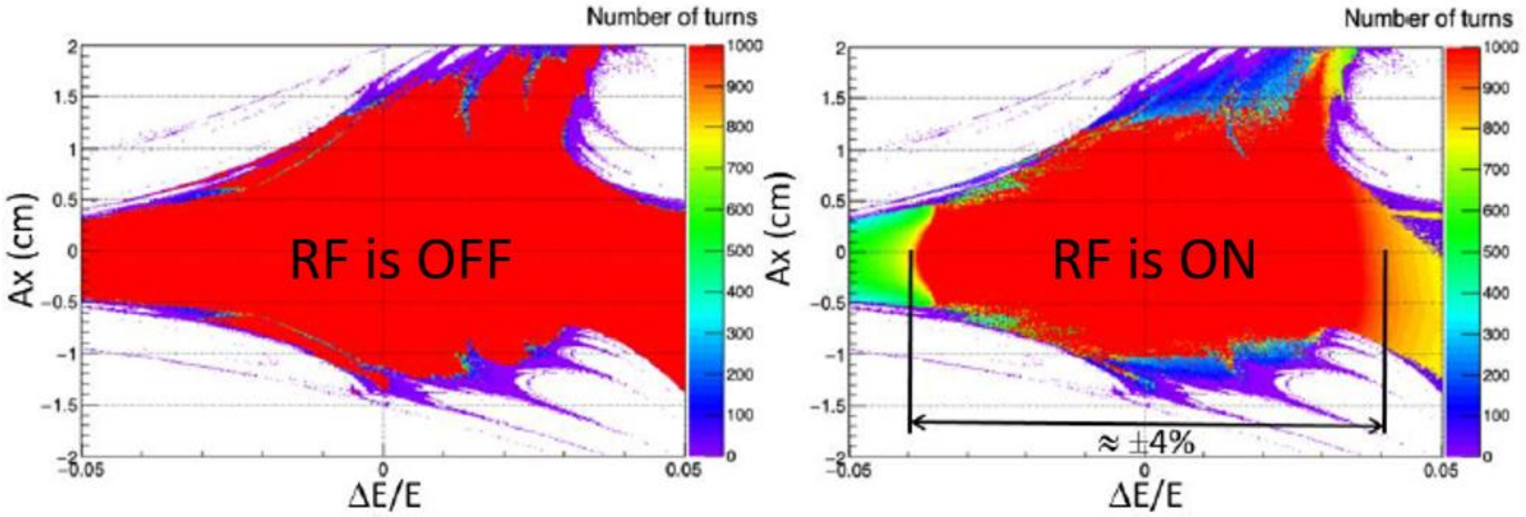
- Nonlinear dynamics performance







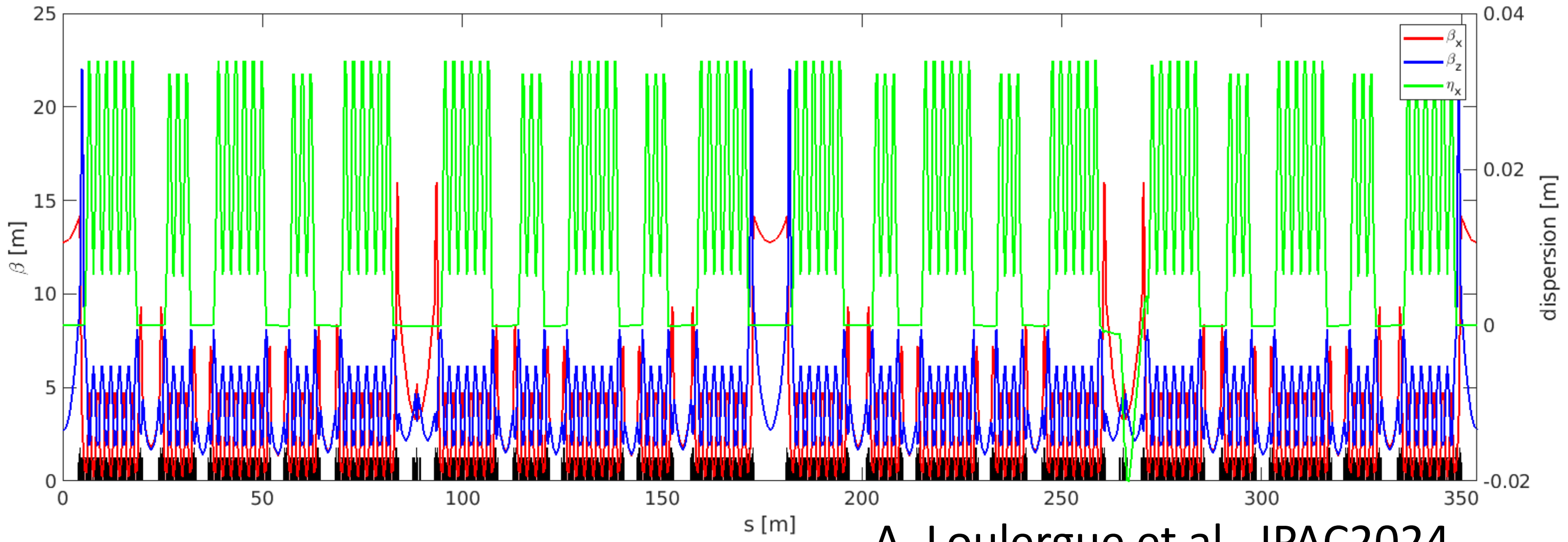




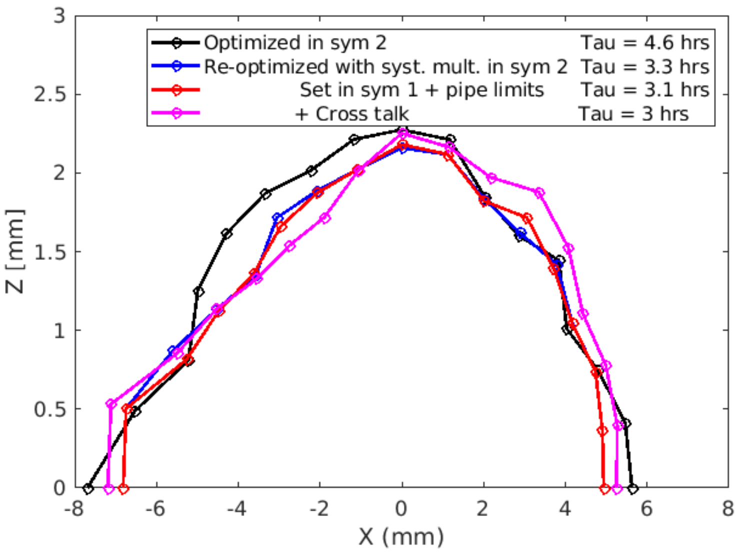
SOLEIL II: 7BA-4BA lattice

- 20 alternating 7BA and 4BA cells
  - The natural solution to best fit the current beamline positioning and leave the tunnel shielding wall unchanged
  - Lower natural emittance: 85 pm·rad @ 2.75 GeV
  - Short and medium straight sections: βx ≈ βy ≈ 1.5 m
  - Very strong magnets (sextupole B''=16000 T/m^2)

	Present	SOLEIL II
H-Emittance (2.75 GeV)	4 nm.rad	85 pm.rad
Circumference	354.10 m	353.96 m
Straight section number	24	20
Long straight length	12.00 m	8.07 / 9.00 m
Medium straight length	7.00 m	3.71 / 4.21 m
Short straight length	3.80 m	3.14 m
Straight length ratio	46 %	25 %
Betatron tunes H/V	18.16 / 10.2	54.2 / 18.3
Mom. comp. factor	$4.18 10^{-4}$	$9.9\ 10^{-5}$
RMS energy spread	0.102 %	0.095 %
Energy loss per turn	917 keV	477 keV
Damping times s/x/z (ms)	3.3/3.3/6.6	7.4 /13.5 /12.1
RMS Nat. bunch length	15.2 ps	8.9 ps
RF main cavity voltage	2.8 MV	1.7 MV

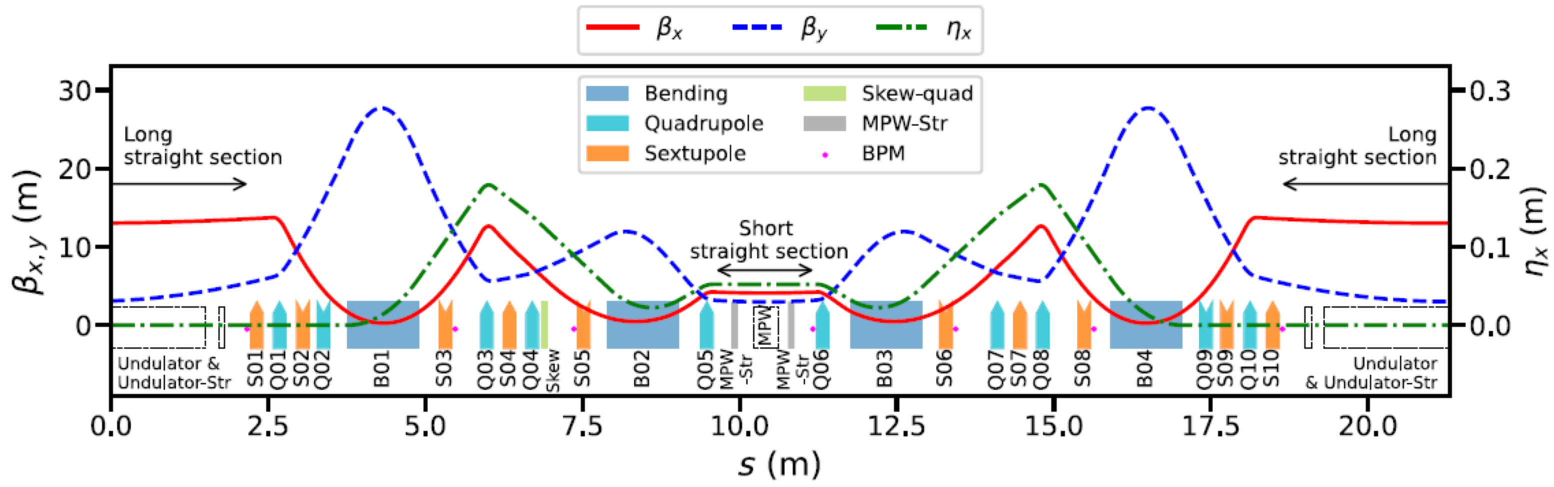


A. Loulergue et al., IPAC2024



NanoTerasu: DDBA (4BA) lattice

- 16 × DDBA with long and short straight sections
  - Natural emittance: 1.14 nm·rad @ 3 GeV
  - Undulators installed in long straight sections and multipole wigglers in short ones
  - 3 families of chromatic sextupoles + 2 families of harmonic sextupoles

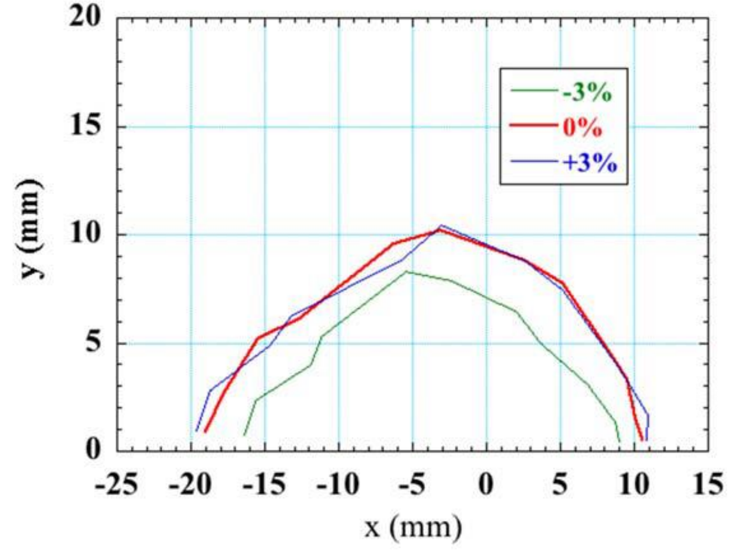


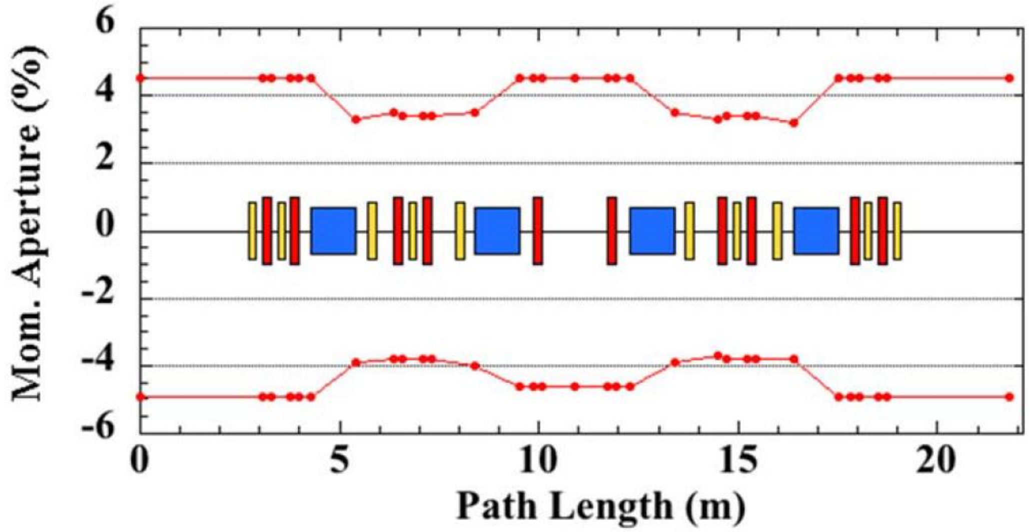
Beam energy	2.998 GeV
Lattice	Four-bend achromat
Circumference length	348.843 m
Number of cells	16
Number of bending magnets	4 × 16
Number of quadrupole magnets	10 × 16
Number of sextupole magnets	10 × 16
Long straight section for undulator	5.44 m × 14
Short straight section for MPW	1.64 m × 14
Betatron tune (ν_x, ν_y)	(28.17, 9.23)
Natural horizontal emittance (ε_x)	1.14 nm rad
Momentum compaction factor (α)	4.3×10^{-4}
Momentum spread (σ_E/E)	0.0843%
Bunch length (@ 0 mA)	2.92 mm
Optics at undulator center $(\beta_x, \beta_y, \eta_x)$	(13.0, 3.0, 0.0 m)
Optics at MPW center $(\beta_x, \beta_y, \eta_x)$	(4.1, 3.0, 0.05 m)
Beam current	400 mA
Energy loss in bends	0.621 MeV/turn
Accelerating radio frequency	508.759 MHz
Harmonic number	592

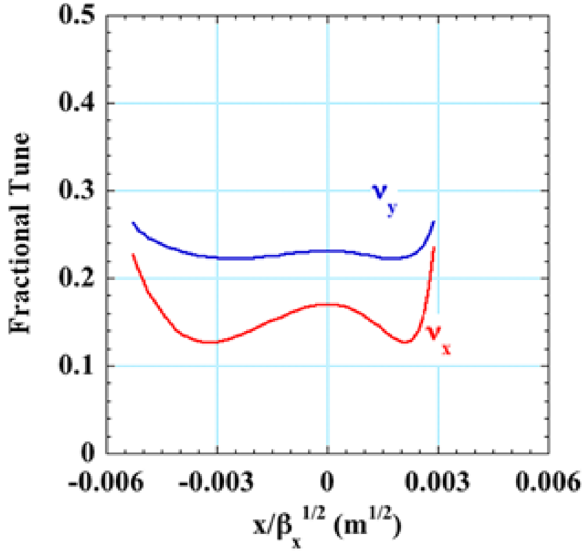
Shuhei Obara et al., PRAB, 2025

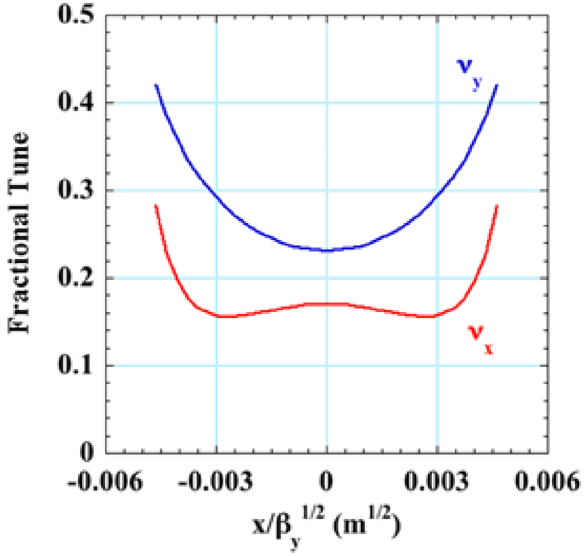
NanoTerasu: DDBA (4BA) lattice

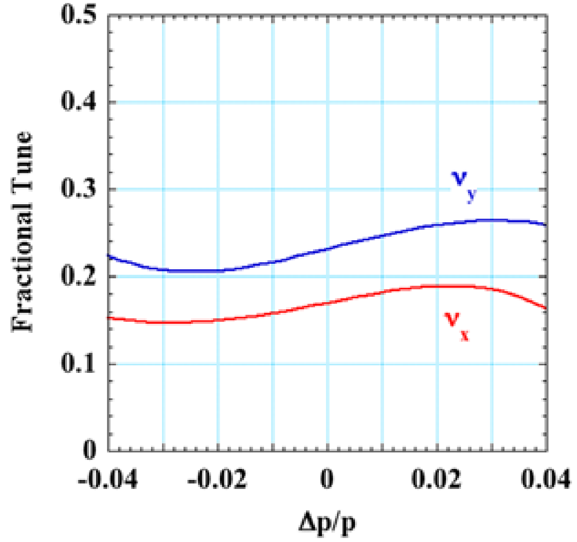
- Nonlinear dynamics performance







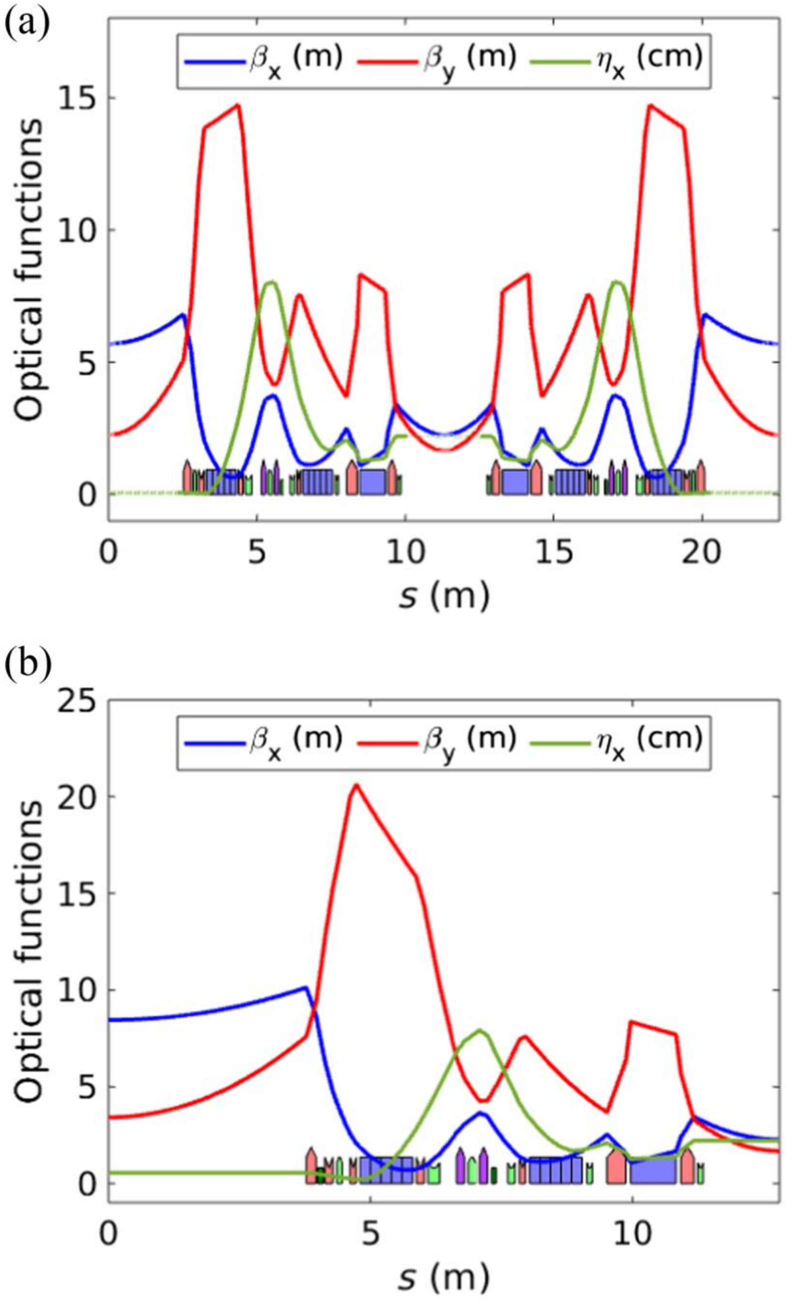




Diamond-II: H6BA lattice

- 24 × H6BA with long and short straight sections
  - Replacing the central bend of H7BA by a short straight section with low beta functions
  - RBs in dispersion bumps: increasing dispersion for lowering sextupole strengths & reducing emittance
  - Natural emittance: 162 pm·rad @ 3.5 GeV

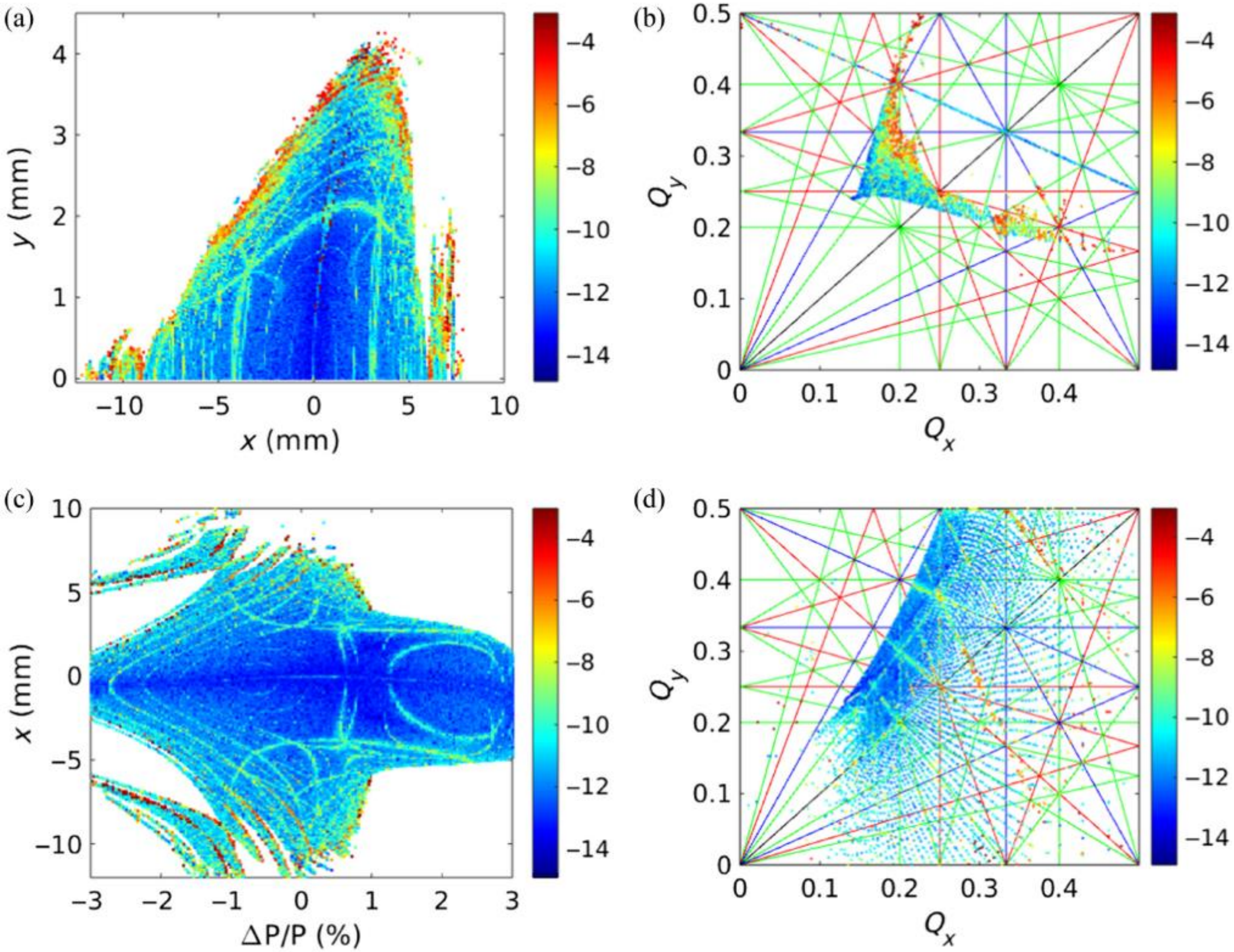
Parameters	Symbol	Unit	Diamond	Diamond-II
Circumference	C	m	561.571	560.561
Energy	E	GeV	3.0	3.5
Number of straight sections	⋯	⋯	6 LSS + 18 SSS + 1 MSS	6 LSS + 18 SSS + 24 MSS
Percentage of ring with SS	⋯	%	38.3	36.8
rf frequency	f_{rf}	MHz	499.680	499.511
Positive bending angle	θ_P	deg.	360	374.4
Reverse bending angle	θ_R	deg.	0	14.4
Total bending angle	θ_T	deg.	360	388.8
Lattice type	⋯	⋯	23 DBA + DDBA	MH6BA
Betatron tune	$[Q_x, Q_y]$	⋯	[28.19, 13.28]	[54.14, 20.24]
Horizontal, Vertical emittance	$[\epsilon_x, \epsilon_y]$	pm	[2690, 8]	[162, 8]
Natural chromaticity	$[\xi_{x0}, \xi_{y0}]$	⋯	[−78.4, −39.4]	[−68.2, −89.1]
Corrected chromaticity	$[\xi_x, \xi_y]$	⋯	[1.7, 2.2]	[2.5, 2.7]
Energy loss per turn	U_0	MeV	0.987	0.723
Natural energy spread	δ	%	0.096	0.094
Mom. comp. factor ($\times 10^{-4}$)	α	⋯	1.56	1.03
Horizontal, vertical, longitudinal damping time	$[\tau_x, \tau_y, \tau_s]$	ms	[11.3, 11.4, 5.7]	[9.6, 18.1, 16.1]
Harmonic number	h	⋯	936	934

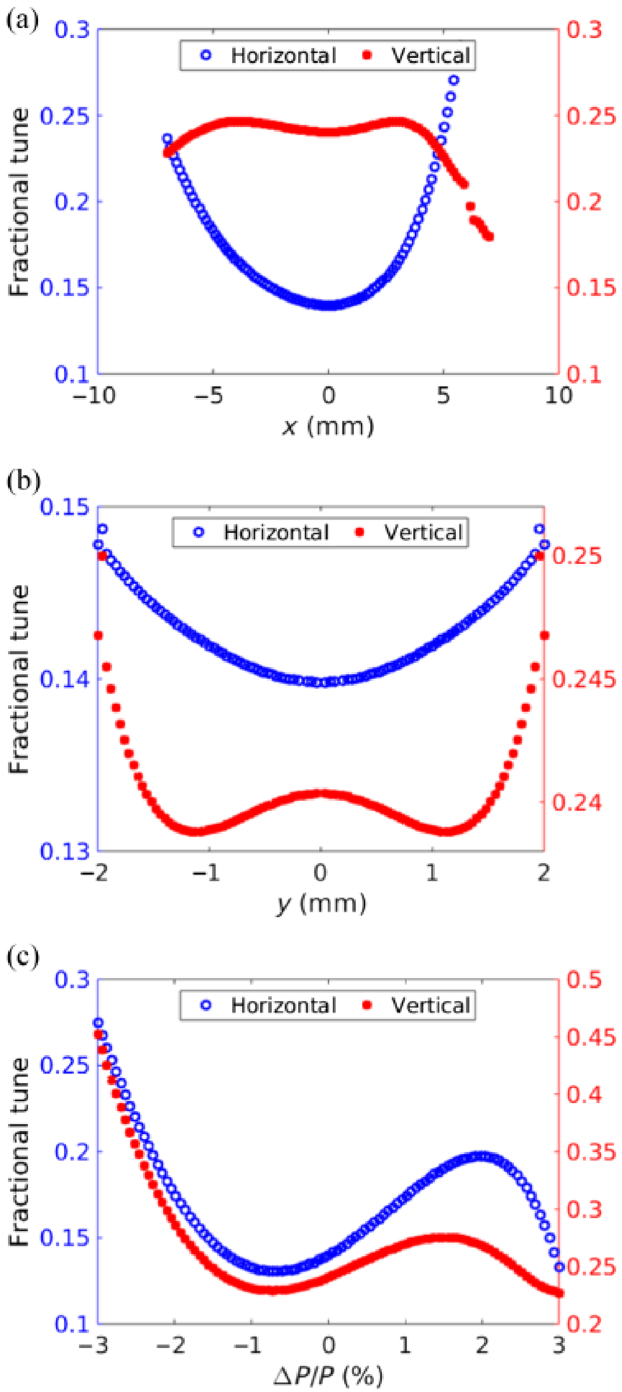


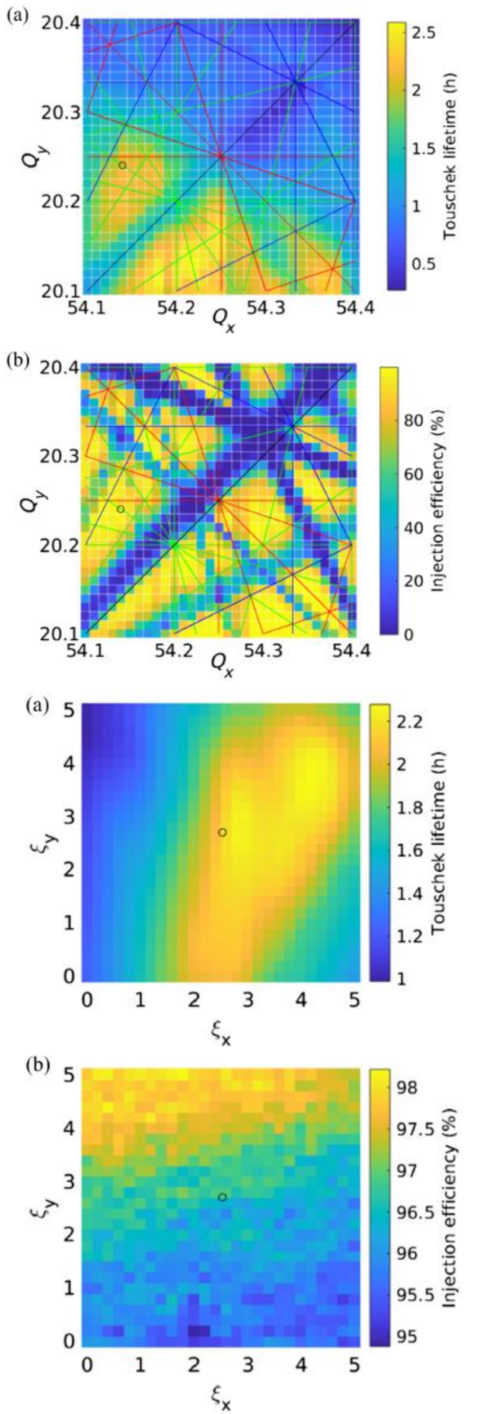
H. Ghasem et al., PRAB, 2024

Diamond-II: H6BA lattice

- Nonlinear dynamics performance

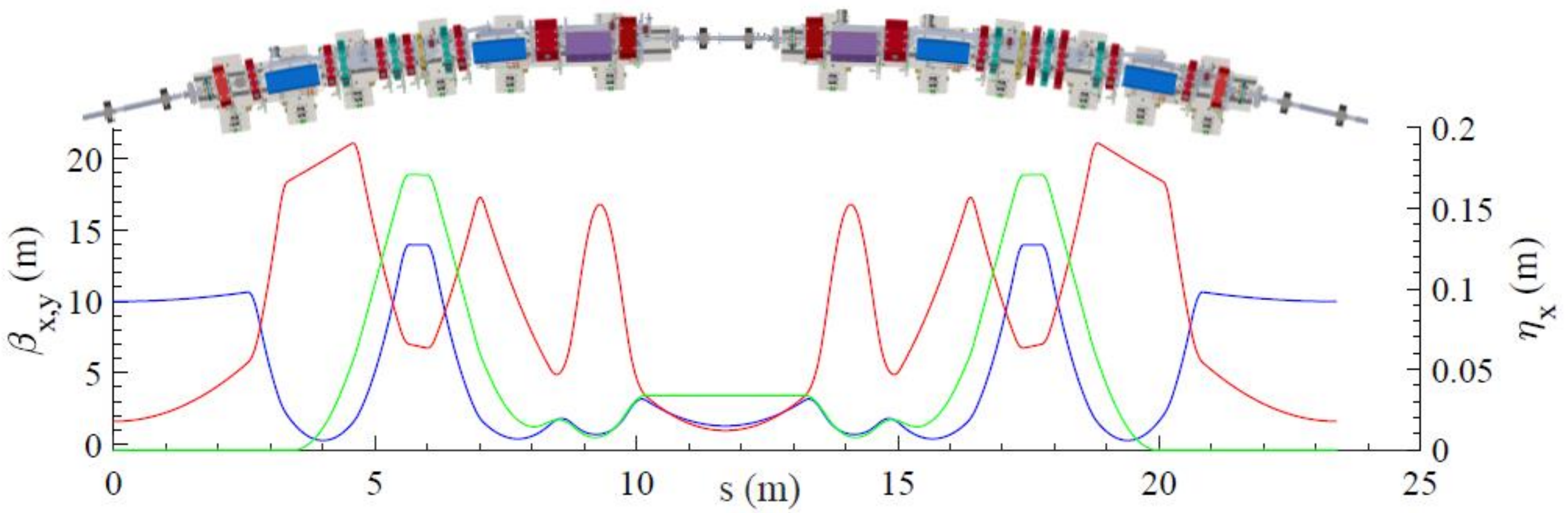






SPS-II: DTBA (H6BA) lattice

- 14×DTBA with long and short straight sections, originated from Diamond II study
  - Low beam emittance < 1 nm·rad @ 3 GeV
  - Maximized space usage up to 35%
  - The ability to inject the beam off-axis
  - Moderate hardware requirements

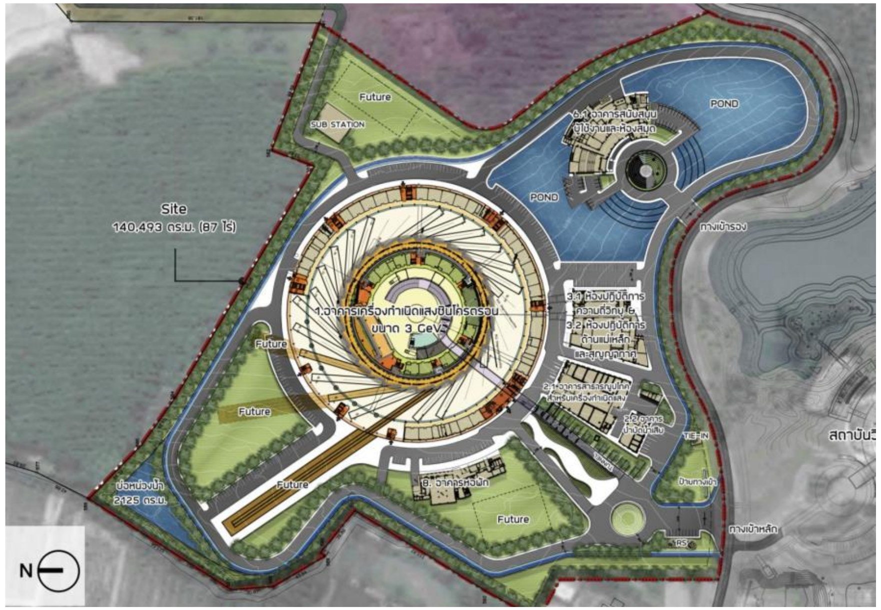


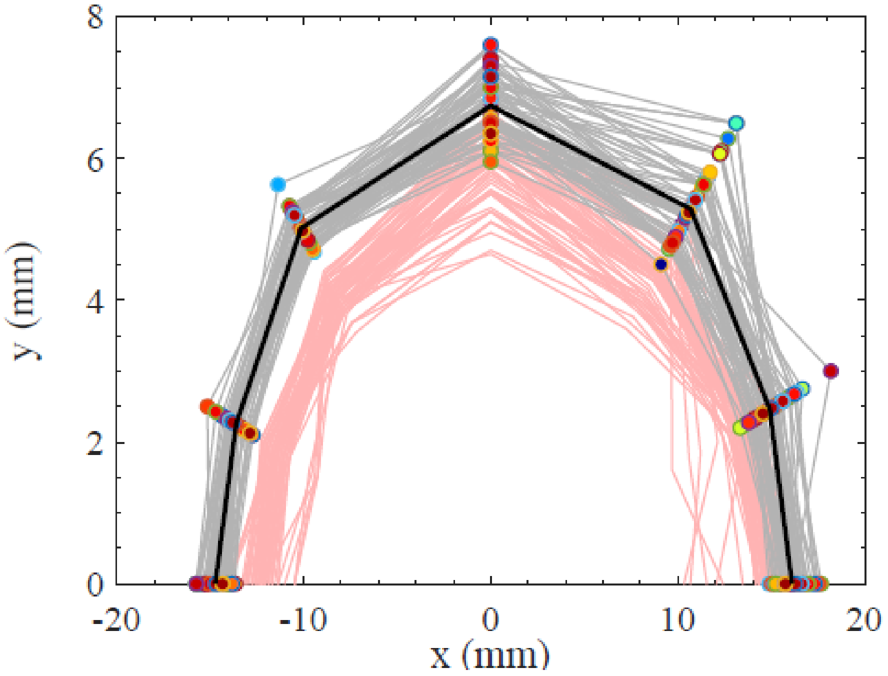
T. Pulampong et al., IPAC2025

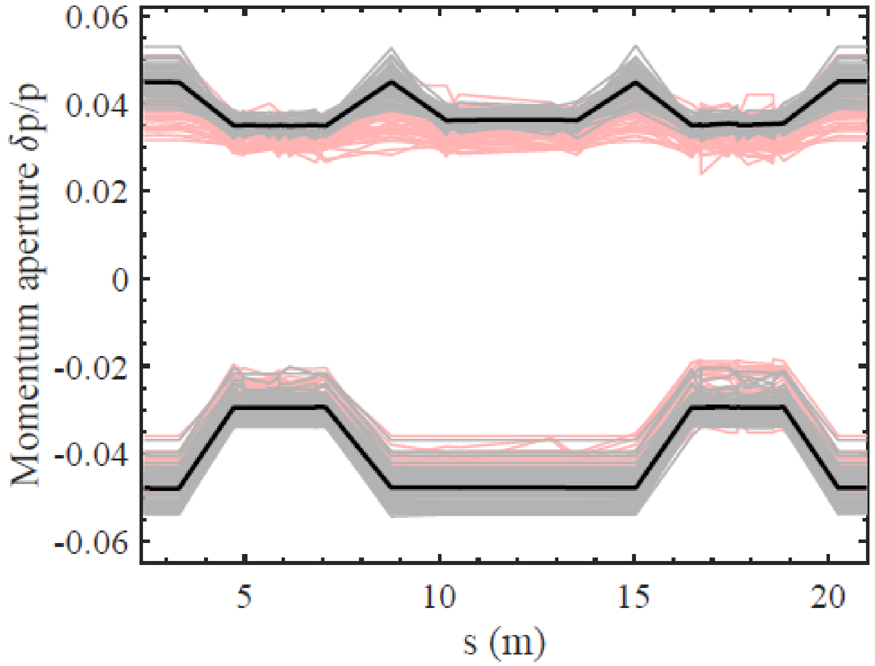
Parameters	Value
Beam energy (GeV)	3.0
Circumference (m)	327.6
Hor. beam emittance (nm·rad)	0.96
Nat. energy spread σ_E (%)	0.077
Nat. chromaticity ξ_x/ξ_y	-69.1/-69.7
Tune Qx/ Qy	34.16/12.42
Momentum compaction α_c	3.24E-04
Damping times hor./ver./long. (ms)	9.8/11.3/6.1
Straight/circumference	0.35
Energy loss per turn U0 (MeV)	0.578
RF frequency (MHz)	499.654096
RF voltage (MV)	2.2
Harmonic number	546

SPS-II: DTBA (H6BA) lattice

- Nonlinear dynamics performance
  - Nonlinear cancellation: -I within a lattice cell + HOA over 7 lattice cells, note that cell tunes ≈ (2+3/7, 6/7)
  - Averaged DA of -14.8 mm: comfortably accepted for injection offset of -9.0 mm with nonlinear kicker
  - Touschek lifetime: 8.7 hrs without harmonic cavity

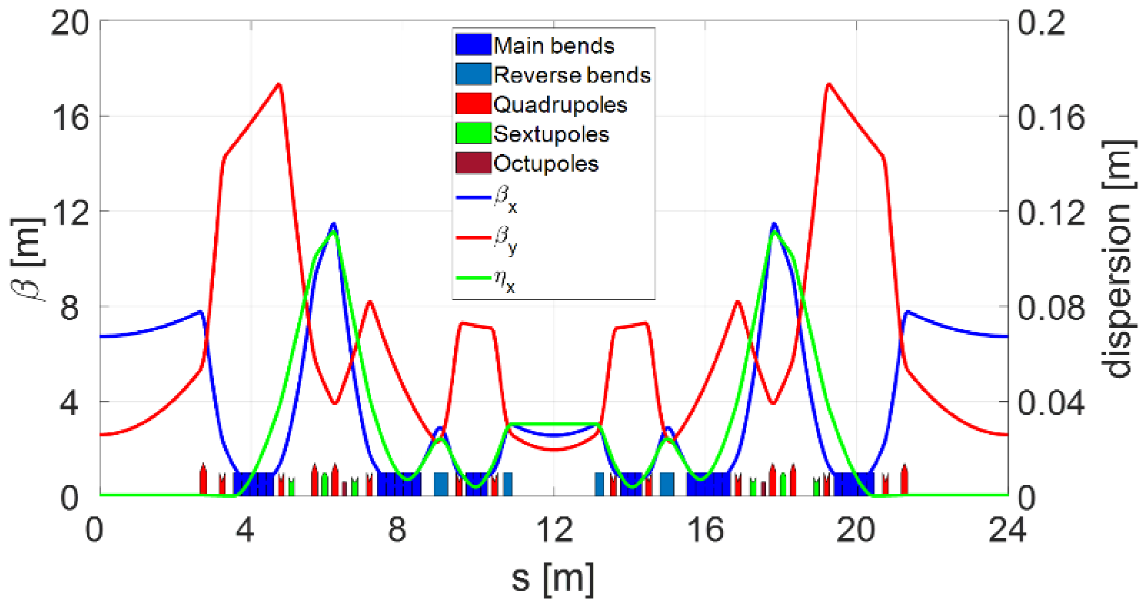


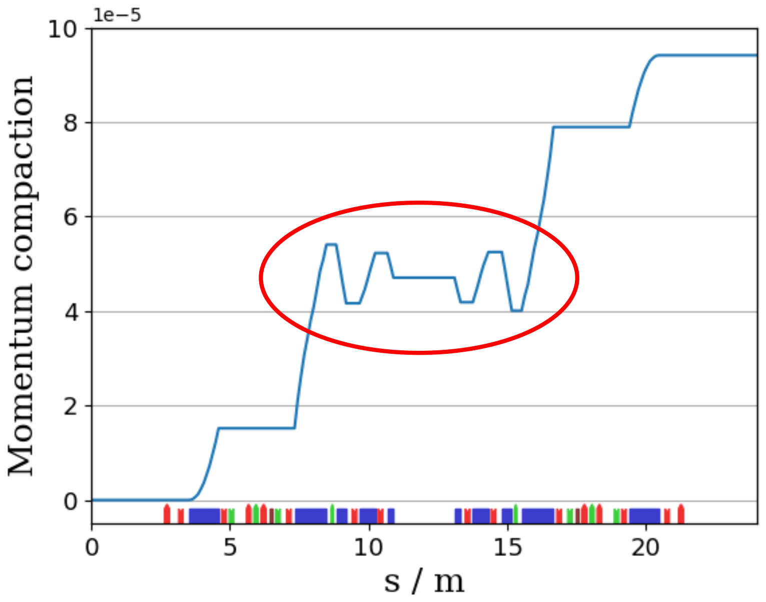




HALF: H6BA lattice

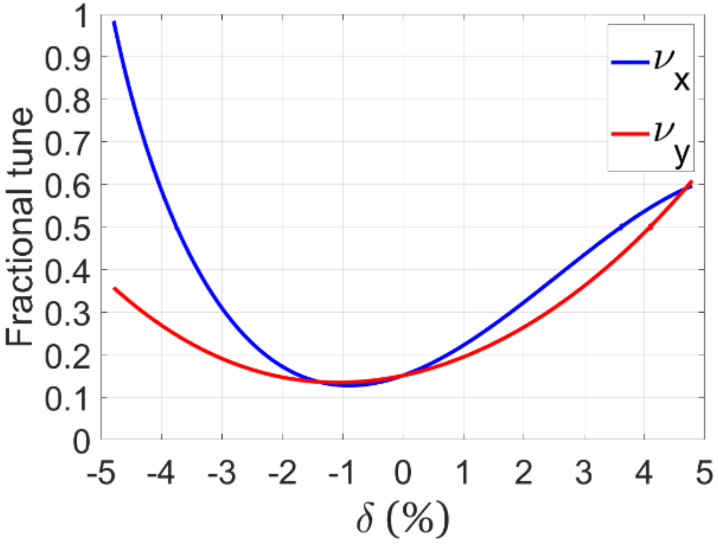
- 20×H6BA with long and short straight sections
  - Replacing the three CB unit cells of H7BA by two LGB/RB unit cells and a short straight section
  - LGB/RB unit cells: reducing emittance and damping times
  - Natural emittance: 86 pm·rad @ 2.2 GeV
  - Full coupling beam: working point on the linear difference resonance line
  - Two damping wigglers: further reducing damping times and emittance
  - Beam energy: increased to 2.4~2.5 GeV in the future



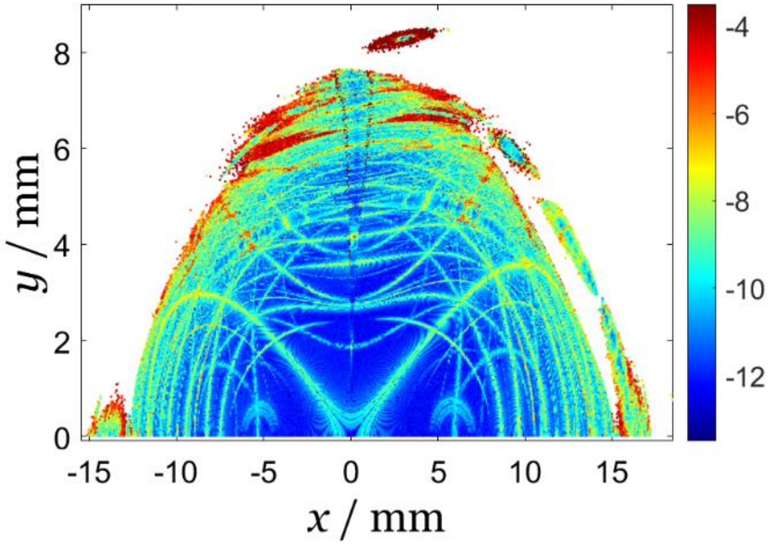


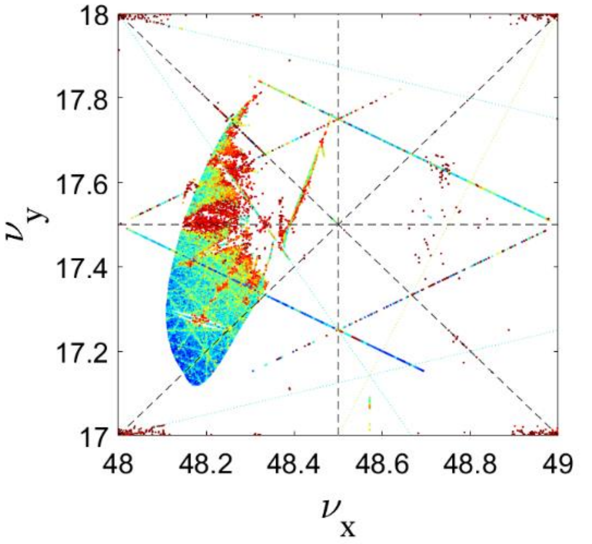
HALF: H6BA lattice

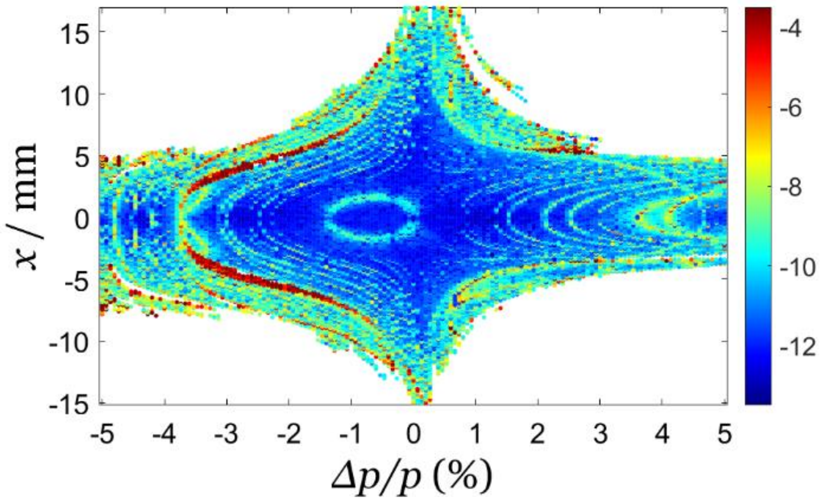
- Nonlinear dynamics performance
  - Nonlinear cancellation: -I within a lattice cell + HOA over 5 lattice cells, note that cell tunes ≈ (2.4, 0.9)
  - 3 sextupole families + 1 octupole family



Parameter	Value
Energy	2.2 GeV
Circumference	479.86 m
Number of cells	20
Natural emittance (w/o DWs)	85.9 pm·rad
Betatron tunes (H/V)	48.15/17.15
Natural chromaticities (H/V)	-80.8/-56.4
Momentum compaction factor	0.94×10^{-4}
Damping partition (H/V/L)	1.36/1.0/1.64
Natural damping times (H/V/L)	28.5/38.8/23.7 ms
Natural energy spread	0.61×10^{-3}
Energy loss per turn	181.4 keV
Total absolute bending angle	438.6°



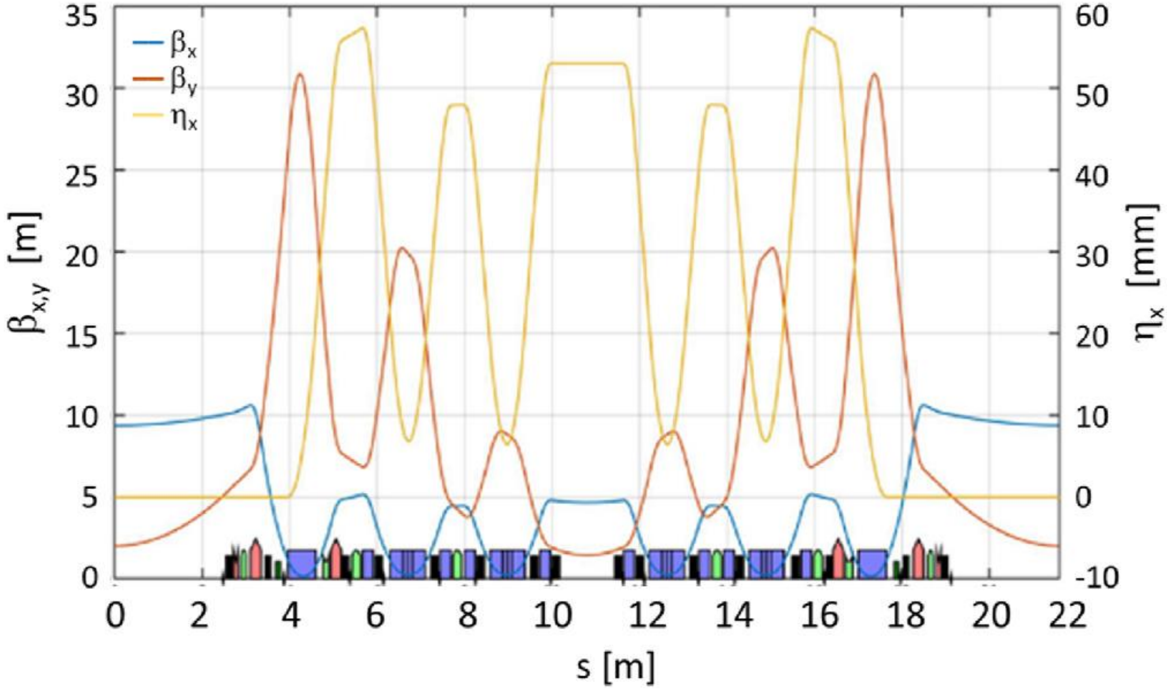


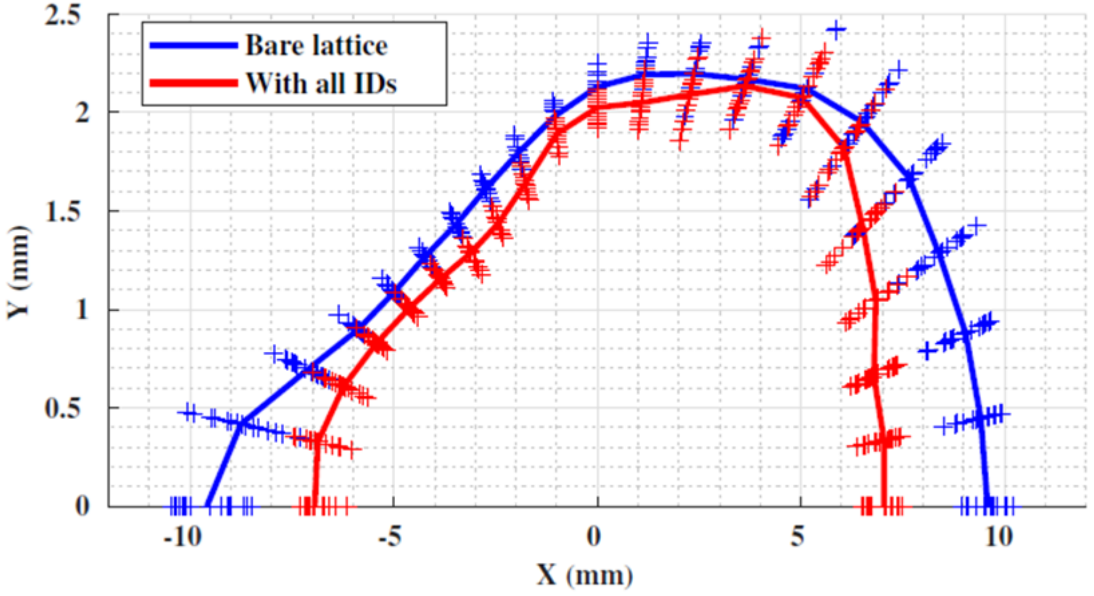


Elettra 2.0: 6BA lattice

- 12 × 6BA with long and short straight sections
  - Natural emittance: 212 pm·rad @ 2.4 GeV (stronger emittance reduction than H6BA)
  - Short straight section: higher dispersion than that of H6BA

Parameter	Value	Units
Energy	2.4	GeV
Circumference	259.2	m
Harmonic number	432	
Average current	400	mA
Horizontal emittance (bare)	212	pm rad
Coupling factor	3	%
Energy loss/turn	<670	keV
RF peak voltage	2	MV
Main radiofrequency	499.654	MHz
Bunch duration, RMS	15–20	ps
Rel. energy spread, RMS	0.09	%





E. Karantzoulis et al., NIMA, 2024

Summary of low-emittance lattice design philosophy

- The nonlinear dynamics presents the primary challenge in low-emittance lattice design, but the critical factor for success resides in the linear optics design. (Lattice设计的难点在于非线性动力学, 关键在于线性光学设计.)
  - RDTs are strongly related to linear optics.
  - Linear optics offers extensive degrees of freedom for nonlinear optimization.
- In low-emittance lattice design, phase-advance based nonlinear cancellation, RDT minimization, and FMA provide the basis for nonlinear dynamic optimization, with powerful intelligence based algorithms taking it to the next level.
- When encountering the issues about the interplay of storage ring parameters in low-emittance lattice design, the combination of properly increasing beam energy and employing damping wigglers is a more effective solution.

谢 谢！
ขอบคุณ！