An Introduction to Accelerators

Part I

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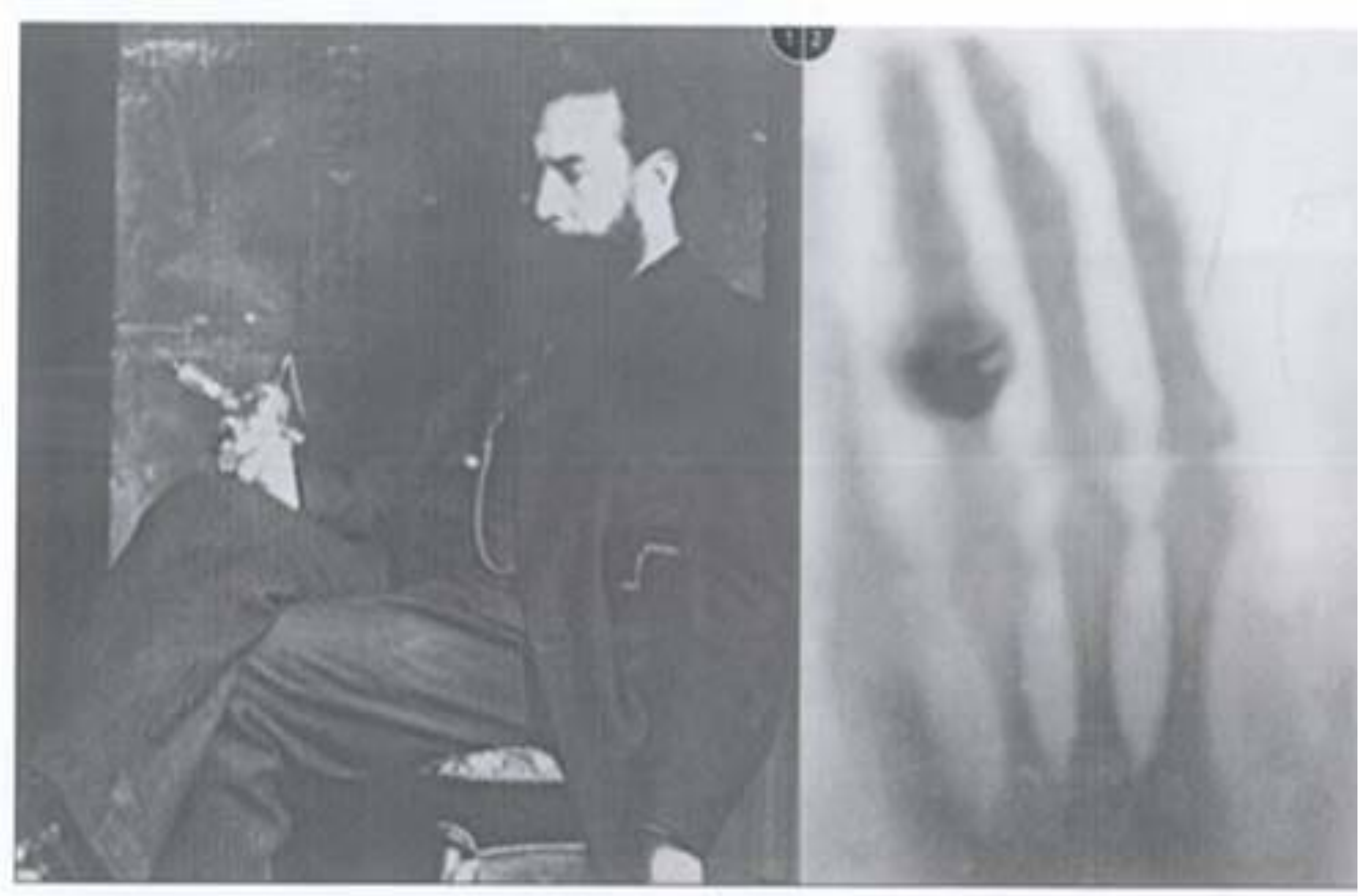
Outline

1. Principles of Accelerators
2. Contributions to science and others applications
3. A Simple Introduction of Accelerator Physics

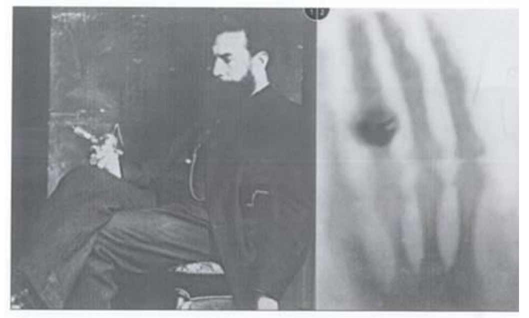
Discovery of X-ray



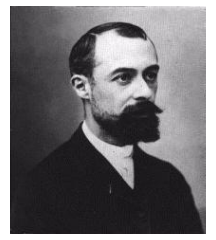
“Thirty-five years ago, on November 8, **1895**, **Wilhelm Conrad Roentgen** made the revolutionary discovery of a mysterious new kind of ray which he called the ‘x-ray’.” —by Otto Classer, W.C. Roentgen and the Discovery of the Roentgen Rays, AJR (American Journal of Roentgenology) 1931, 25:437-450, also be adapted at ARJ 1995, 165:1033-1040.

A brief history of nuclear science

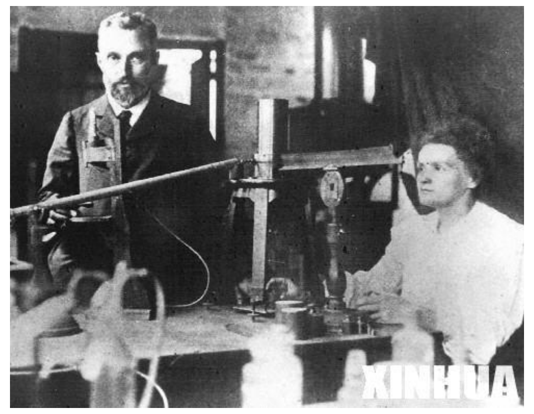
核科学发展简史



1895 年，德国物理学家伦琴（Röntgen WC）发现 X 射线，并于 1901 年获得首届诺贝尔物理学奖，同时成为核科学与技术一系列重大发现的开端。



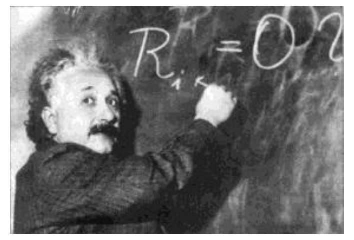
1896 年法国物理学家贝克勒尔(Becquerel H)发现铀化物的天然放射性，开启了核科学的百年历史。



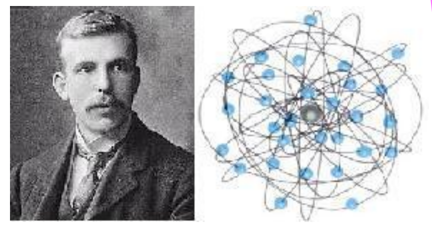
1898 年，居里夫妇发现镭，并开创了放射化学学科。1903 年，居里夫妇与贝克勒尔获得诺贝尔物理学奖

英国科学家卢瑟福（Rutherford E）发现α和β粒子；1908 年诺贝尔化学奖。1900 年法国物理学家维拉德(Villard PV) 发现γ射线。

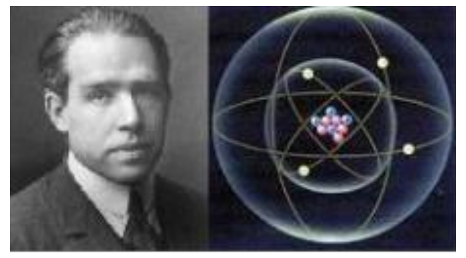
1897 年，英国物理学家汤姆逊(Thomson J J)发现电子，并确定其质量是氢原子的 1/1837。1906 年获诺贝尔物理学奖。



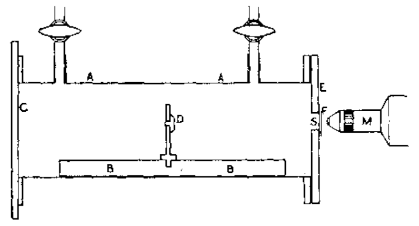
1905年，爱因斯坦(Einstein A)给出了质能转换关系$E=mc^2$。



1911 年卢瑟福用α粒子轰击金箔，提出了有核原子模型，并估计出原子核半径为 10^{-14}m。

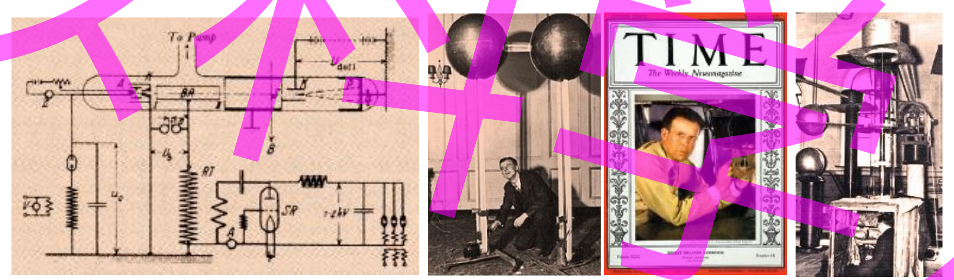


1913 年丹麦物理学家玻尔(Bohr N)利用量子概念提出了原子结构壳层模型。1922 年诺贝尔物理学奖。



1919 年卢瑟福利用α粒子轰击氮，通过 $^{14}N+^{4}He=^{17}O+p$,得到了质子，并提出了建造粒子加速器的需求，从此带来了加速器的时代。

1932 年查德威克(Chadewick J)发表文章宣布发现了中子，1935 年获诺贝尔物理学奖。

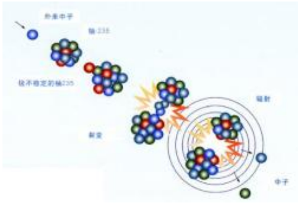


1924 年，Gustav Ising 提出直线加速器的概念，4 年后 Rolf Wideröe 在德国亚琛建成世界上第一台加速器。
1929 年，Van de Graaff 发明静电加速器；
1930 年，Ernest Lawrence 发明回旋加速器，并开展核物理实验，1939 年诺贝尔物理学奖；
1932 年，Cockcroft-Walton 加速器发明，开展了第一次人工粒子核反应，并获 1951 年诺贝尔物理学奖。
1940 年,Donald Kerst 在美国 illinois 大学建成第一台电子感应加速器，其原理是由 Joseph Slepian 于 1920s 提出。
1943 年，Marcus Oliphant 给出了同步加速器的概念。
1944 年，前苏联的 Vladimir Veksler 与美国的 Edwin McMillan 同时提出了加速器的自动稳相原理。
1946 年，英国的 William Walkinshaw 建成第一台电子直线加速器；Frank Goward 建成第一台电子同步加速器；随后在 GE 的一台电子同步加速器上发现了同步辐射光，开启了光源的新时代。

人工放射性

1931 年,伊伦.居里(Curie I)和约里奥(Joliot F)夫妇用 a 粒子照射铝箔，生成了放射性核素。

德国科学家海森堡（Heisenberg W）和前苏联科学家伊万年柯（Ivanenko D）提出了原子核是由质子和中子组成的假设。



裂变

1939 年奥地利女物理学家麦特纳(Meitner L)和弗里施(Frishi O)实验证实了核裂变现象。

匈牙利物理学家西拉德(Sczilard L)提出链式反应，1939 年约里奥-居里夫妇发现铀裂变产生中子后，1942 年，费米(Fermi）在芝加哥大学建成世界上第一座原子核反应堆。

曼哈顿

1945 年 7 月 16 日，世界上第一颗试验性原子弹在新墨西哥州爆炸成功。

广岛

1945年8月6日，美国投到日本广岛的第一颗原子弹

核电站

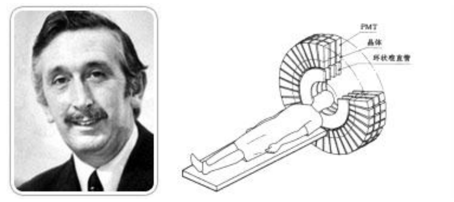
1954 年，世界上第一座核电站在前苏联建成，发电功率 5000 千瓦。标志着和平利用核能的开始。

氢弹

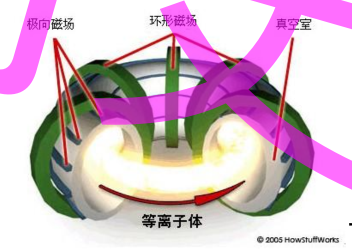
1952 年，美国在太平洋马绍尔群岛的埃卢盖拉布小岛首次爆炸试验成功

核医学

1960 年， Yalow RS 和 Berson 创立了放射免疫分析技术，并获 1977 年诺贝尔医学奖。

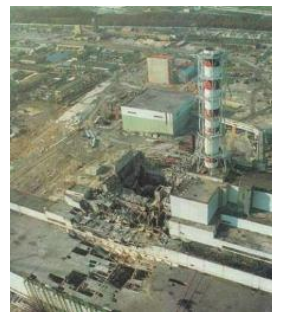


1973 年，EMI 公司第一台 CT 扫描机问世。G.N. Hounsfield 获 1979 年诺贝尔医学奖。
1975 年，第一台 PET 研制成功。

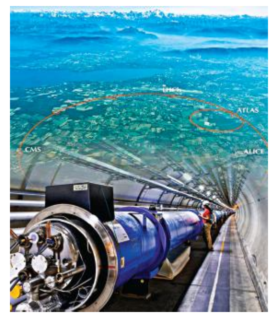


可控核聚变

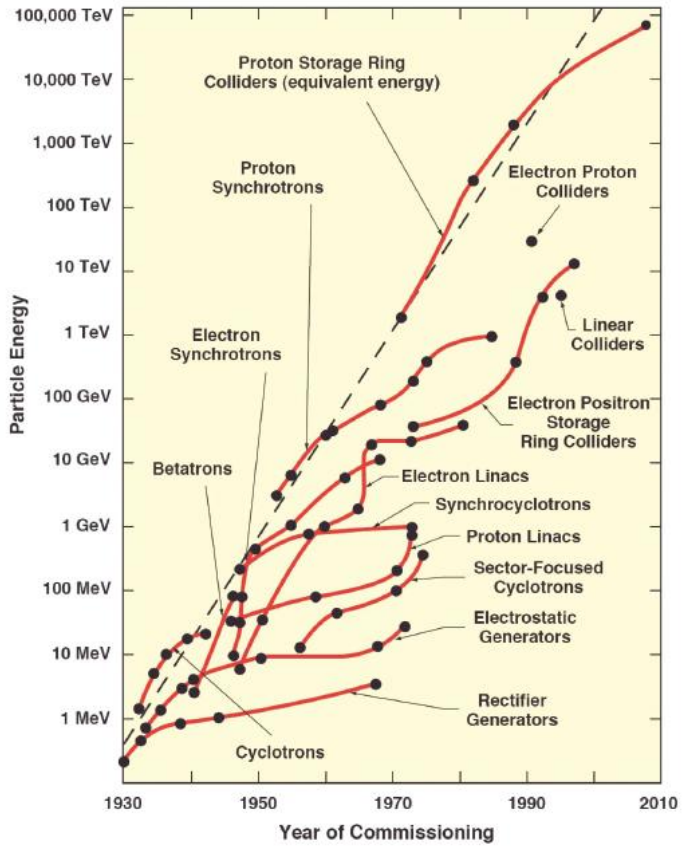
20世纪下半叶，提出磁约束核聚变装置托卡马克(Tokamaka)。1991 年欧共体的欧洲联合环(JET)第一次实现了氘-氚聚变反应，1997年它又创造了核聚变输出功率为12.9MW的新纪录，随后提高到16.1MW。

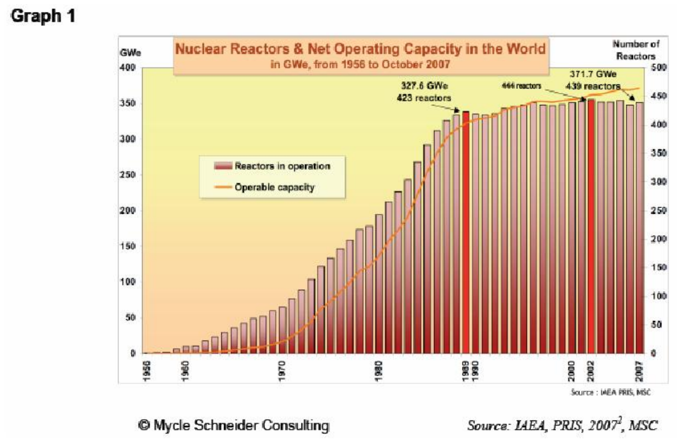


1986 年 4 月 26 日，位于乌克兰北部切尔诺贝利核电站发生爆炸。核能建设进入低潮。



2008 年，世界上最大的加速器装置周长 27km 的 LHC 开始运行。





世界核能的发展在切尔诺贝利后进入低潮，近十年随着能源危机的加强及核能安全的进一步发展，核能又开始了新一轮的大发展。

1895 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000 2010

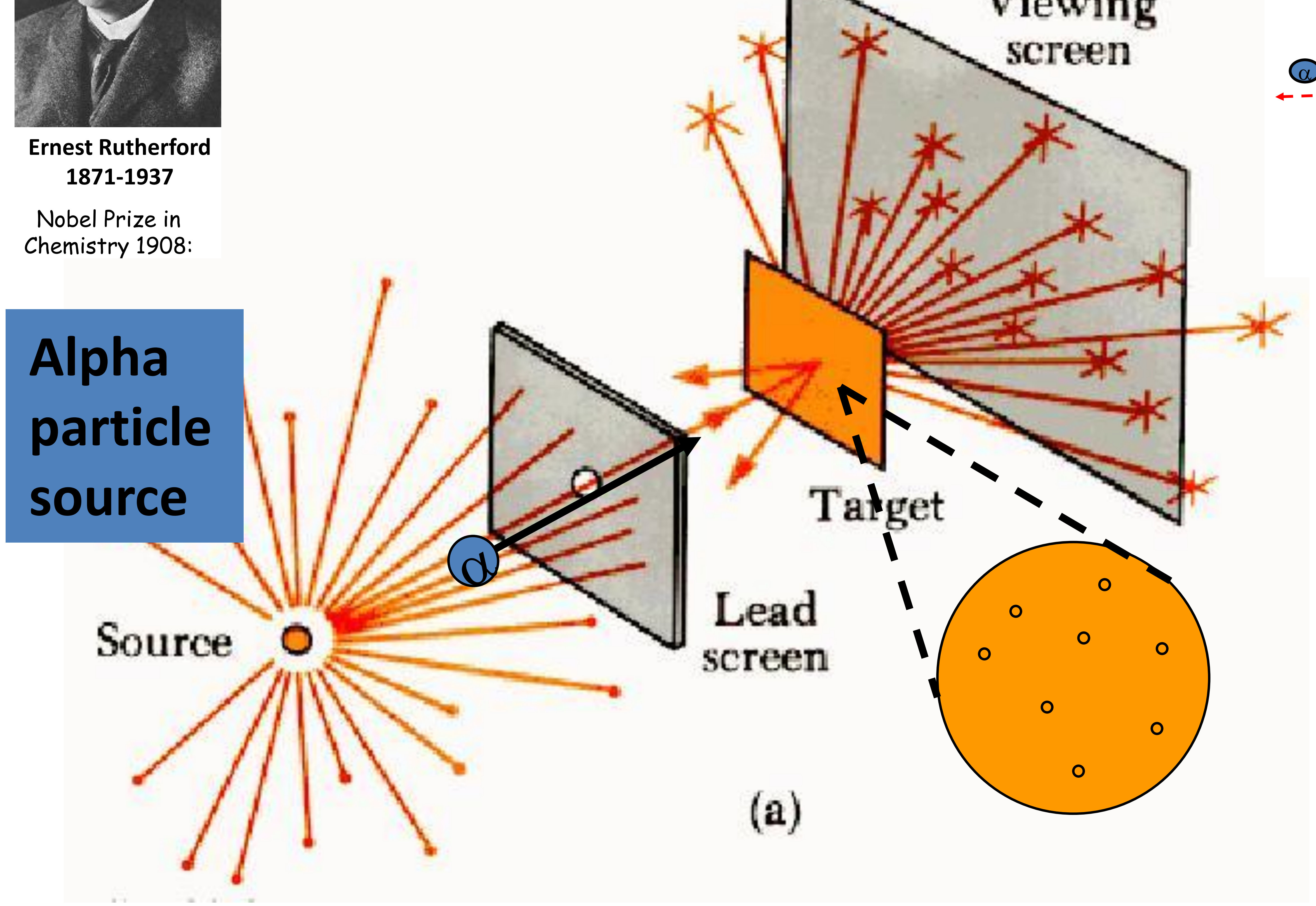
“粒子加速器的发明年代”

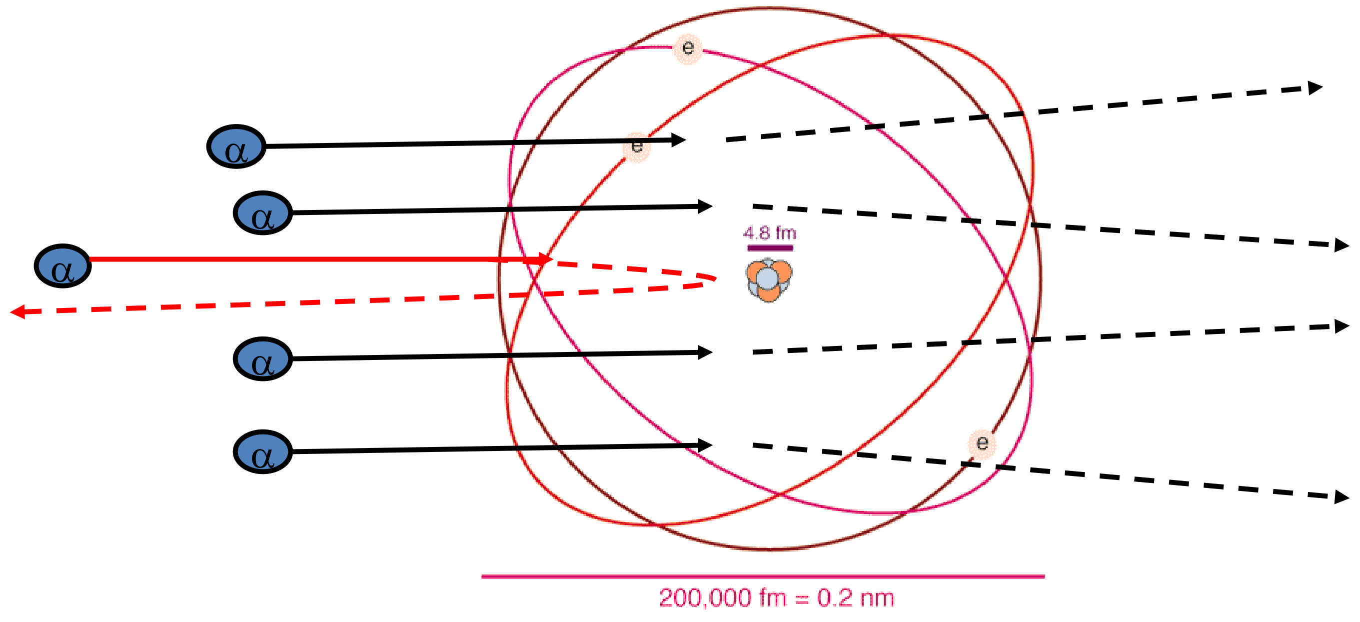
唐传祥 2010年11月11日

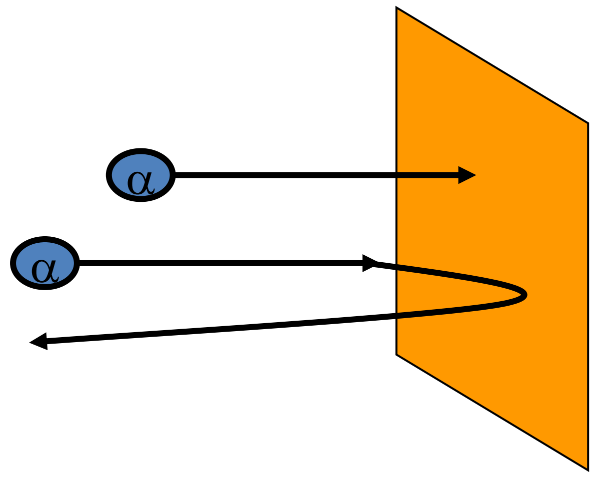
Scattering Experiment and Atomic Nucleus

Ernest Rutherford
1871-1937

Nobel Prize in
Chemistry 1908:



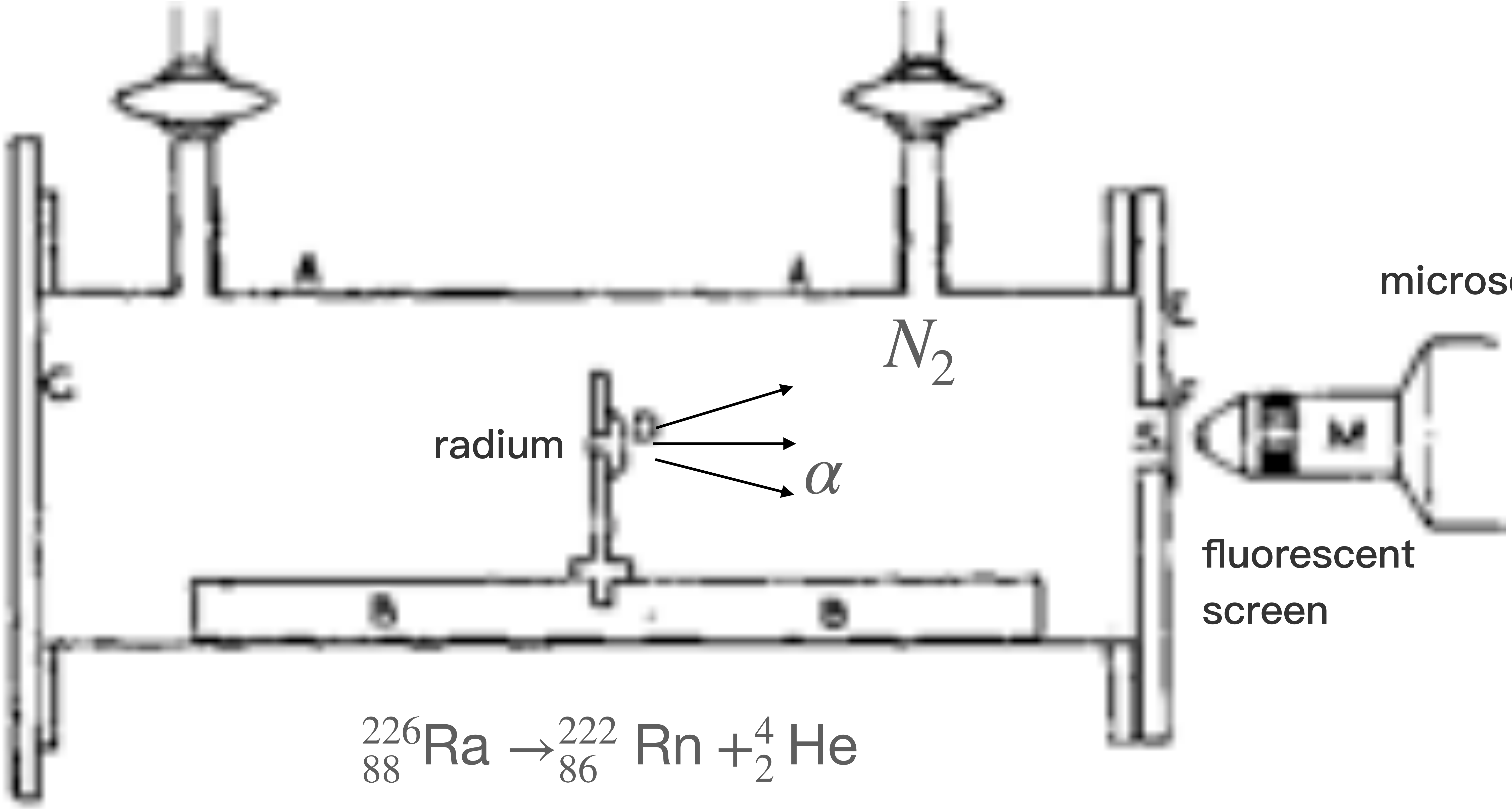




In Rutherford's words...
"It was quite the most incredible event that ever happened to me in my life. It was as if you fired a 15-inch naval shell at a piece of tissue paper and the shell came right back and hit you."

The birth of particle accelerators

In 1919, Ernest Rutherford used the α from the decay of Ra hitting N. With the nuclear reaction $^{14}N+^{4}He=^{17}O+p$, he discovered proton. Later, he calls for “a copious supply” of particles more energetic than the natural sources. The accelerator era is born.



$$^{226}_{88}Ra \rightarrow ^{222}_{86}Rn + ^{4}_{2}He$$

氡（Radon）

An Era of Accelerator Inventions

Rutherford’s plea for particle accelerators, and the start point of accelerators

1919

G. Ising, and Rolf Wiederoe: **Linear accelerator**

1924

Van De Graaff: **Tandem**

1929

E.T. Lawrence: **Cyclotron**

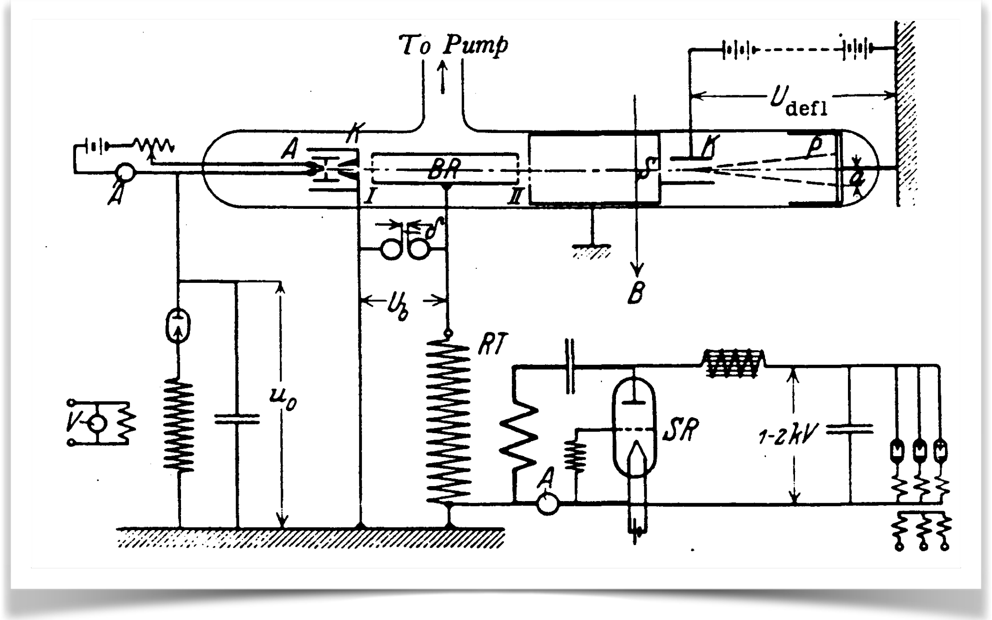
1930

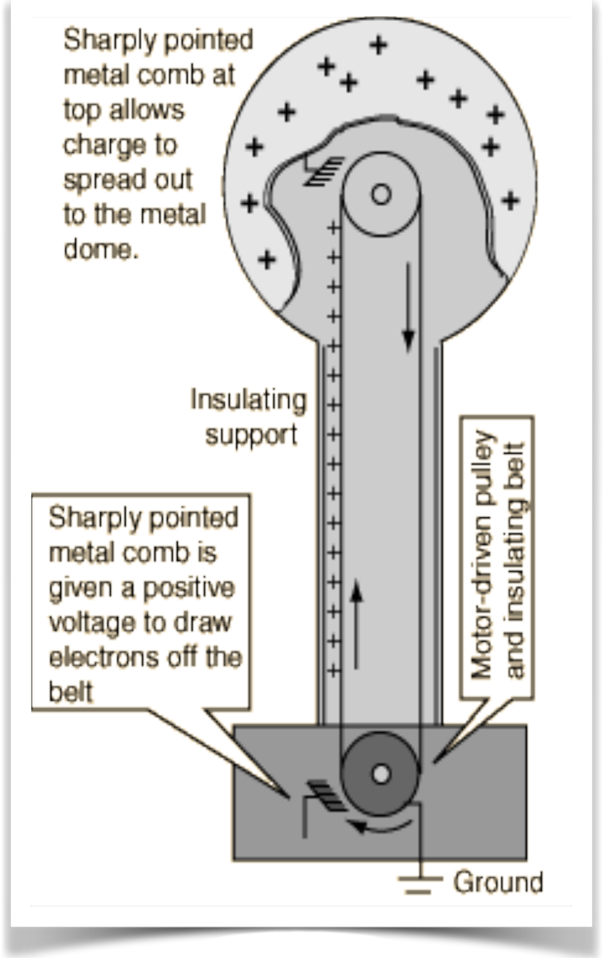
Cockcroft and Walton: **Voltage Multiplying Accelerator**

1932

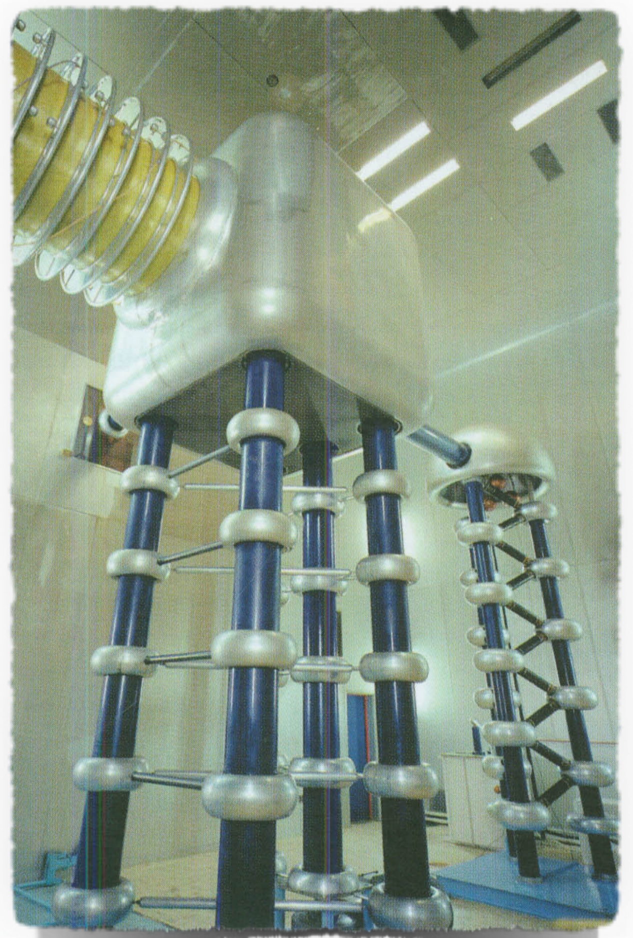
D. Kerst: **Betatron**

1940





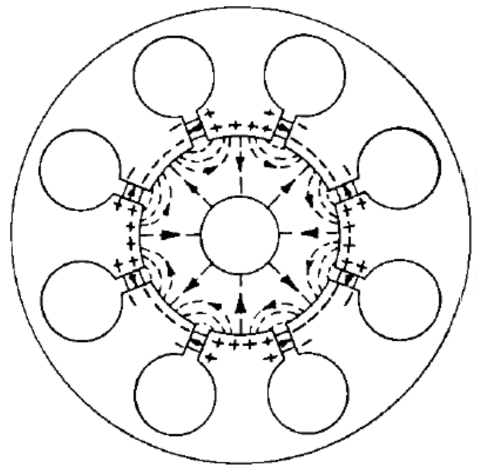
1932 by M. S. Livingston
Proton: 80keV, 1kV

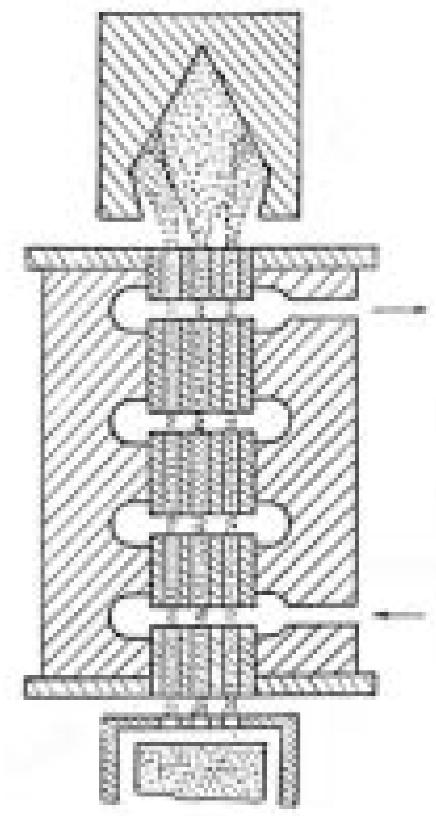


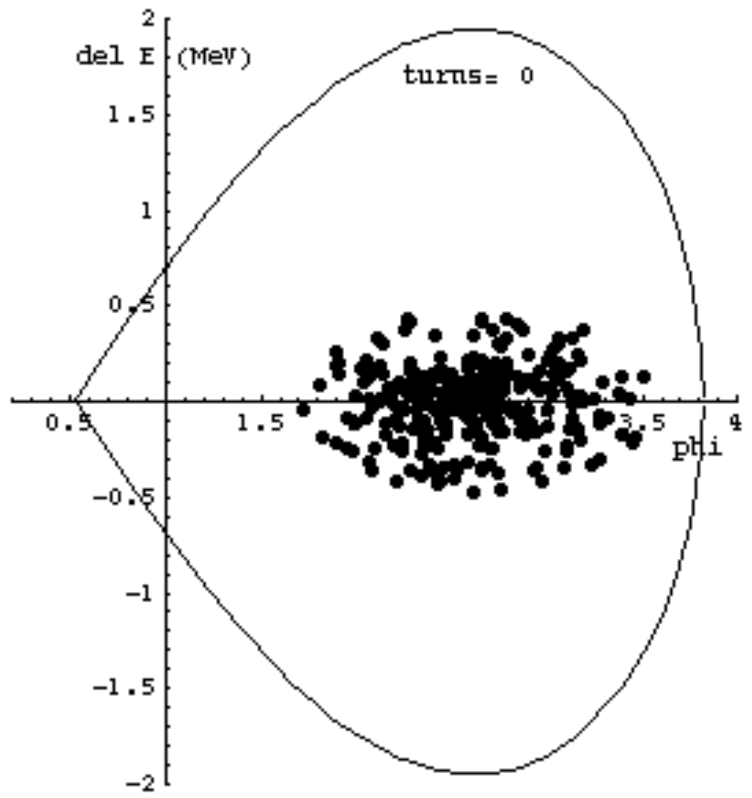
- Engines of Discovery-A Century of Particle Accelerators, by Andrew Sessler and Edmund Wilson, World Scientific, 2006
- A Brief History of Particle Accelerators, Poster, Review of Accelerator Science and Technology, by A. Chao and W. Chou
- Per F. Dahl, Rolf Wideroe: Progenitor of Particle Accelerators, SSCL-SR-1186, March 1992

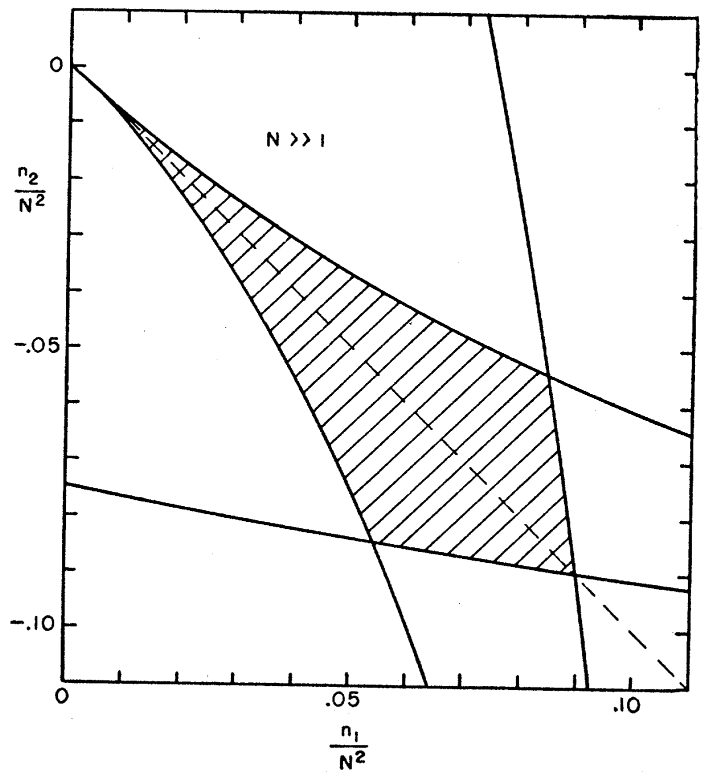
Accelerators of Until Today

- 1921 — A.W. Hull: Magnetron
- 1937 — Rusell and Sigurd Varain and William Hansen: Klystron
- 1940 — Synchrotron
- 1943 — Vladimir Veksler, Edwin McMillan: Phase stability
- 1944 — Electron Linac
- 1946 — E.Courant, S.Livingston and H. Synder: Strong Focusing-AG
- 1966 — SLAC 3km Linac: 50GeV electron, with 240 Klystrons, 64MW, and SLED
- 2008 — LHC: 7TeV Proton Synchrotron. Begin operation 2012: discovered higgs boson





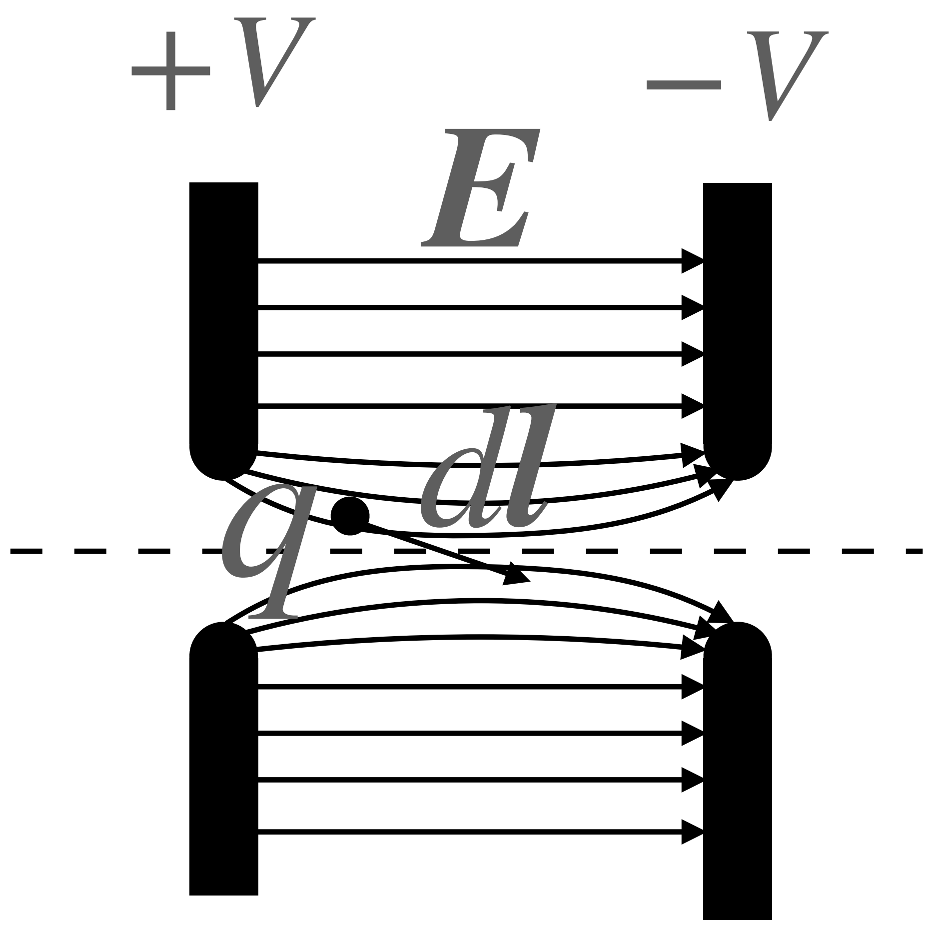




- Hull A W. The Magnetron. Journal of the American Institute of Electrical Engineers, 1921, 40(9):715-723.
- A Brief History of Particle Accelerators, Poster, Review of Accelerator Science and Technology, by A. Chao and W. Chou
- George Caryotakis, The klystron: A Microwave Source of Surprising Range and Endurance, SLAC-PUB-7731, April 1998
- “The strong-focusing synchrotron- A new high energy accelerator”, PHYSICAL REVIEW Vol 88 No.5 Dec. 1952

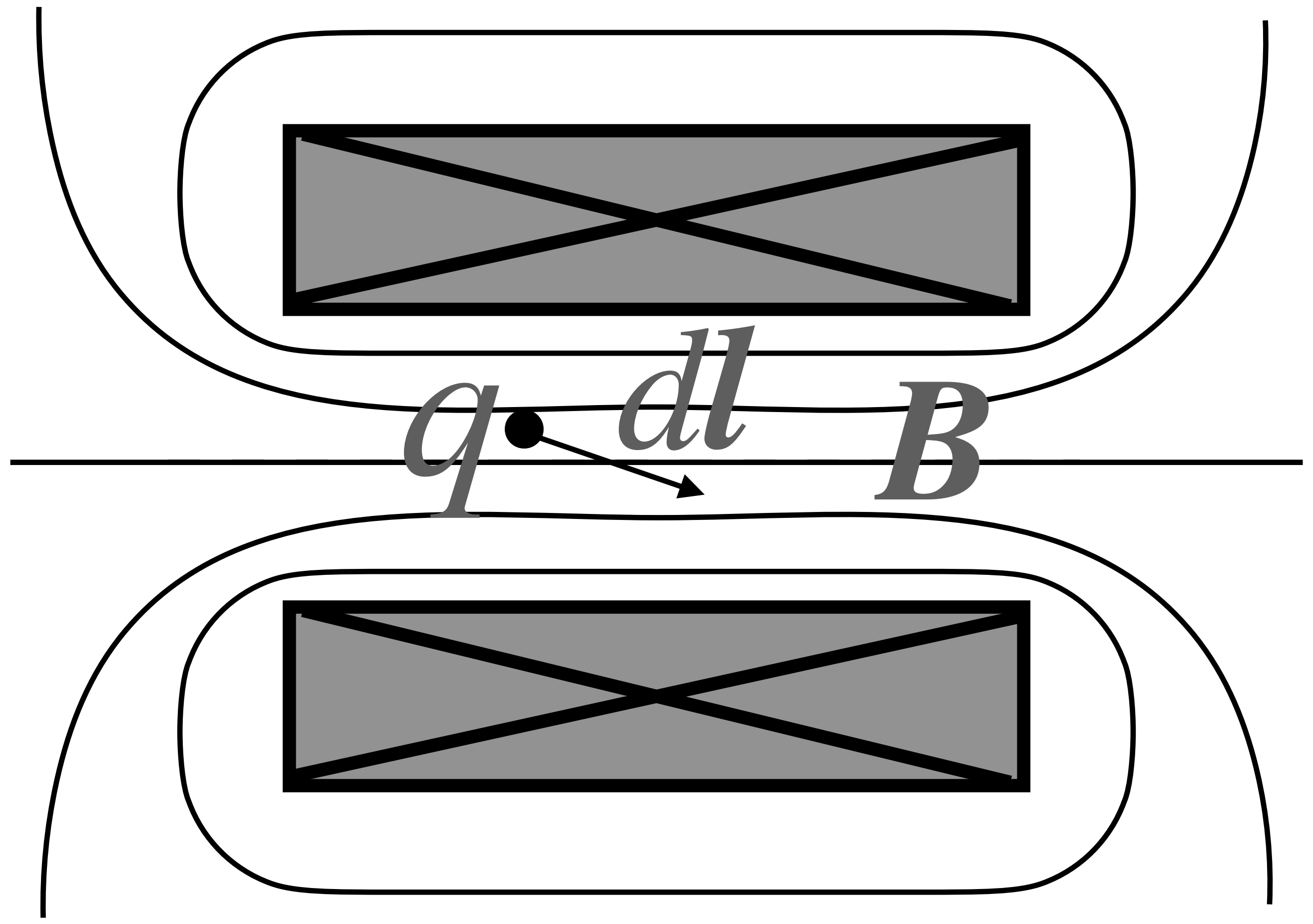
1. Principles of Accelerators

Charged particles can be accelerated or decelerated by electric fields, not magnetic fields



A charged particle is moving in the electric field **E**. The energy it can gain from or loss to the electric field after a distance of dl, is :

$$dW = q\boldsymbol{E} \cdot d\boldsymbol{l} = qE \cdot dl \cos\theta$$



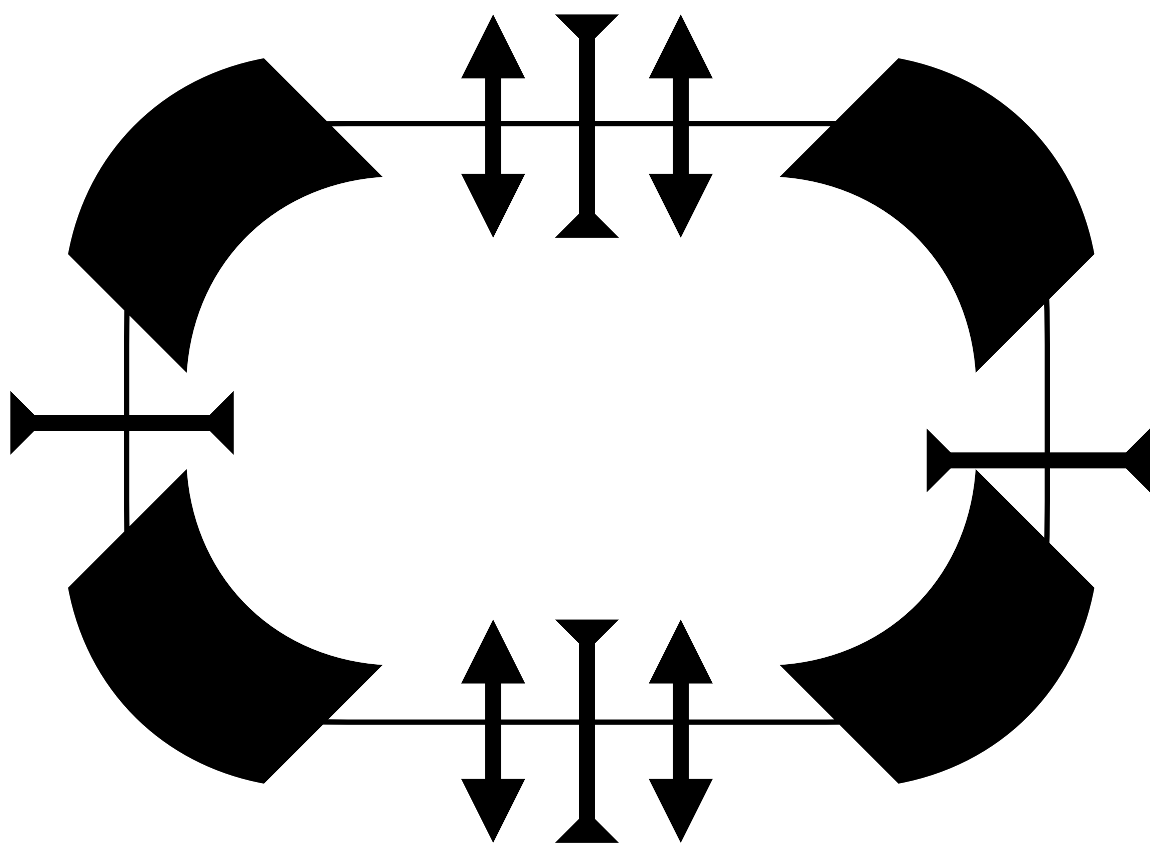
A charged particle is moving in the magnetic field **B**. The energy it can gain from or loss to the electric field after a distance of dl, is :

$$dW = q(\boldsymbol{v} \times \boldsymbol{B}) \cdot d\boldsymbol{l} = q(\frac{d\boldsymbol{l}}{dt} \times \boldsymbol{B}) \cdot d\boldsymbol{l} = q(d\boldsymbol{l} \times \frac{d\boldsymbol{l}}{dt}) \cdot \boldsymbol{B} = 0$$

Both electric and magnetic fields are important for accelerators, because except acceleration, we also need to control the charged particles on the orbit by magnetic fields.

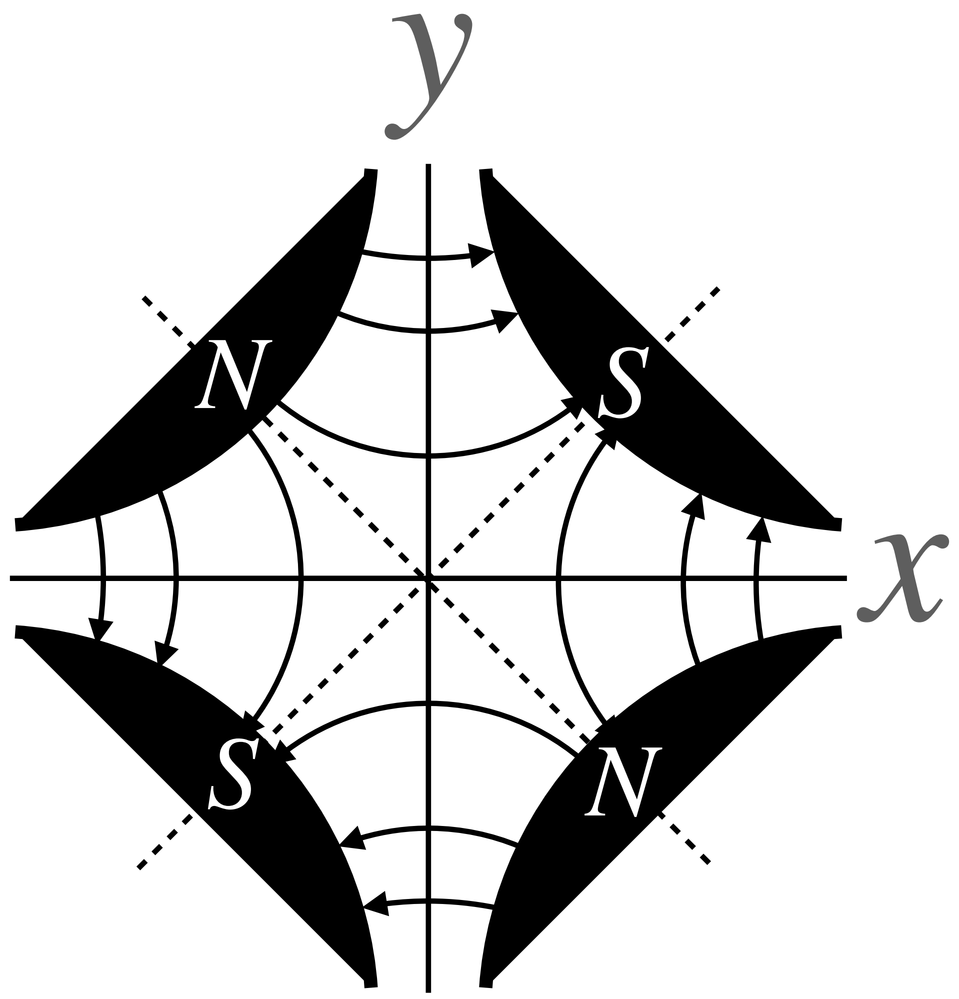
Comparing the strengths of electric force and magnetic force to a particle with one electron charge

Electric force : $\boldsymbol{F} = e\boldsymbol{E}$, $F = eE$, which has nothing to do with the speed of the particle.
Magnetic force: $\boldsymbol{F} = e\boldsymbol{v} \times \boldsymbol{B}$, which is proportional to the speed of the particle.



For a particle moving with the speed of light (which is most of cases in a high energy accelerator, especially for electrons), the magnetic force acting on it by a 1 Tesla magnetic field perpendicular to it is:

$$F = ecB = 3 \times 10^8 eV/m = 300MeV/m.$$



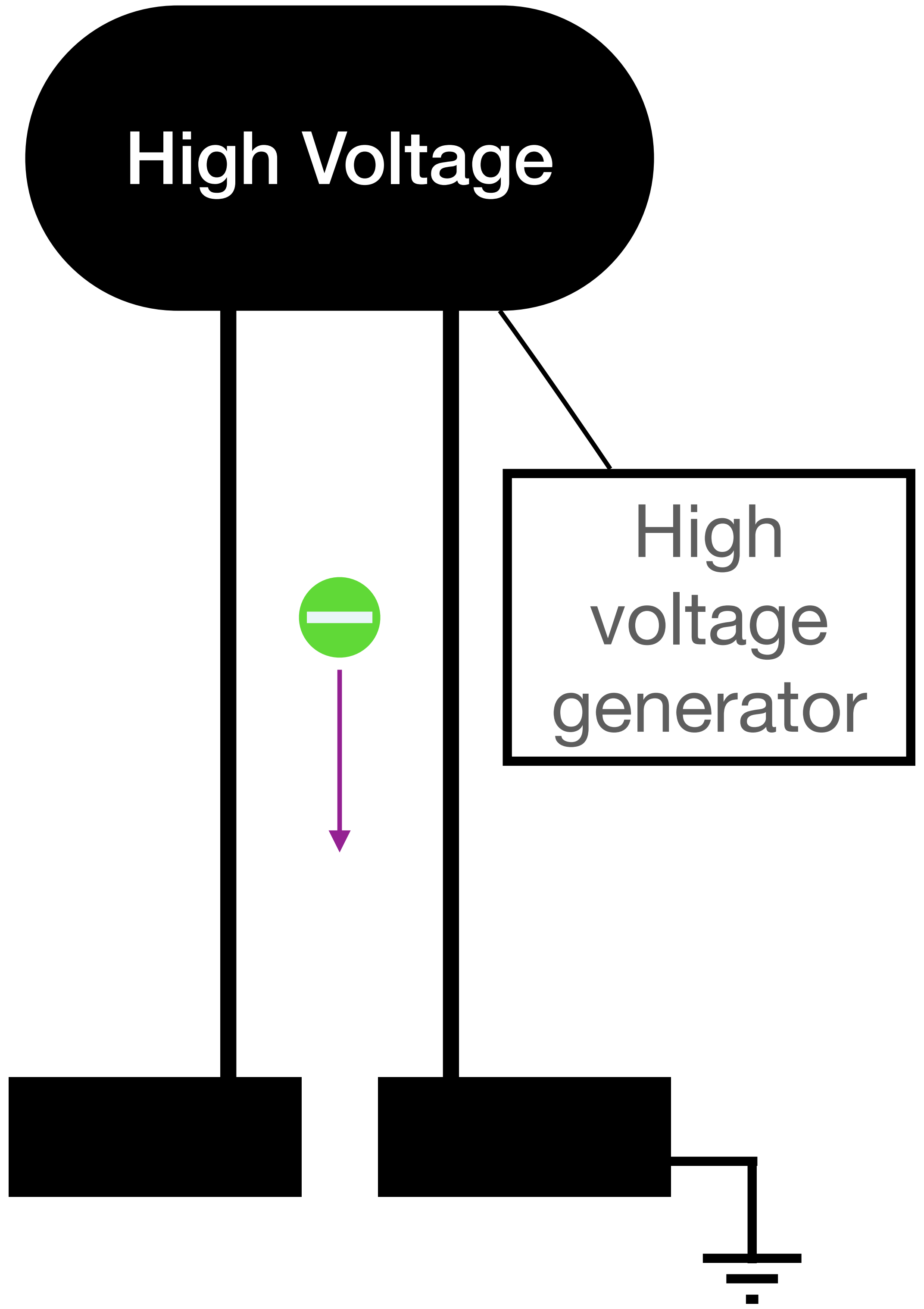
$B_x = Gy$

$B_y = Gx$

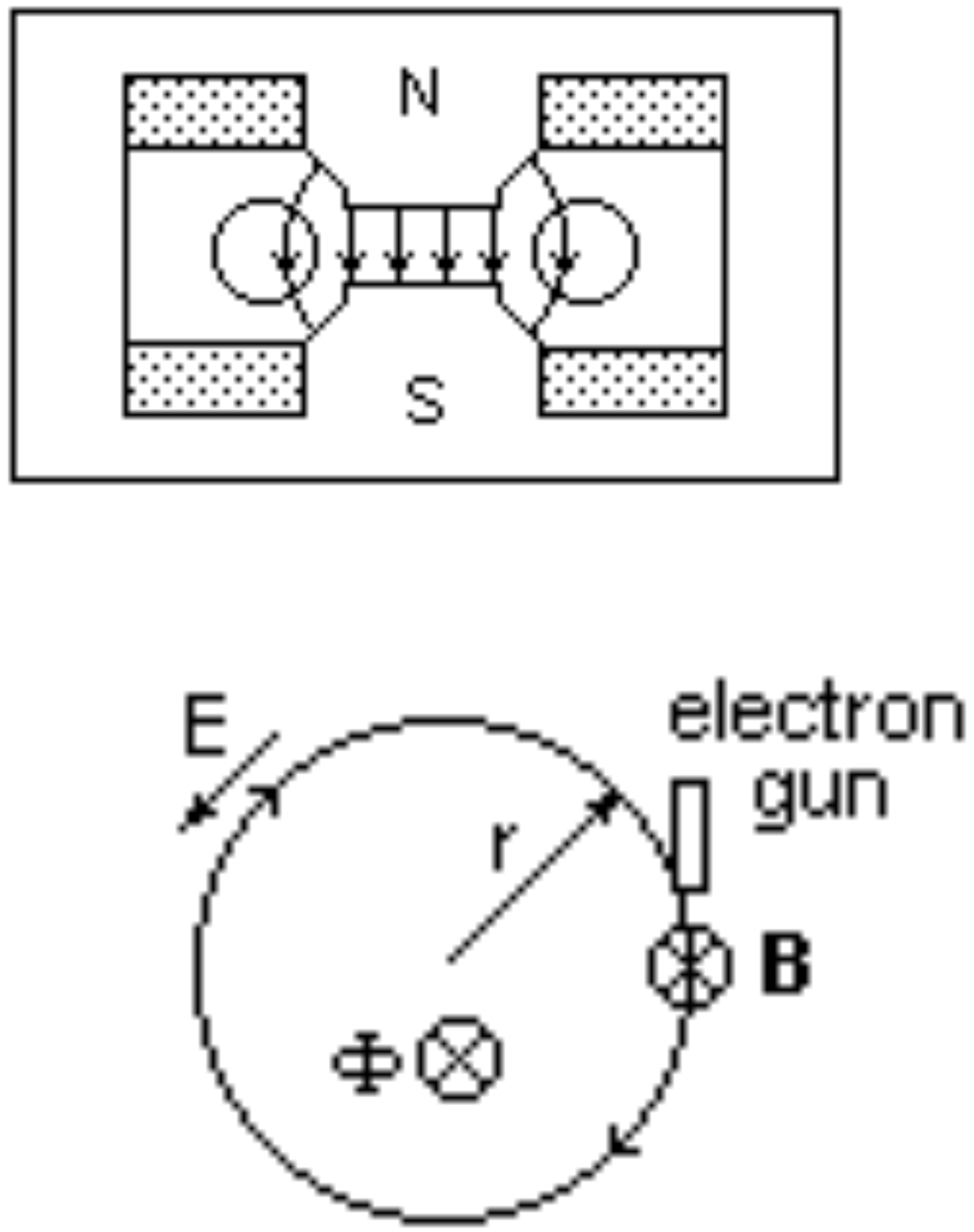
If we use an electric field to give the particle the same force, we need an electric field of 300MV/m, which is almost impossible. But a 1 Tesla magnetic fields can be easily achieved.

This is why we normally use magnetic dipoles to deflect and magnetic quadruple to focus charged particles in accelerators.

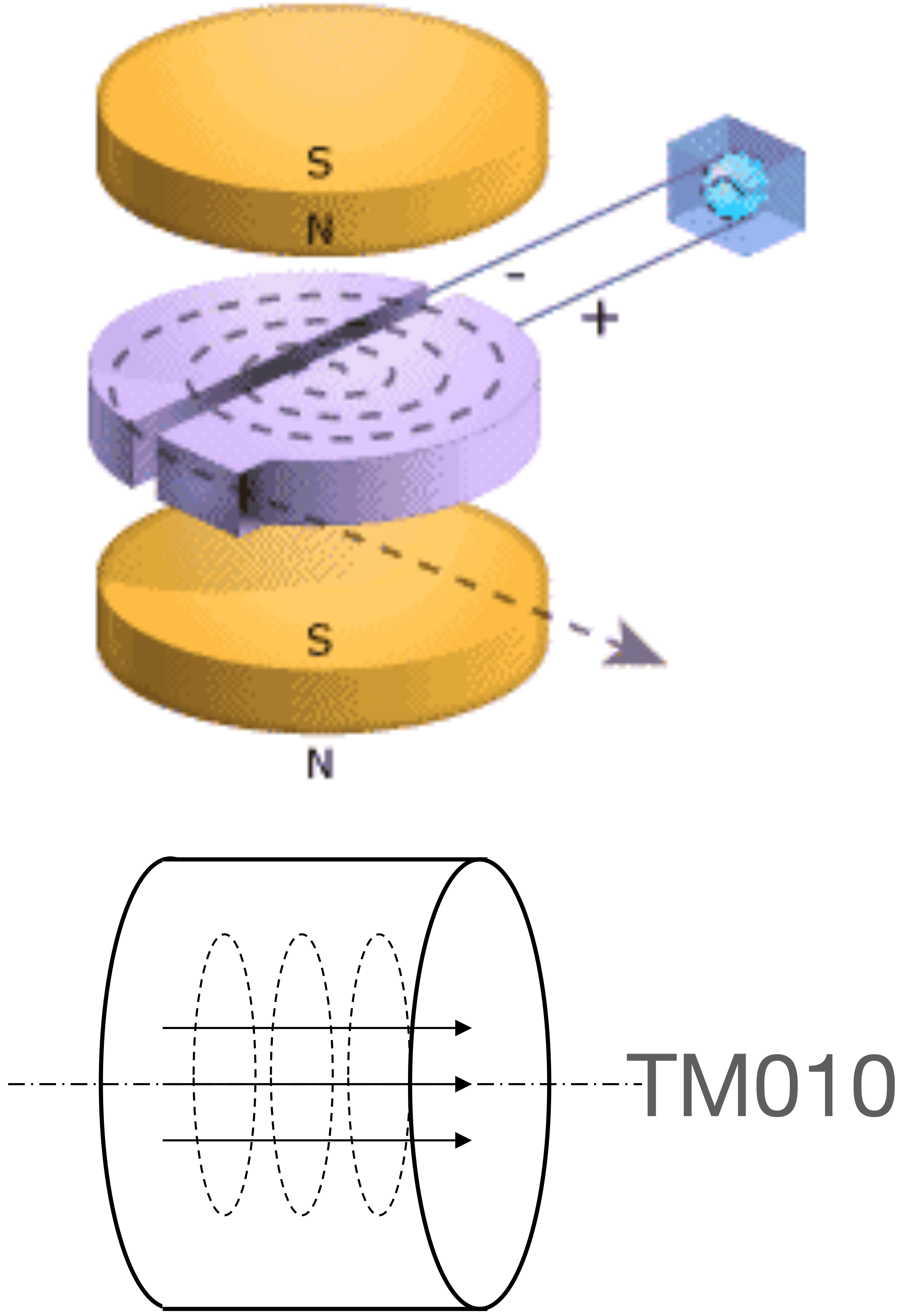
Accelerators can be categorized by its accelerating electric fields , particle orbit or kinds of particles accelerated



DC high voltage
(static electric field)



Induction Accelerator
(Induction electric field)



RF Acceleration
(Oscillating electromagnetic field)

1.1 High Voltage Accelerators

According to its high voltage generator, the high voltage accelerators can be named:

a) High voltage multiplier （Cockcroft-Walton）,

b) Insulated core transformer,

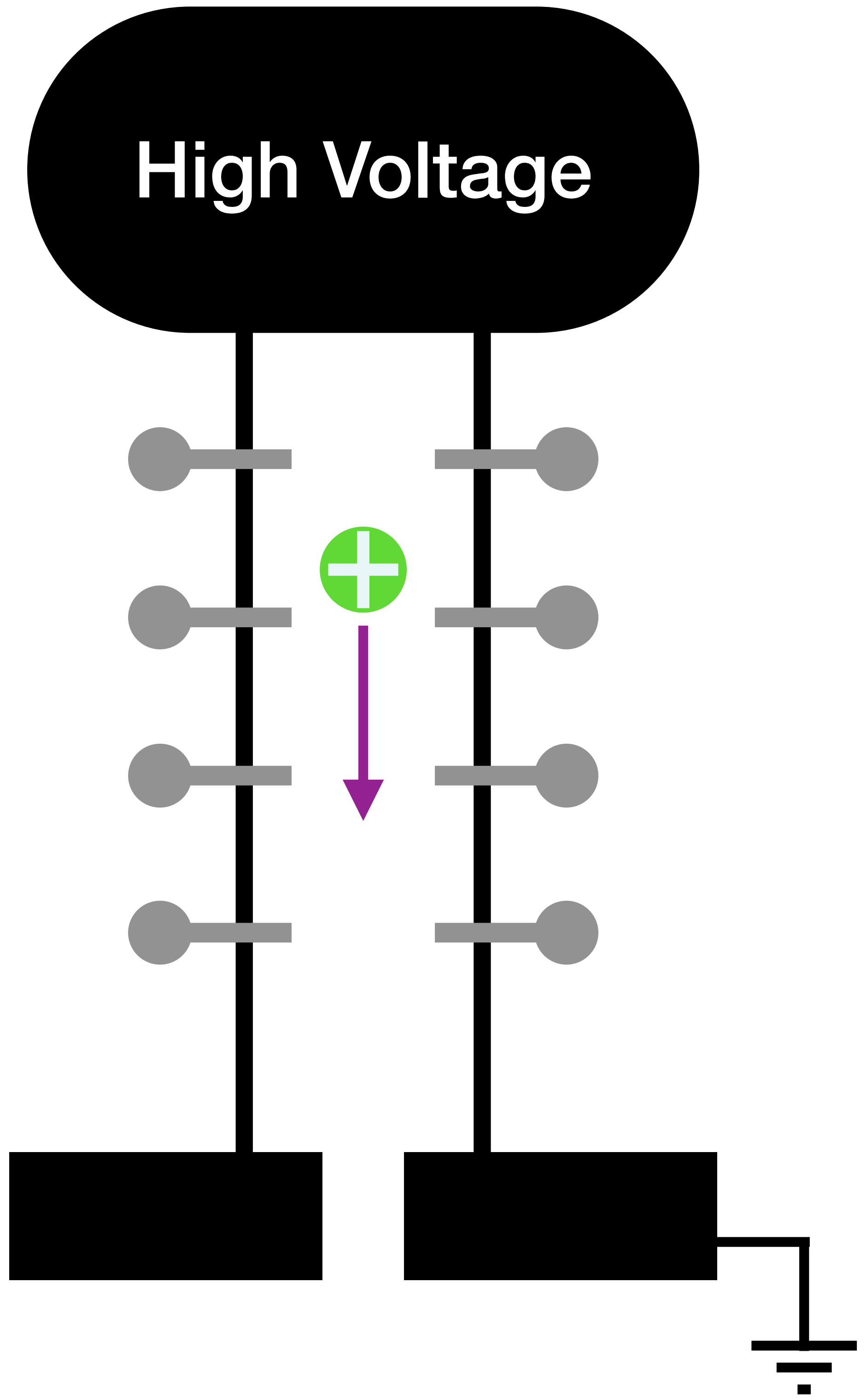
c) Electrostatic accelerator （Van de Graaff）,

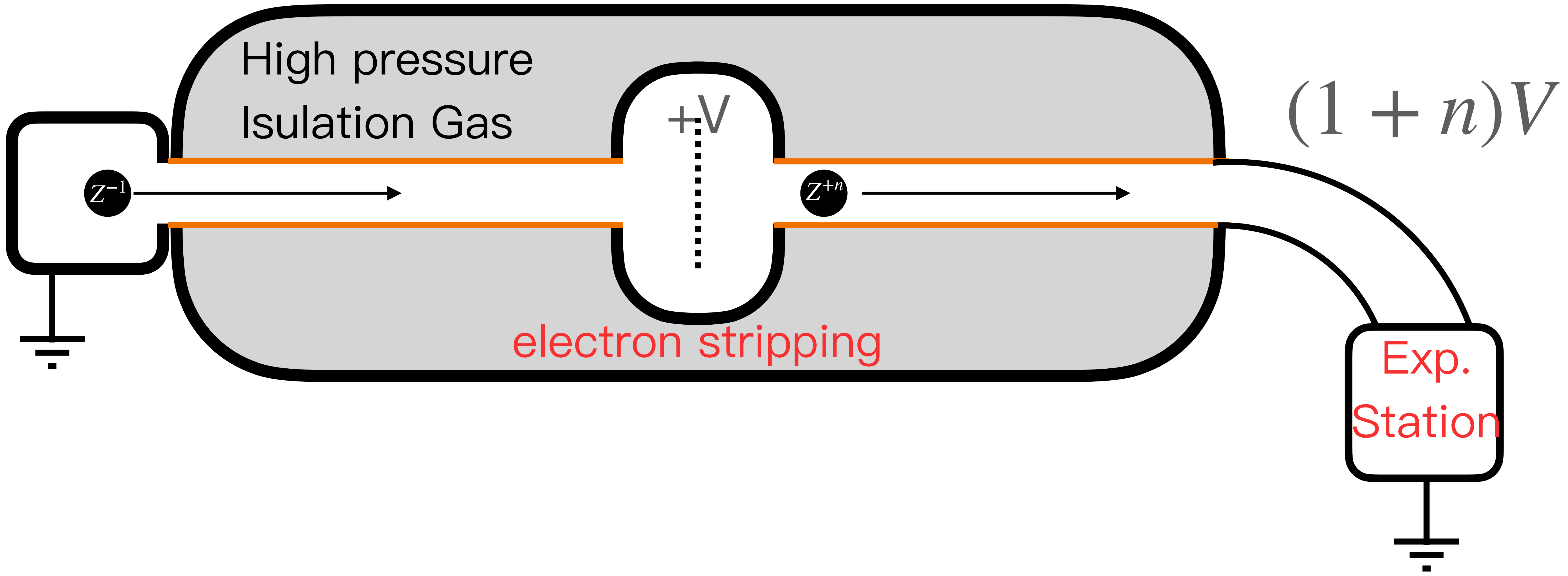
d) Tandem Van de Graaff

e) Dynamitron

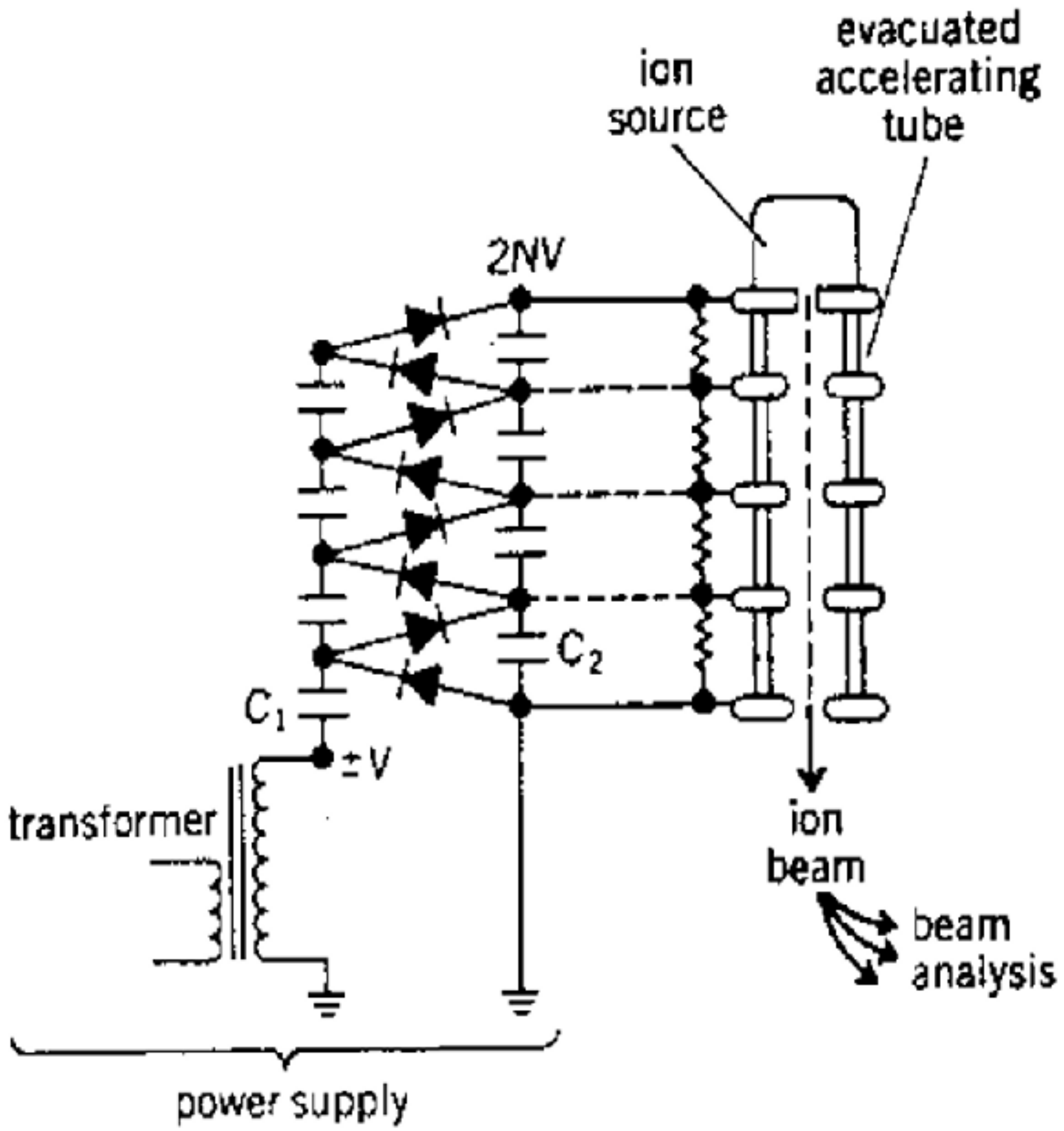
f) ……

High voltage breakdown is the main limitation to get even higher particle energy. Their particle energy is normally less than several MeV, except Tandem Van de Graaff, whose energy can exceed 30MeV.





High voltage multiplier （Cockcroft-Walton）



$$V_{\text{out}} \approx 2nV_0 - \frac{I_b}{fC} \cdot \frac{n(n+1)(2n+1)}{6}$$

V_0: Peak AC input voltage from the transformer (V)

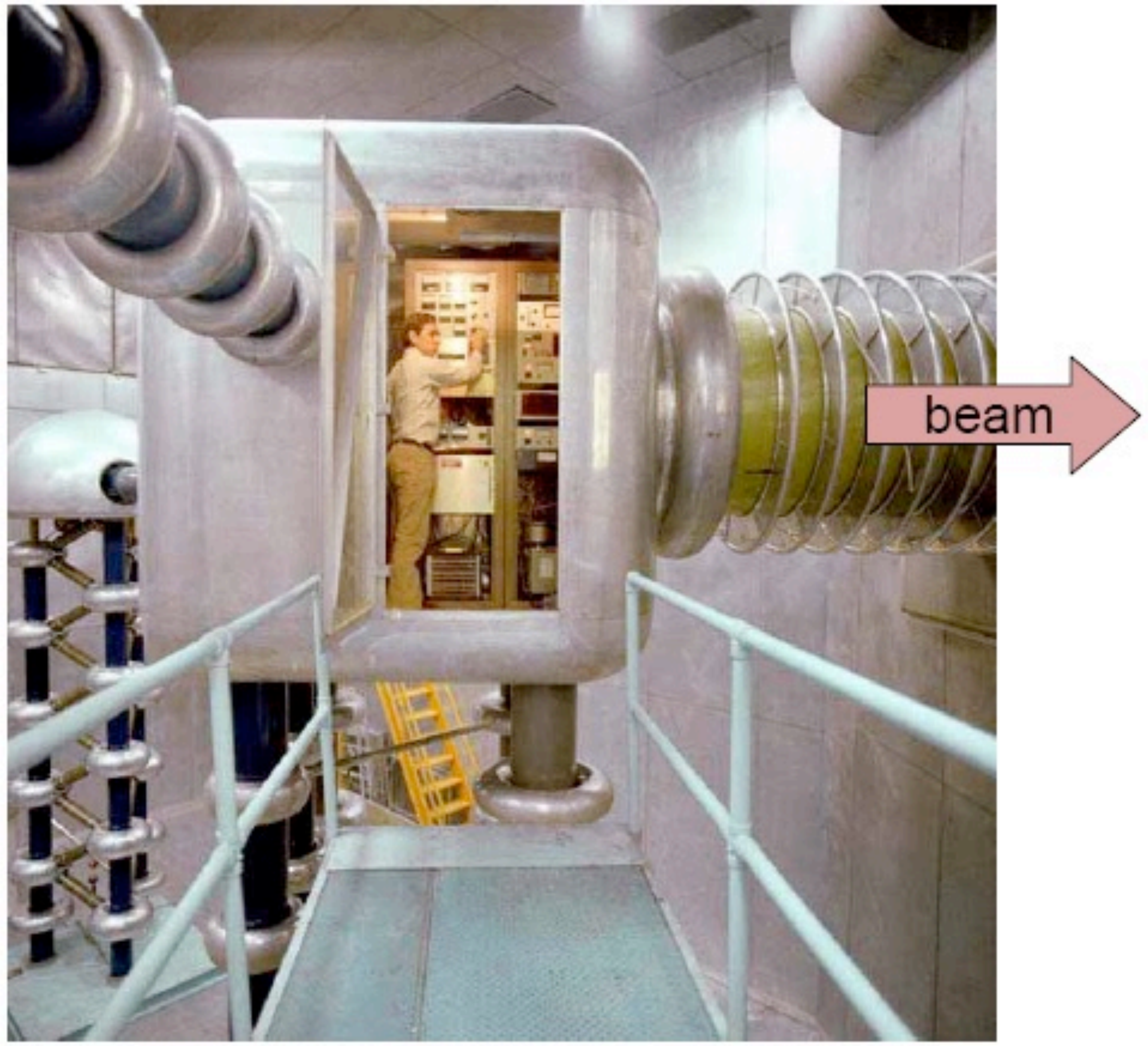
n: Number of multiplier stages

I_b: Beam current (A)

f: AC supply frequency (Hz)

C: Capacitance per stage (F)

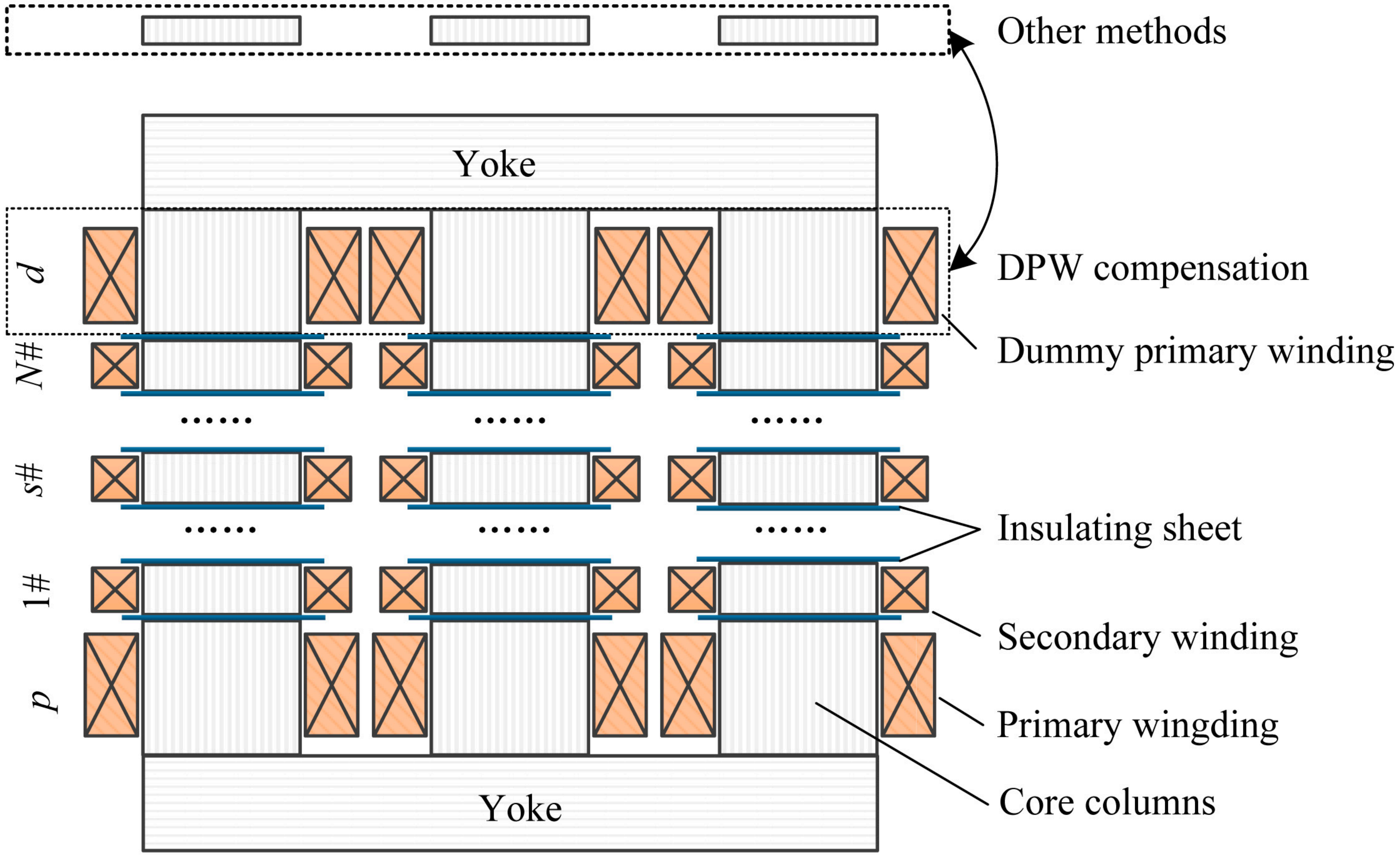
John Cockcroft, Ernest Rutherford, and E.T.S. Walton.



Crockroft-Walton at FNAL accelerates H^- to 750keV

https://history.aip.org/exhibits/lawrence/epa.htm

Insulated core transformer



$$V_{\text{out}} = n \cdot k \cdot V_{\text{prim}} - I_b \cdot R_{\text{eq}}$$

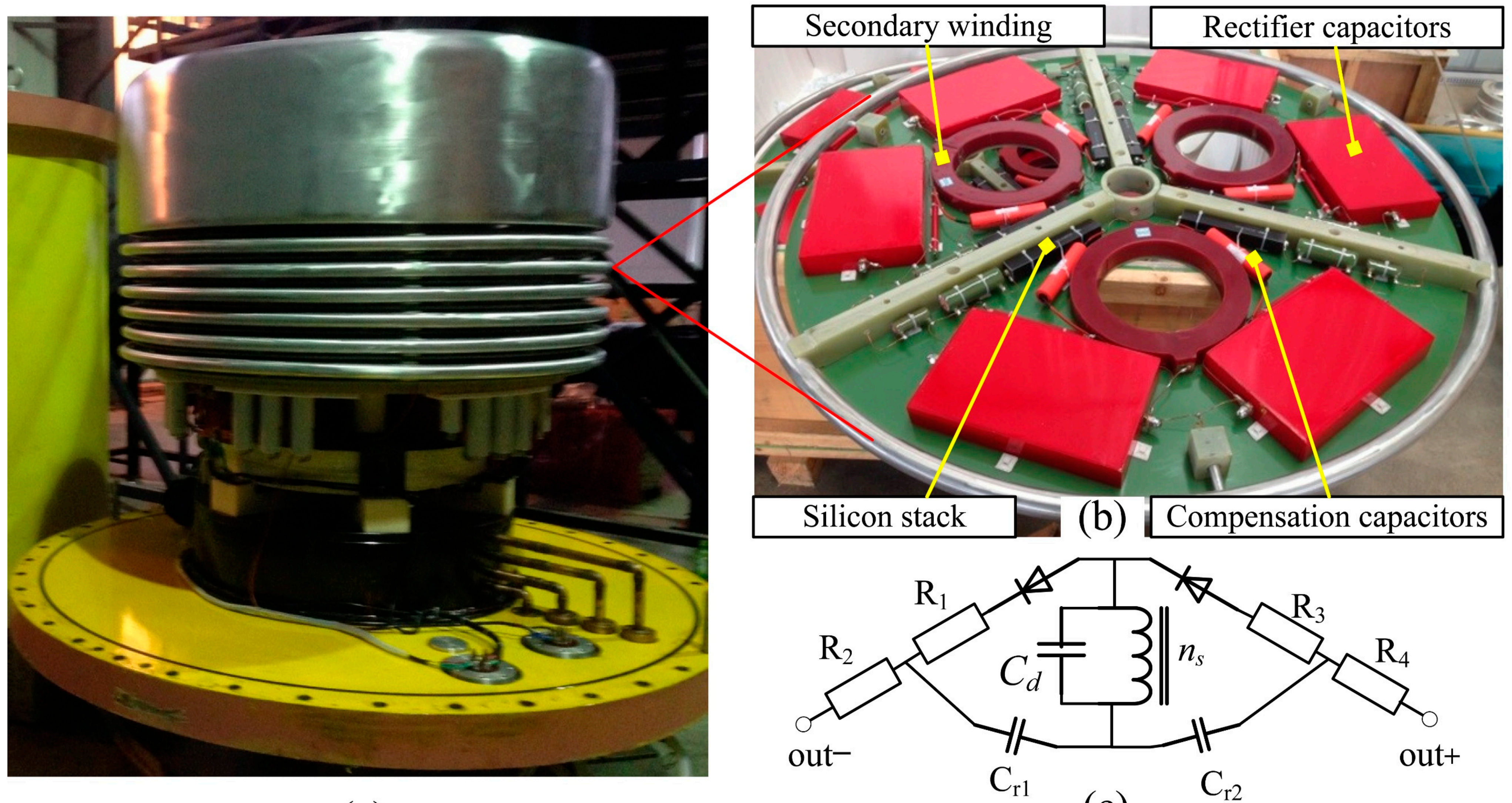
n : Number of stages

k : Magnetic coupling coefficient (≈0.9–0.95)

V_{prim}: Peak primary voltage

I_b : Beam current

R_{eq}: Equivalent resistance (diodes/cables)

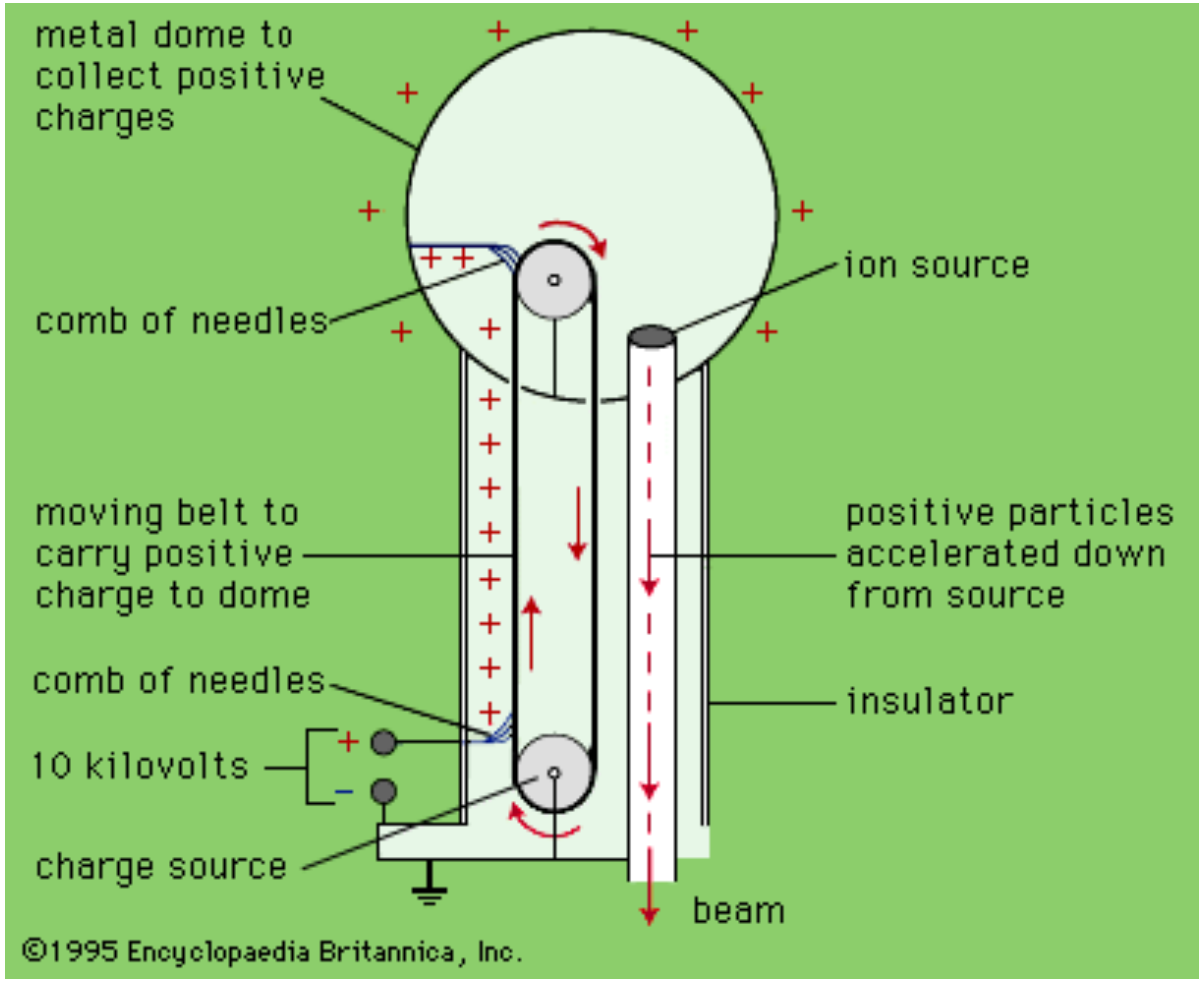


(a) (b) (c)

Voltage drop is linear with I_b, enabling stable high-current operation

• Yang, L.; Liu, X.; Yang, J. Research on Voltage Compensation Methods and Optimization Algorithm for Insulated Core Transformer High-Voltage Power Supply. Energies 2024,17,547. https://doi.org/ 10.3390/en17030547

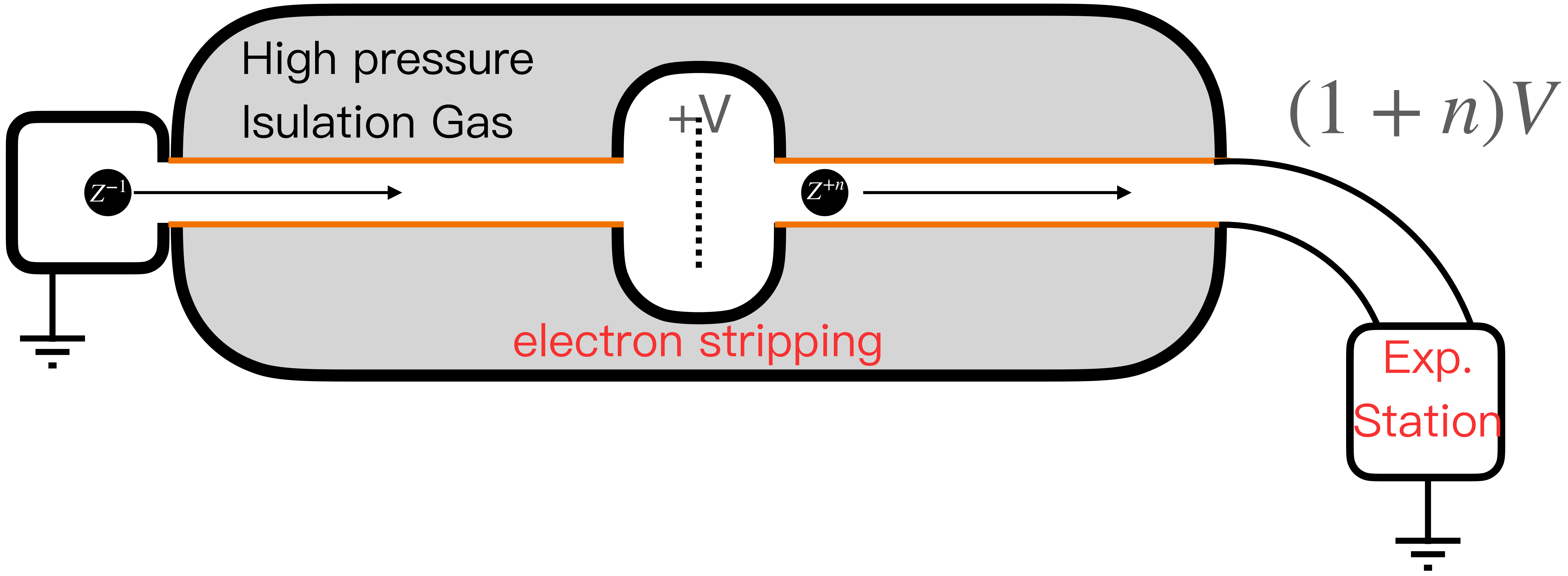
Electrostatic accelerator （Van de Graaff）

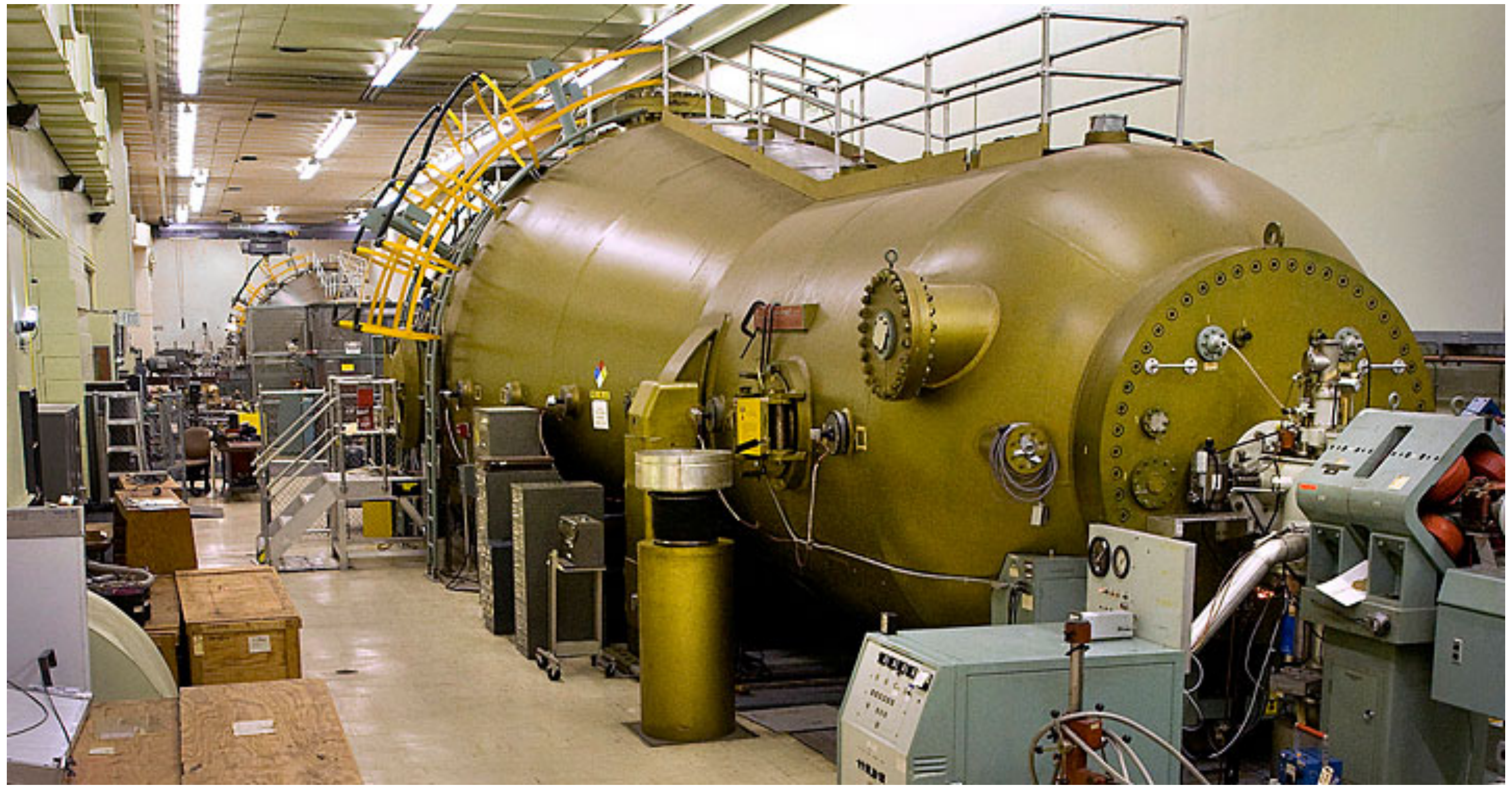


Van de Graaf's very large accelerator built at MIT's Round Hill Experiment Station in the early 1930s. 43feet above the ground.

- "Engines of Discovery-A Century of Particle Acceleators" , A. Sessler, E. Wilson. 2007

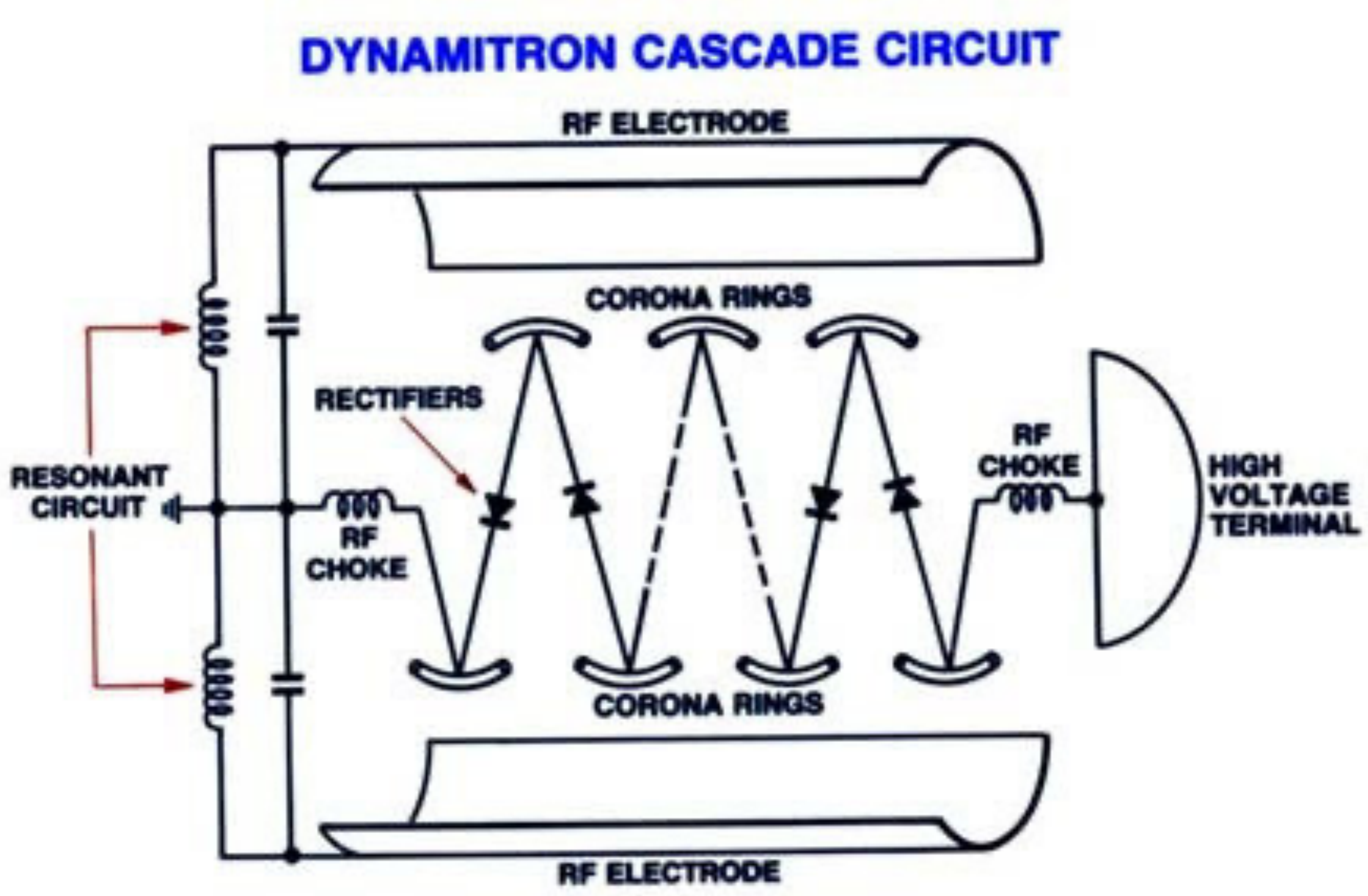
Tandem Accelerator





The Tandem Van de Graaff Facility consists of two 15-megavolt electrostatic accelerators capable of delivering continuous, or high-intensity pulsed ion beams in a wide range of ion species at various energies to experimental chambers that are available to researchers on a full cost-recovery basis

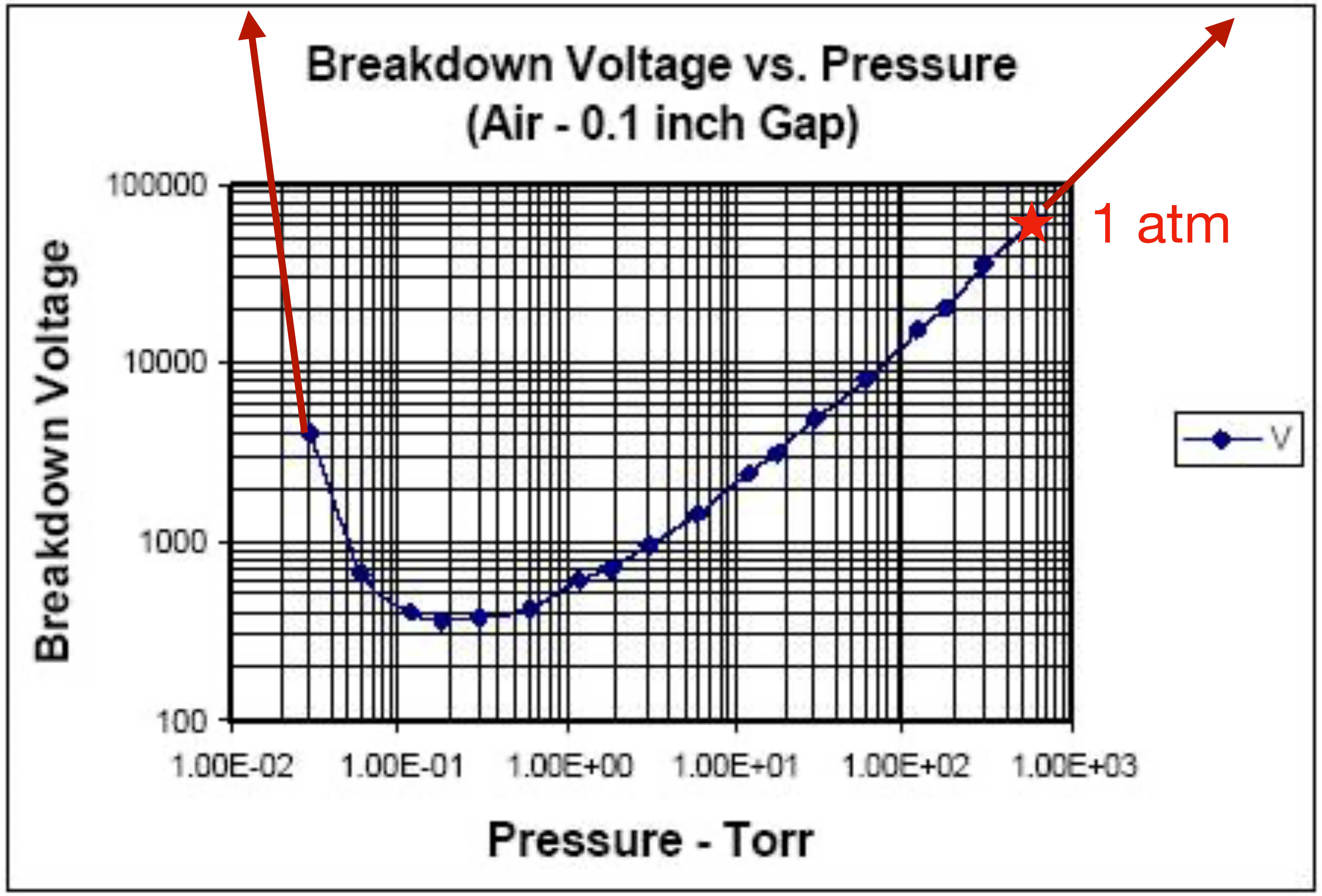
Dynamitron



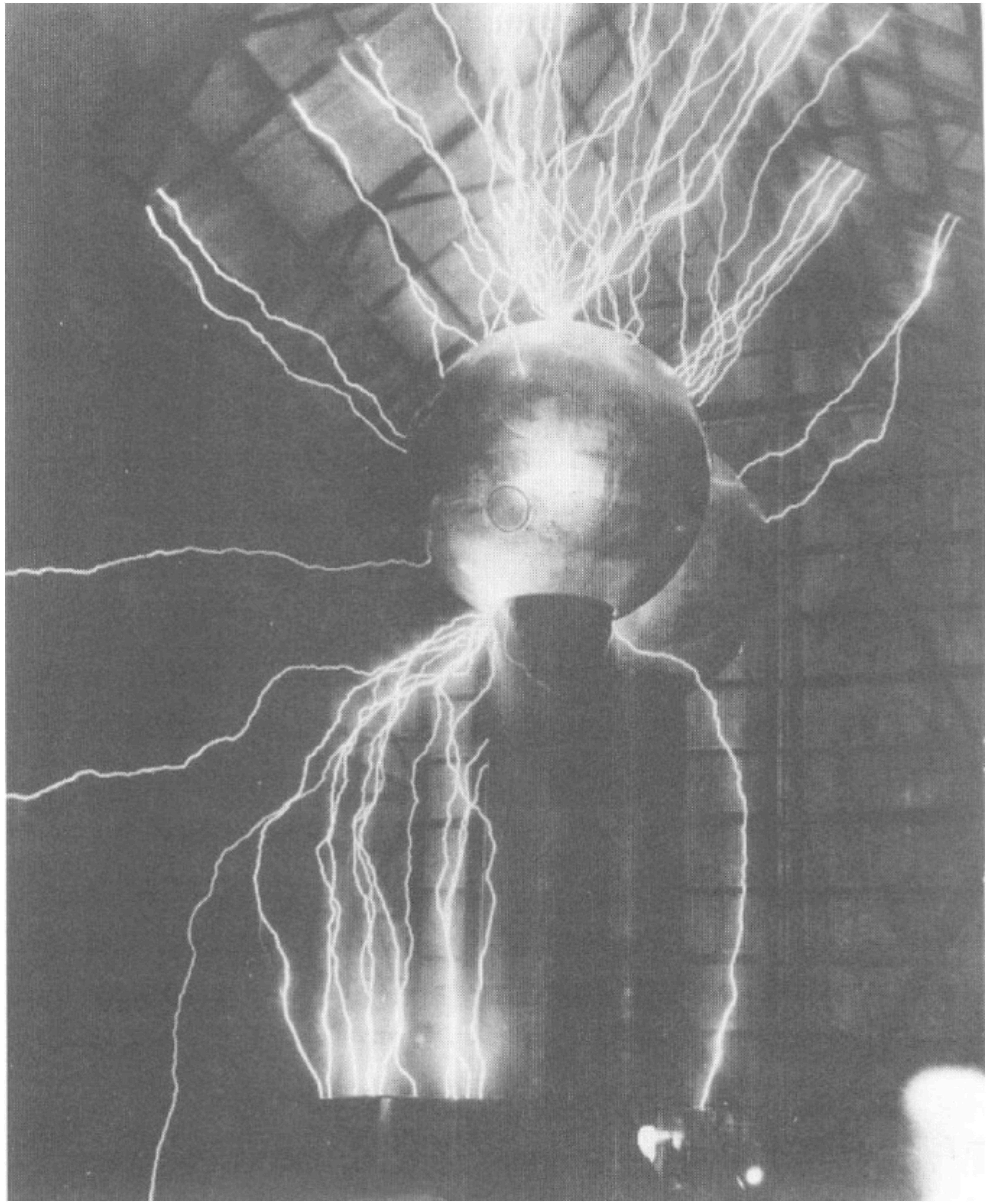
The high-frequency electrode (typically featuring a stepped configuration) generates a radio-frequency alternating electric field, which is capacitively coupled through spatial capacitance to the corona rings surrounding the core column. This signal undergoes multi-stage parallel rectification before being series-stacked to produce a DC high-voltage electric field.

High Voltage Breakdown

Normally air medium is widely use as an insulating medium in different electrical power equipments and over head lines as its breakdown strength is 30kV/cm.

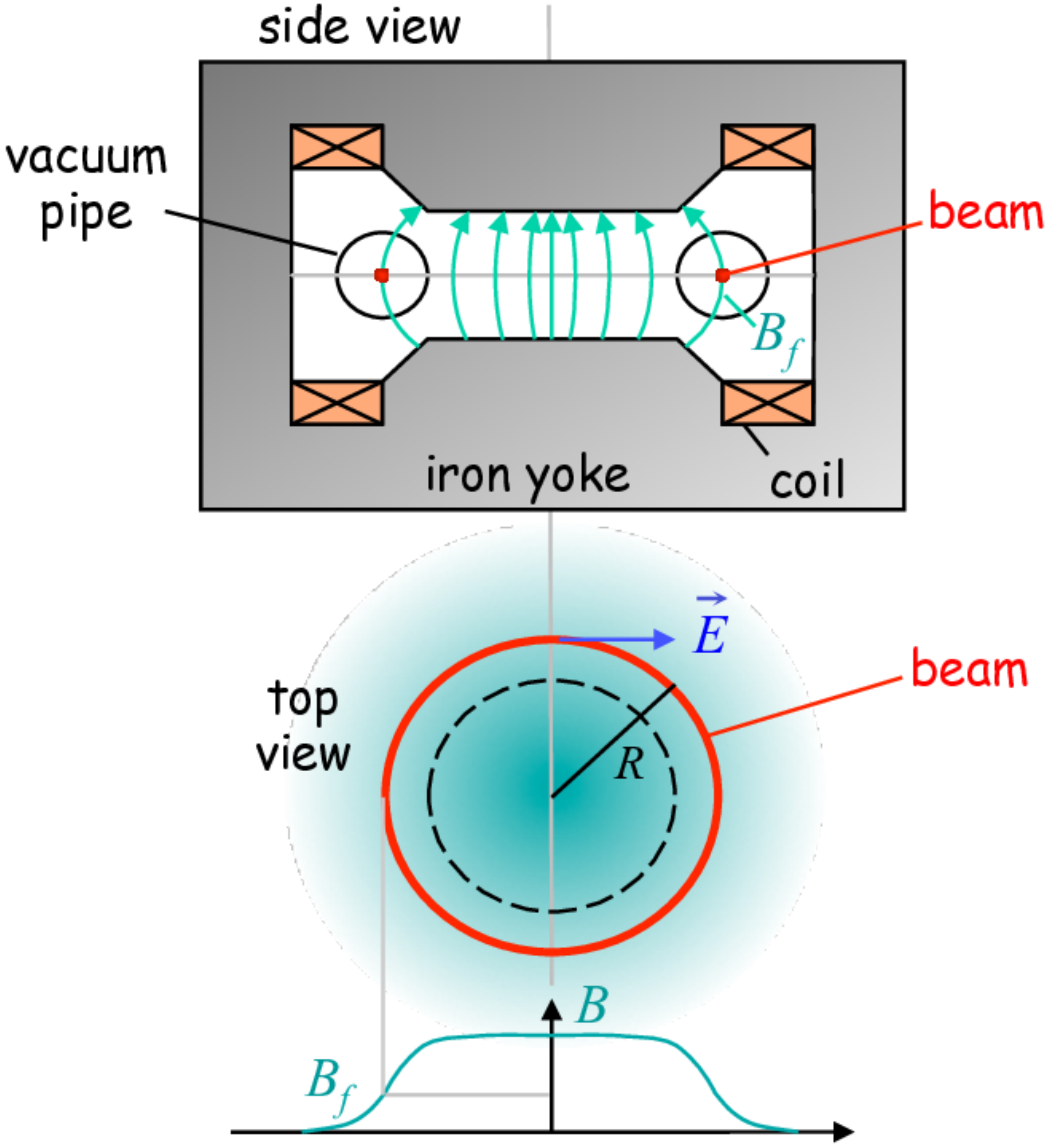


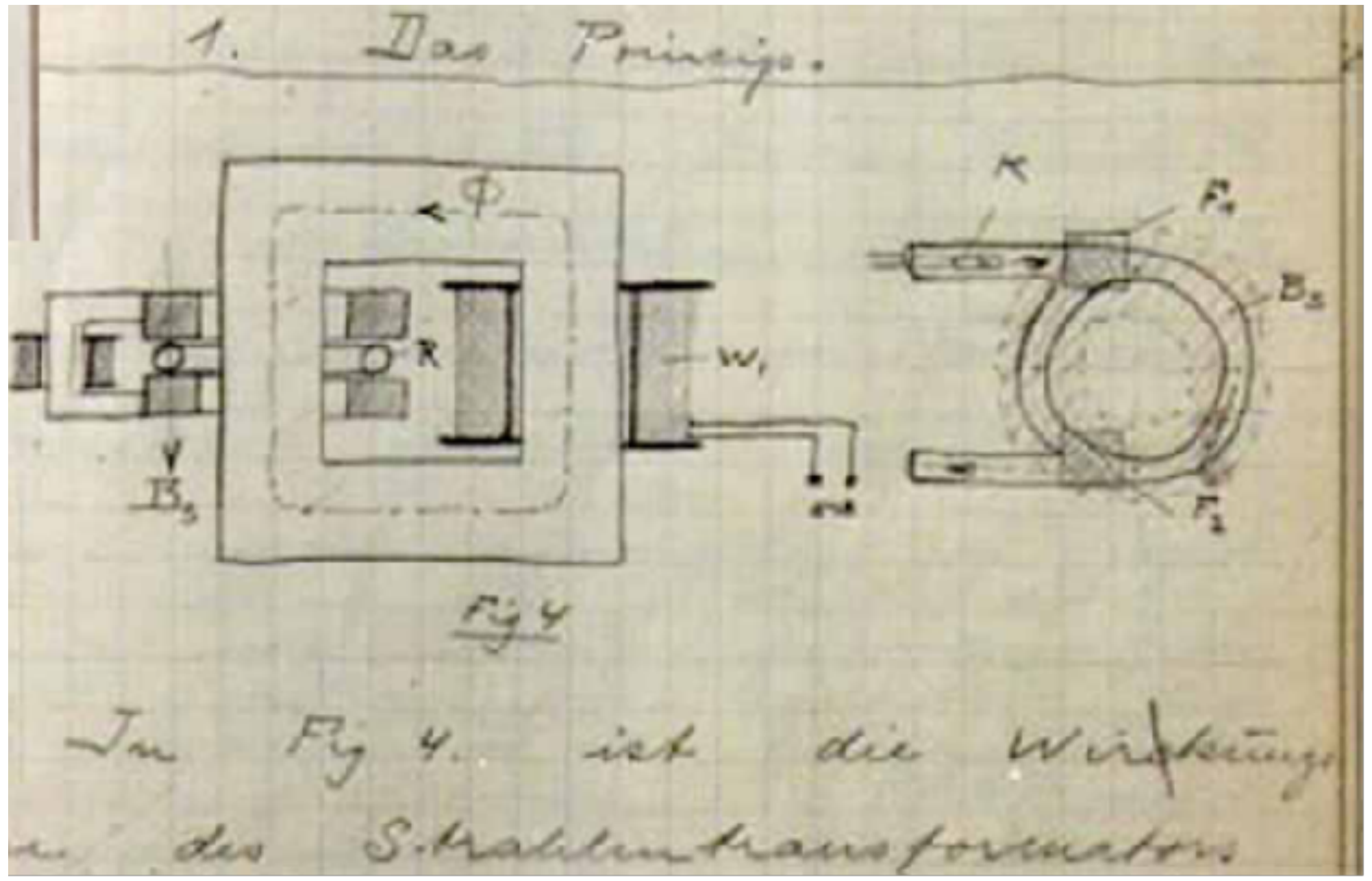
The "Paschen Curve" for air, two flat parallel copper electrodes, separated by 1 inch, for pressures between 3×10^{-2} Torr and 760 Torr.



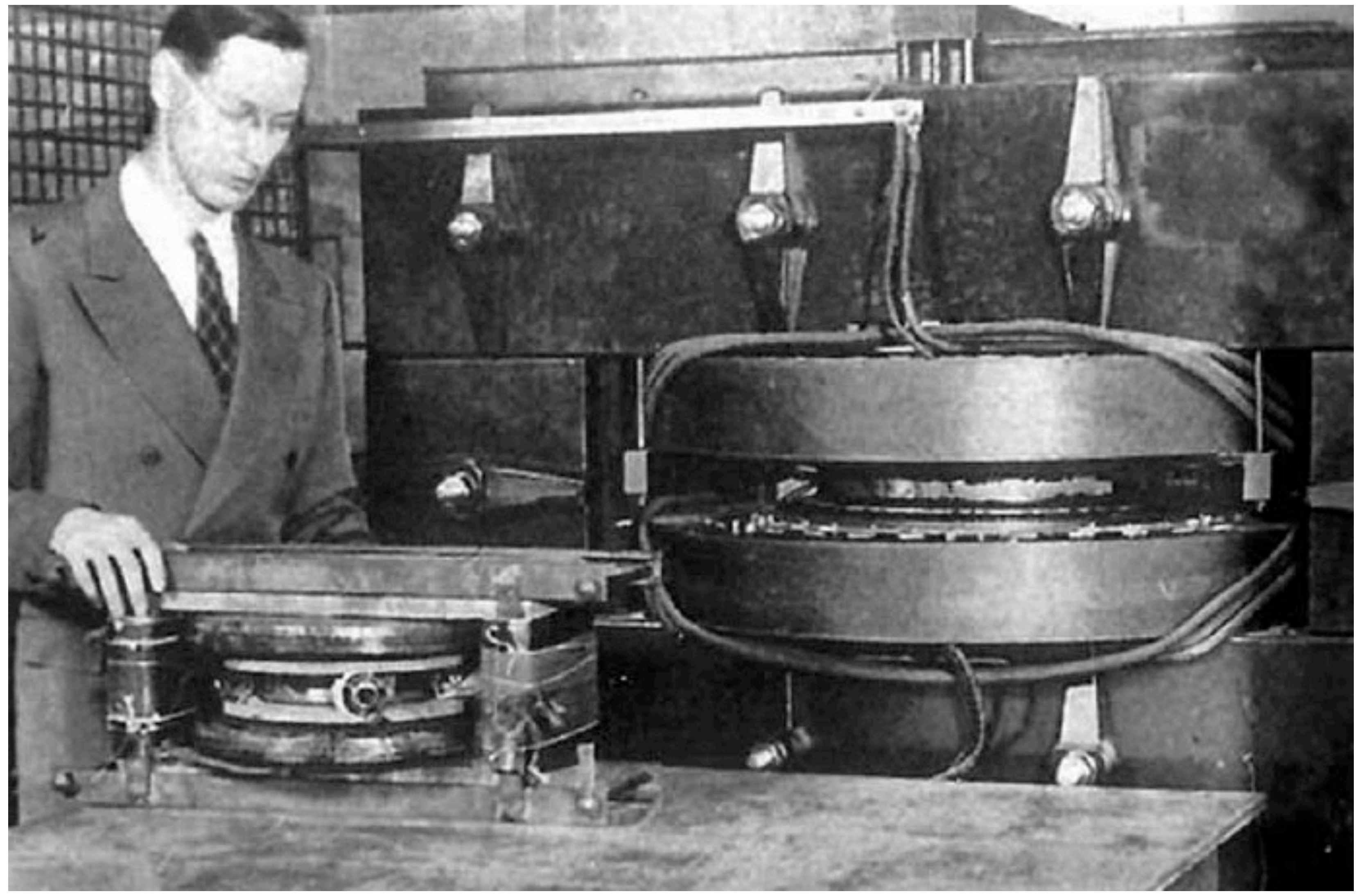
- "Engines of Discovery-A Century of Particle Accelerators" , A. Sessler, E. Wilson. 2007

1.2 Induction Accelerators

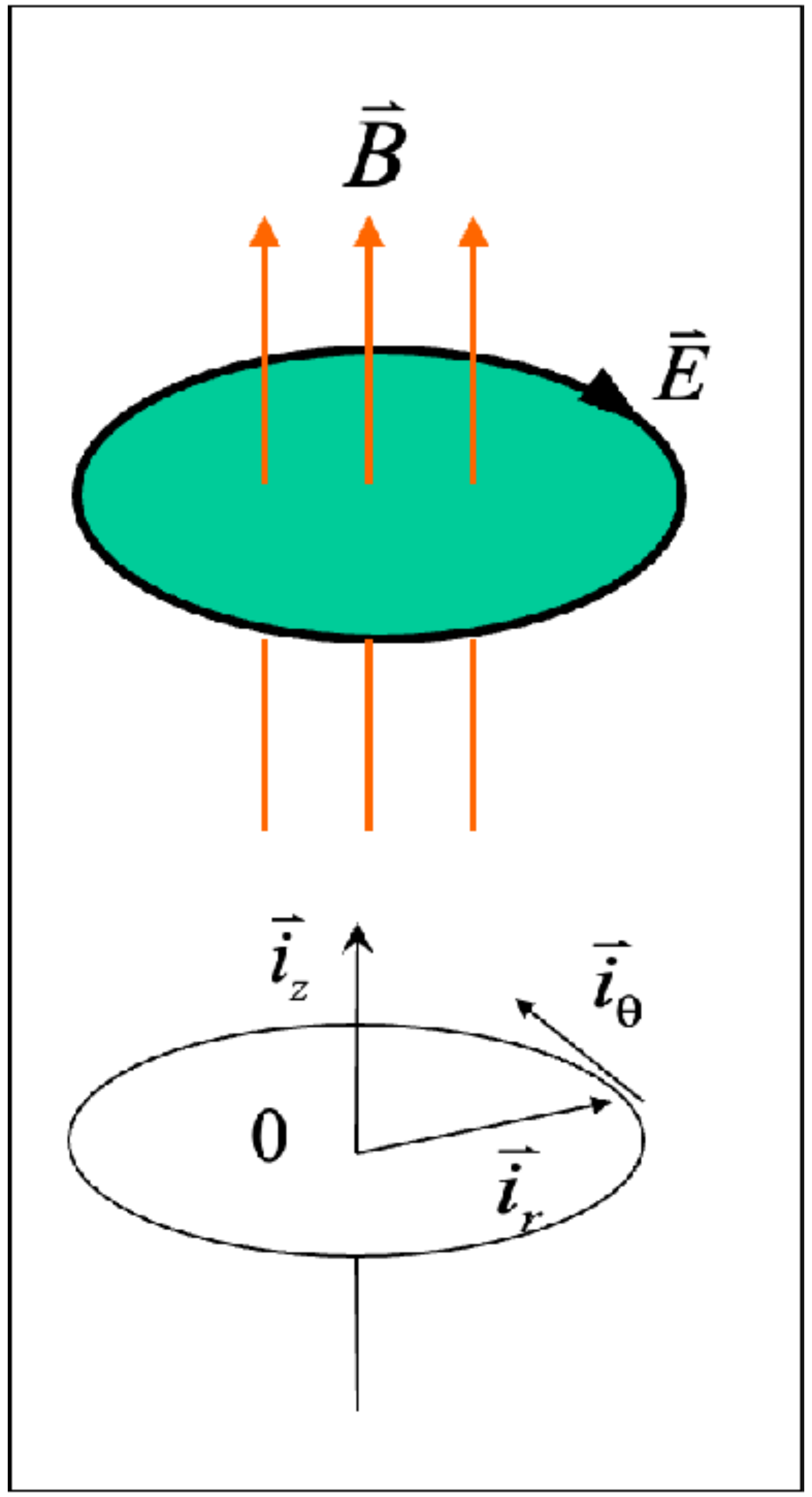




The original idea is given by Wiederoe's ray transformer in 1920s



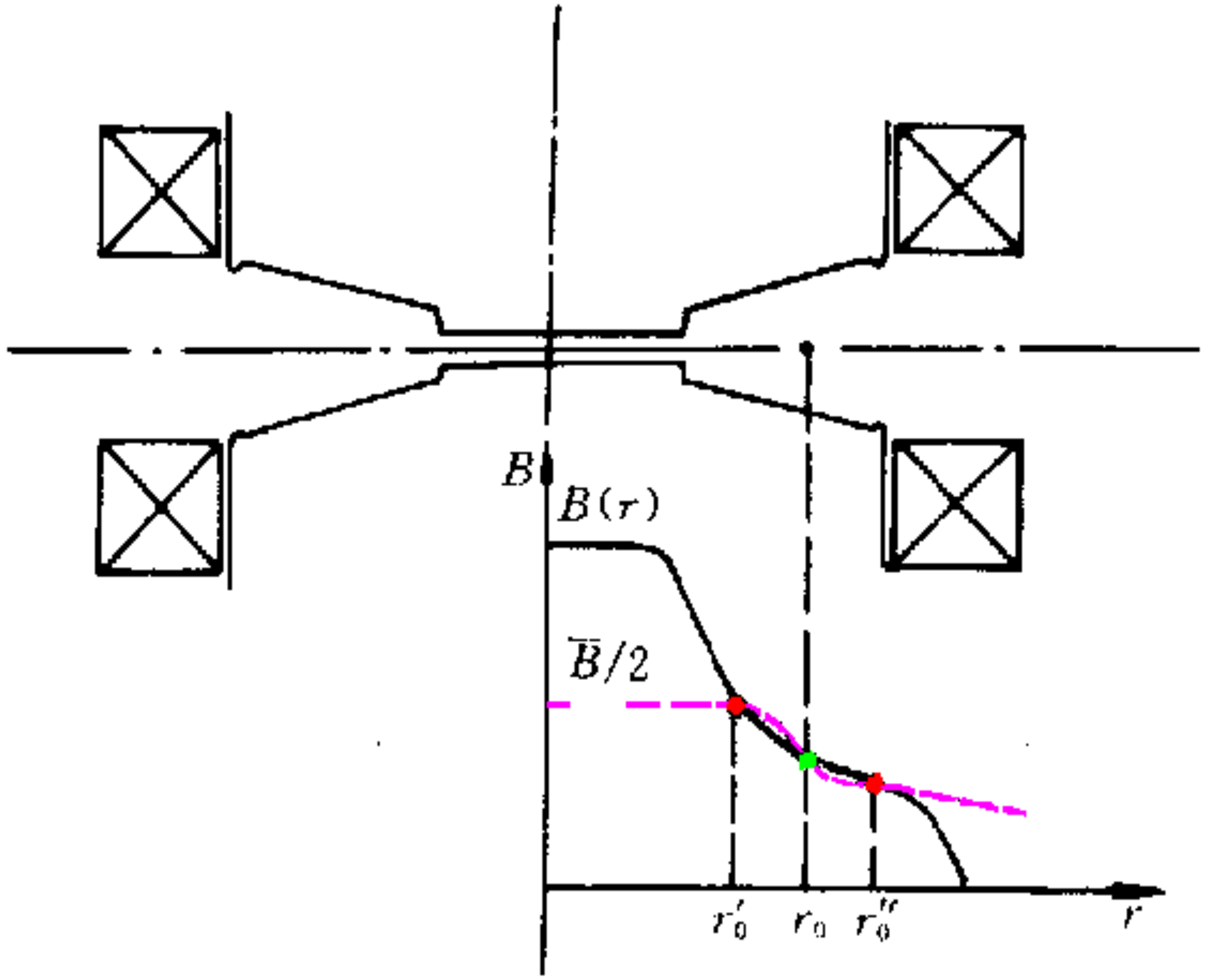
Donald Kerst built the world's first magnetic induction accelerator at the University of Illinois in 1940



$$\oint \vec{E} \bullet d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \bullet d\vec{S} = -\frac{\partial \Phi}{\partial t}$$

To keep the electrons on the orbit, the magnetic field of the orbit and the average magnetic field inside the orbit must satisfy the **2:1 condition.**

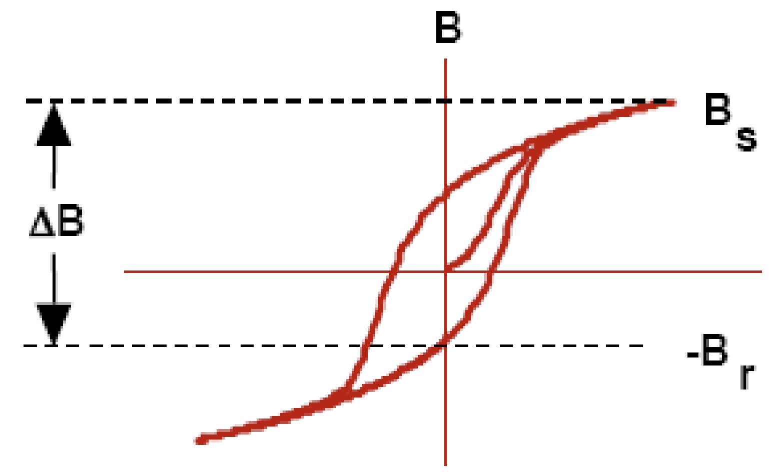
$$\frac{dB_0(t)}{dt} = \frac{1}{2} \cdot \frac{d\bar{B}_0(t)}{dt}$$



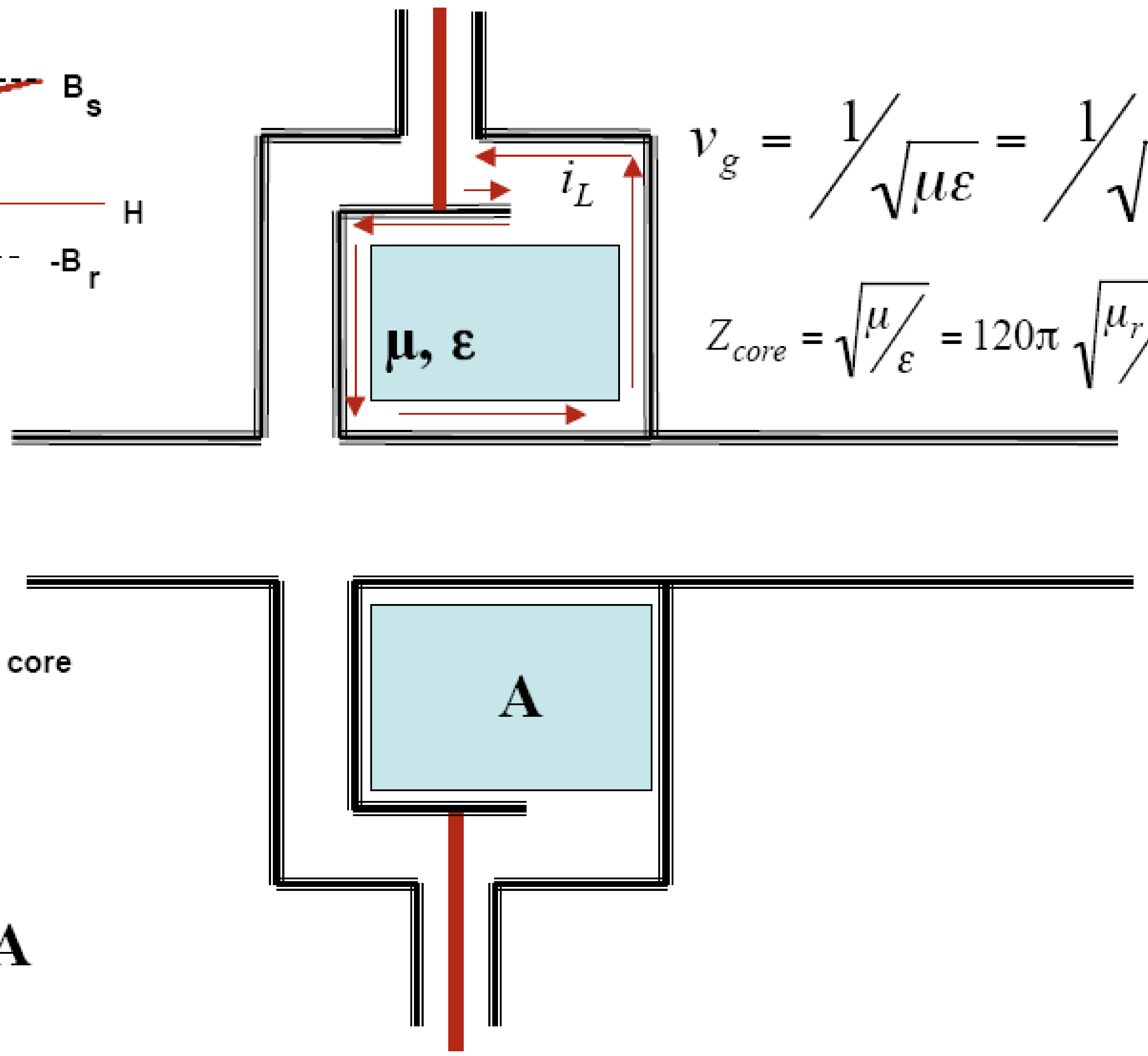
For $B_0(0) = \bar{B}_0(0) = 0,\ B_0(t) = \frac{1}{2}\bar{B}_0(t)$. There are three points satisfying the condition.

But only the middle one is stable in transverse. WHY?

Linear Induction Accelerator



Core hysteresis loop



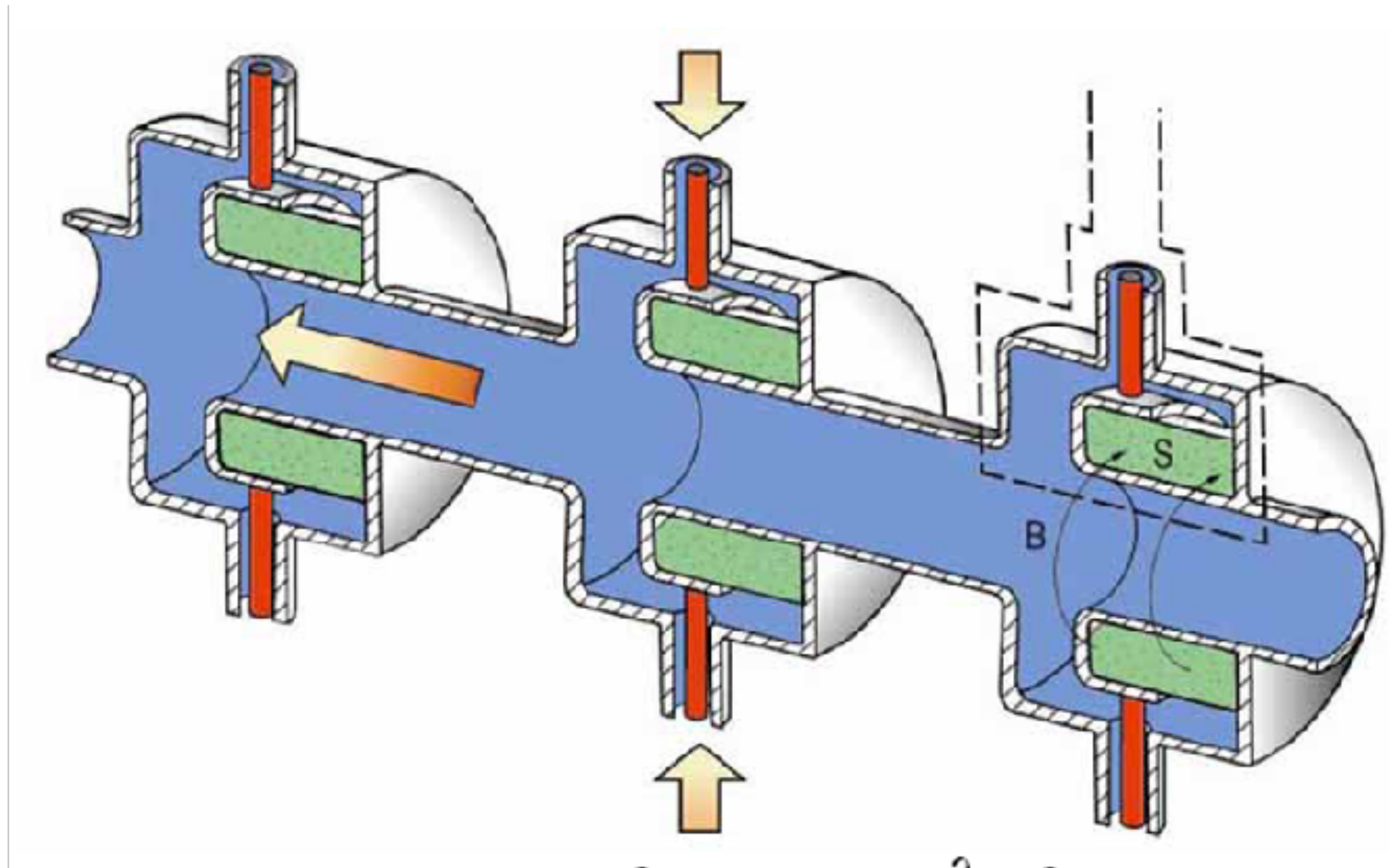
$$v_g = 1/\sqrt{\mu\varepsilon} = 1/\sqrt{\mu_r \varepsilon_r}$$

$$Z_{core} = \sqrt{\mu/\varepsilon} = 120\pi \sqrt{\mu_r/\varepsilon_r} \text{ Ohms}$$

Leakage current magnetizes core

$$i_L = \frac{V}{L_c} t$$

$$V \cdot \Delta t = \Delta B \cdot A$$



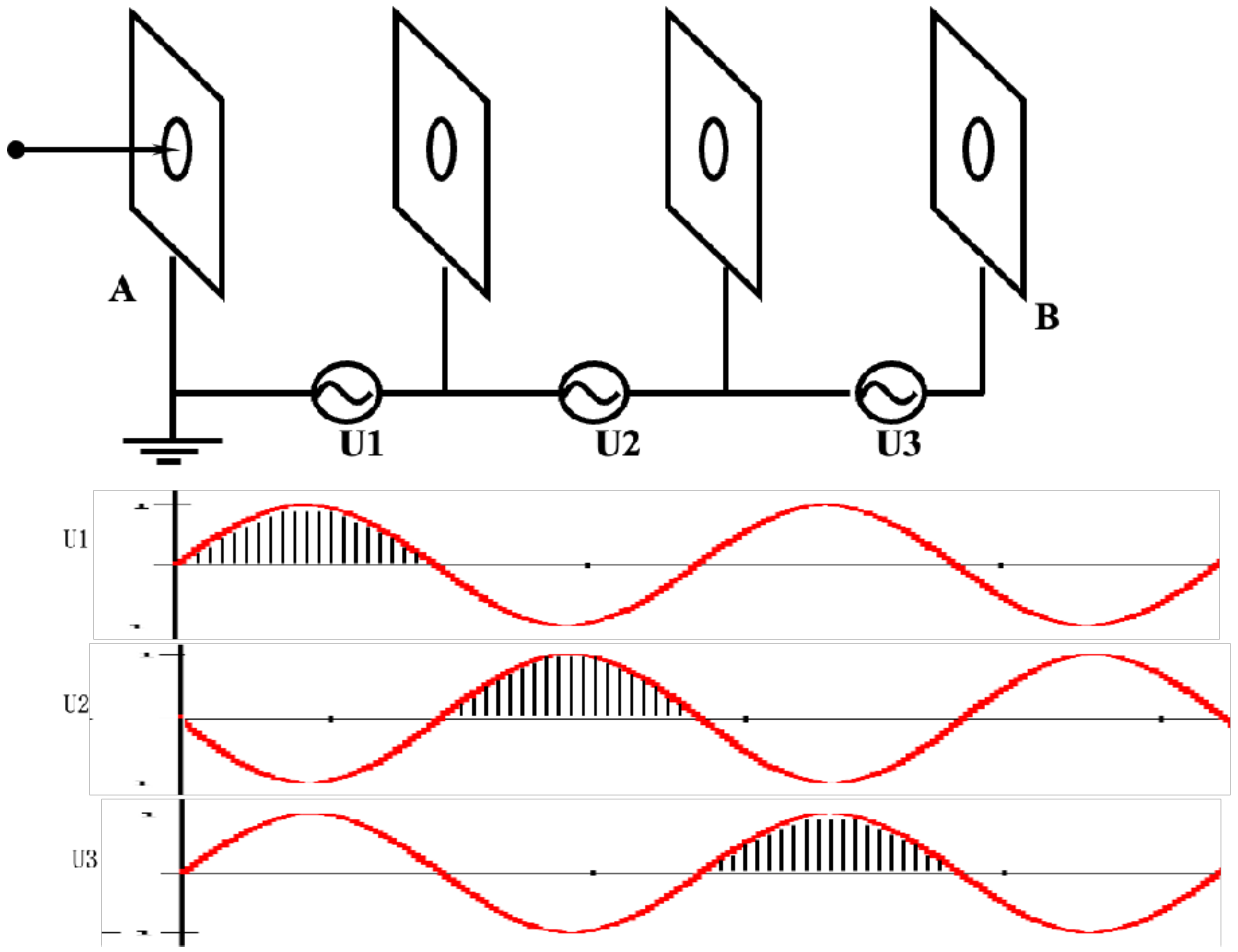
$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{s}$$

The LIA is suitable to accelerate high current electron beams of several kilo-Amperes to tens of mega-electronvolts, with pulse width of about tens of nano-seconds.

1.3 Radio-Frequency Accelerators

The charged particles can be accelerated to a much higher energy than the highest voltage of the machine.

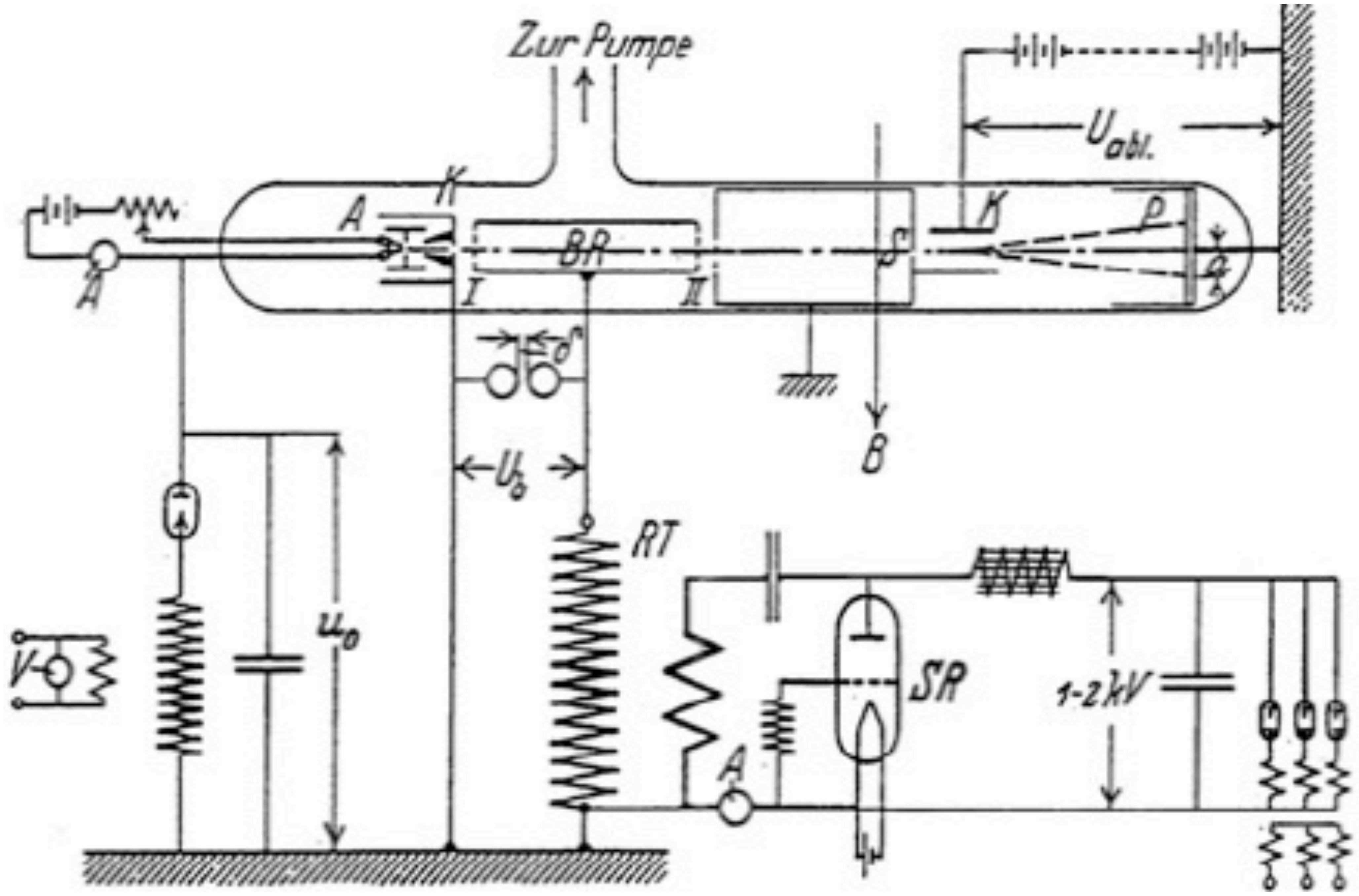
Synchronization is needed for the particles and the RF fields.

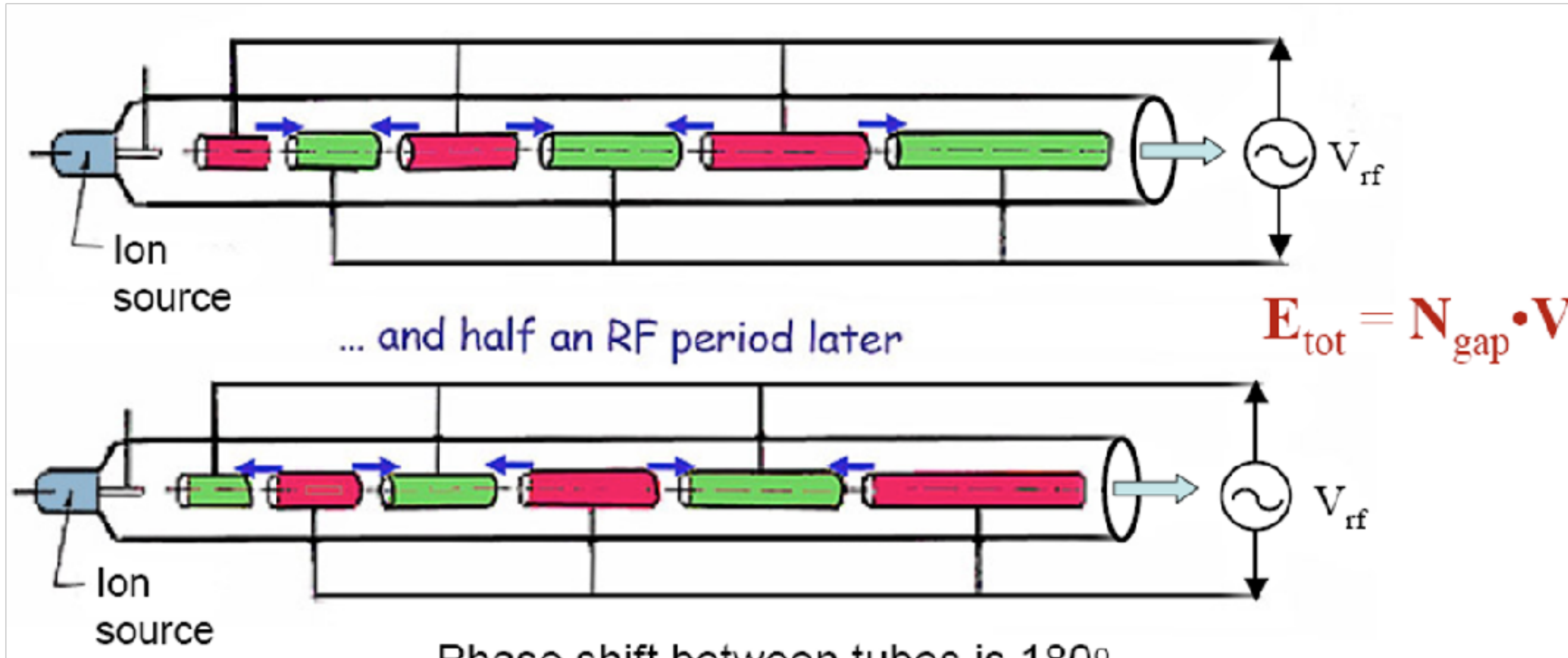


1.3.1 Linear Accelerators:

DTL - Drift Tube Linac
Alvarez linac
RFQ - Radio Frequency Quadruple
SW Linac
TW Linac
……

DTL - Drift Tube Linac (Wiederoe)





Phase shift between tubes is 180°

As the ions increase their velocity, drift tubes must get longer

$$L_{drift} = \frac{1}{2}\frac{v}{f_{rf}} = \frac{1}{2}\frac{\beta c}{f_{rf}} = \frac{1}{2}\beta\lambda_{rf}$$

In 1928, Rolf Widerioe

RF frequency： 1MHz

Voltage of gap： 25kV

Ion： K+

Final Energy： 50kV

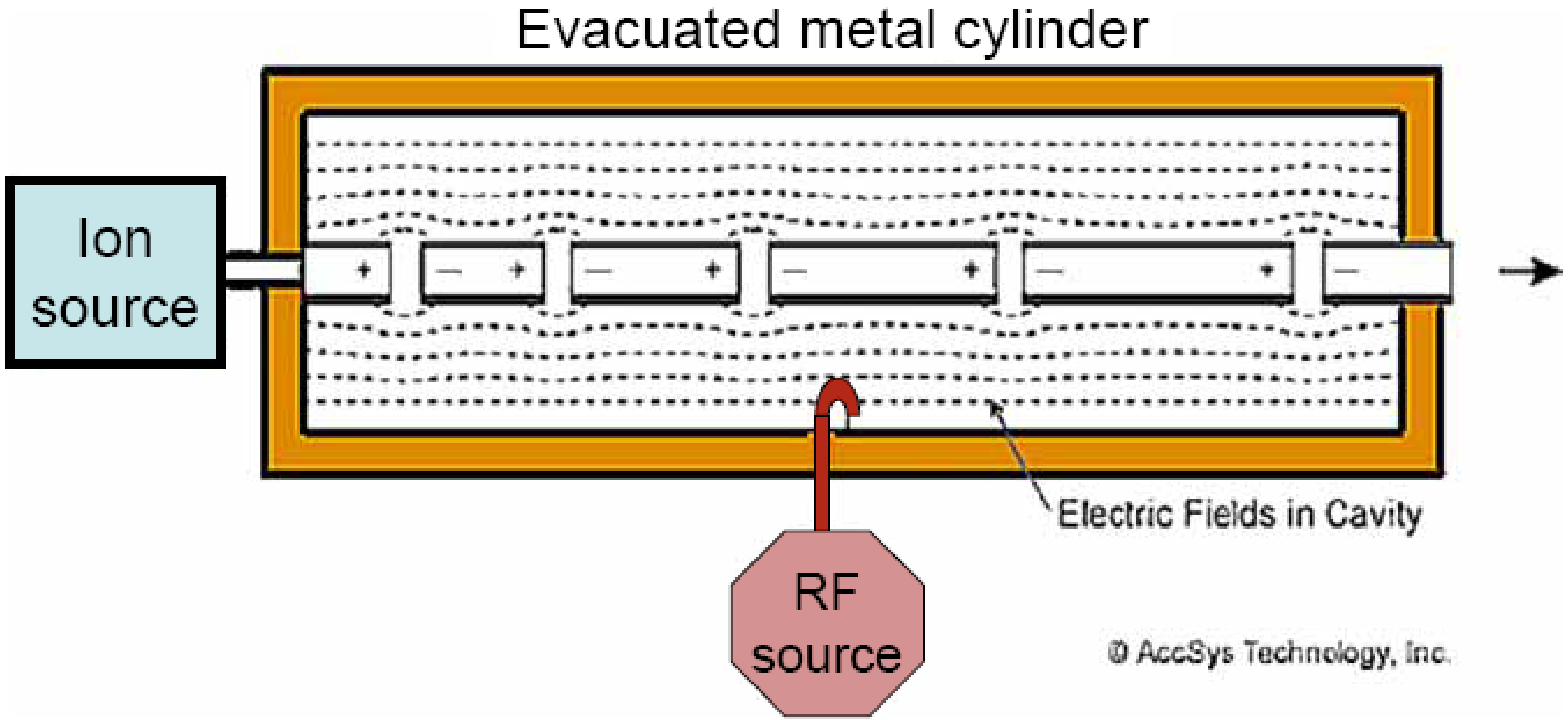
For Widerioe's DTL : Z=19, A=39, E=25keV

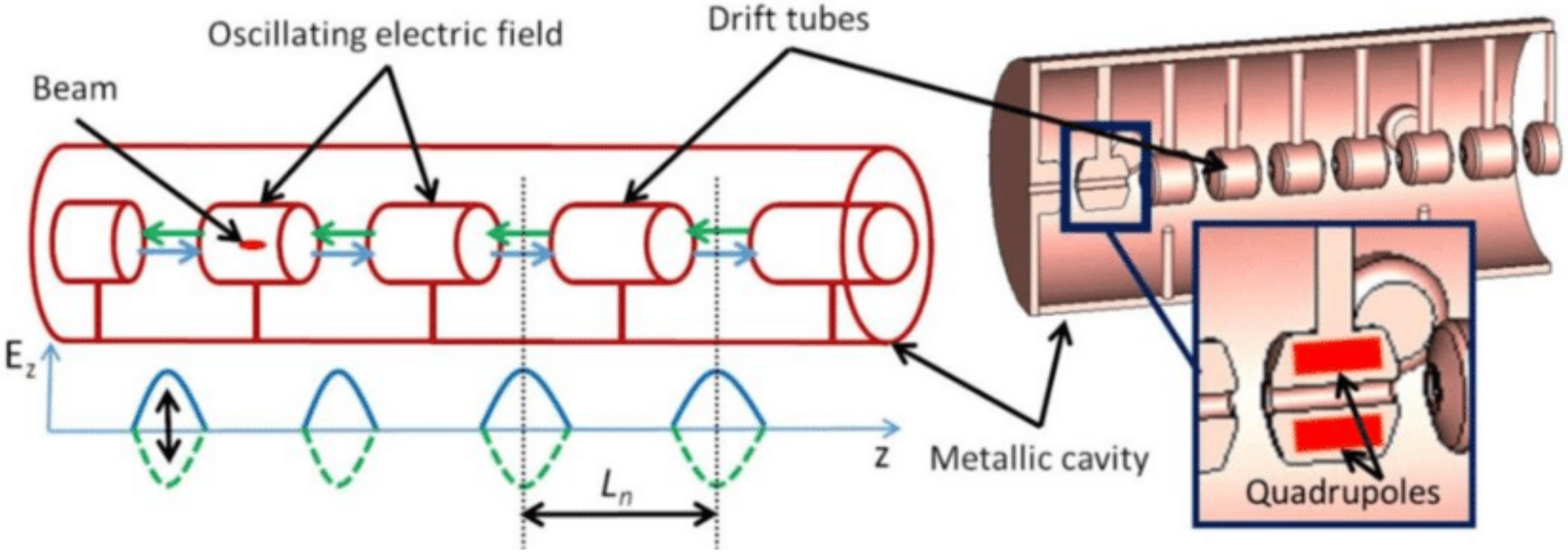
$$\beta = 1.000001337,$$

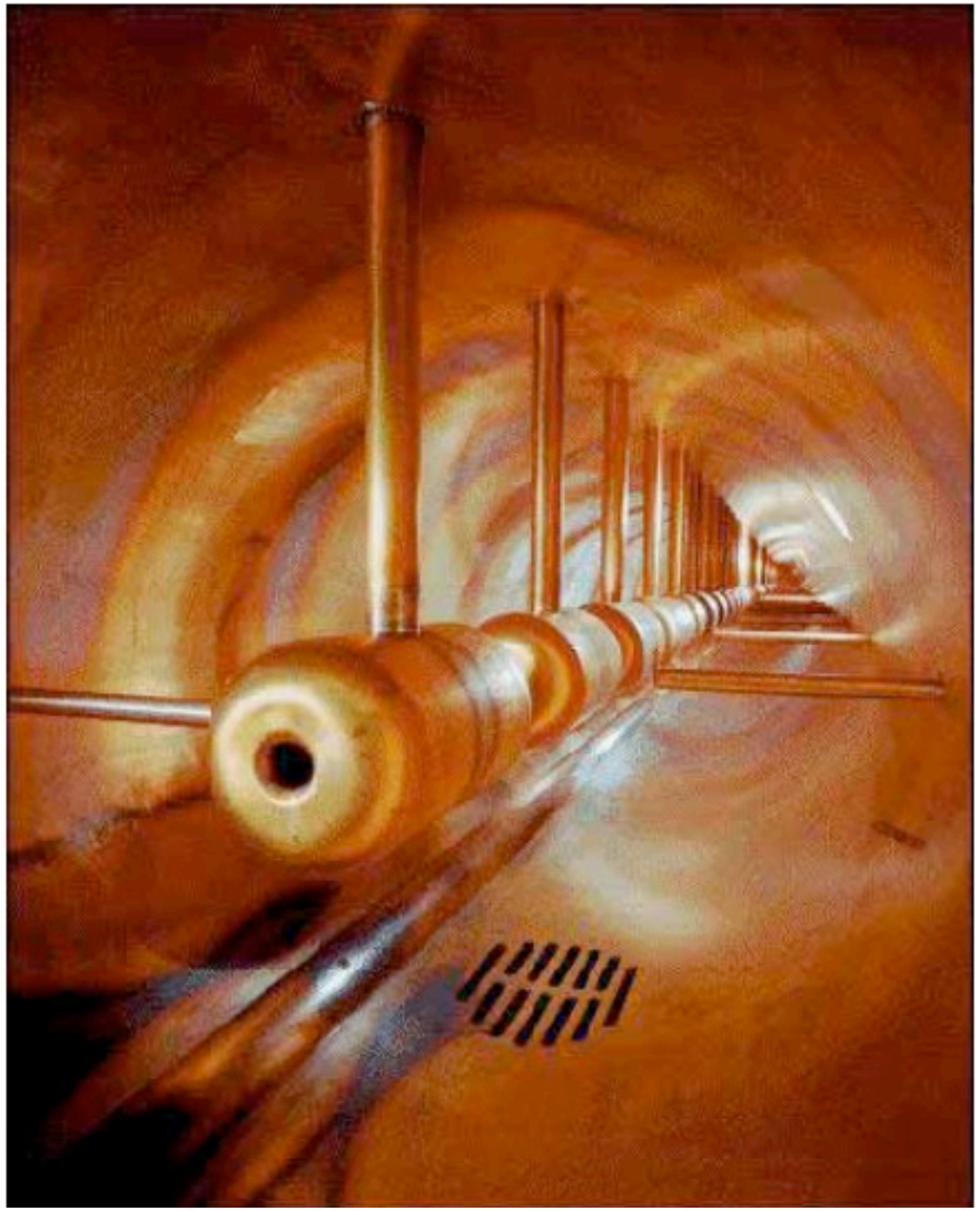
$$\lambda_{rf} = 300m$$

The length of the 2nd drift is about 24.5cm

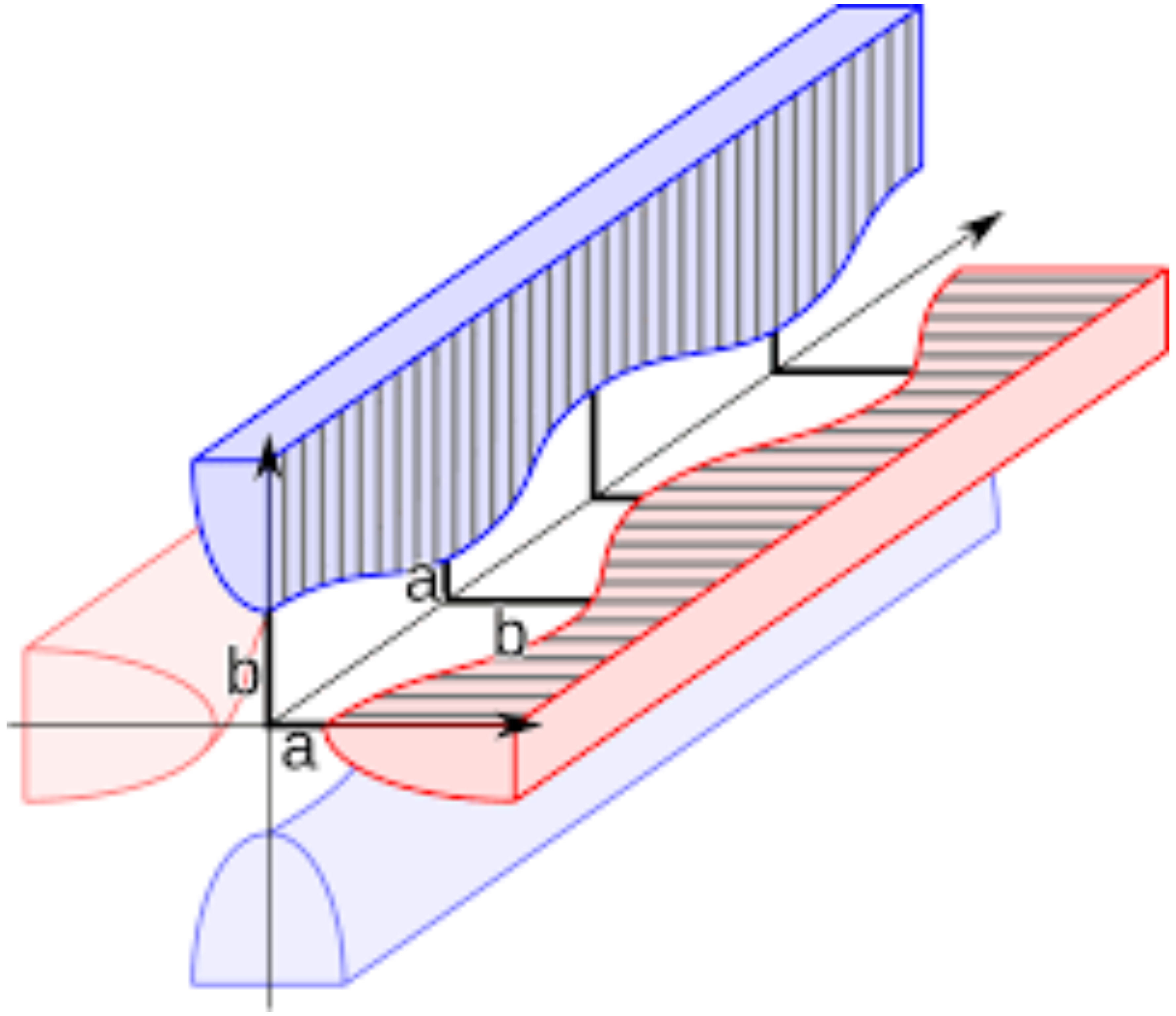
Alvarez linac

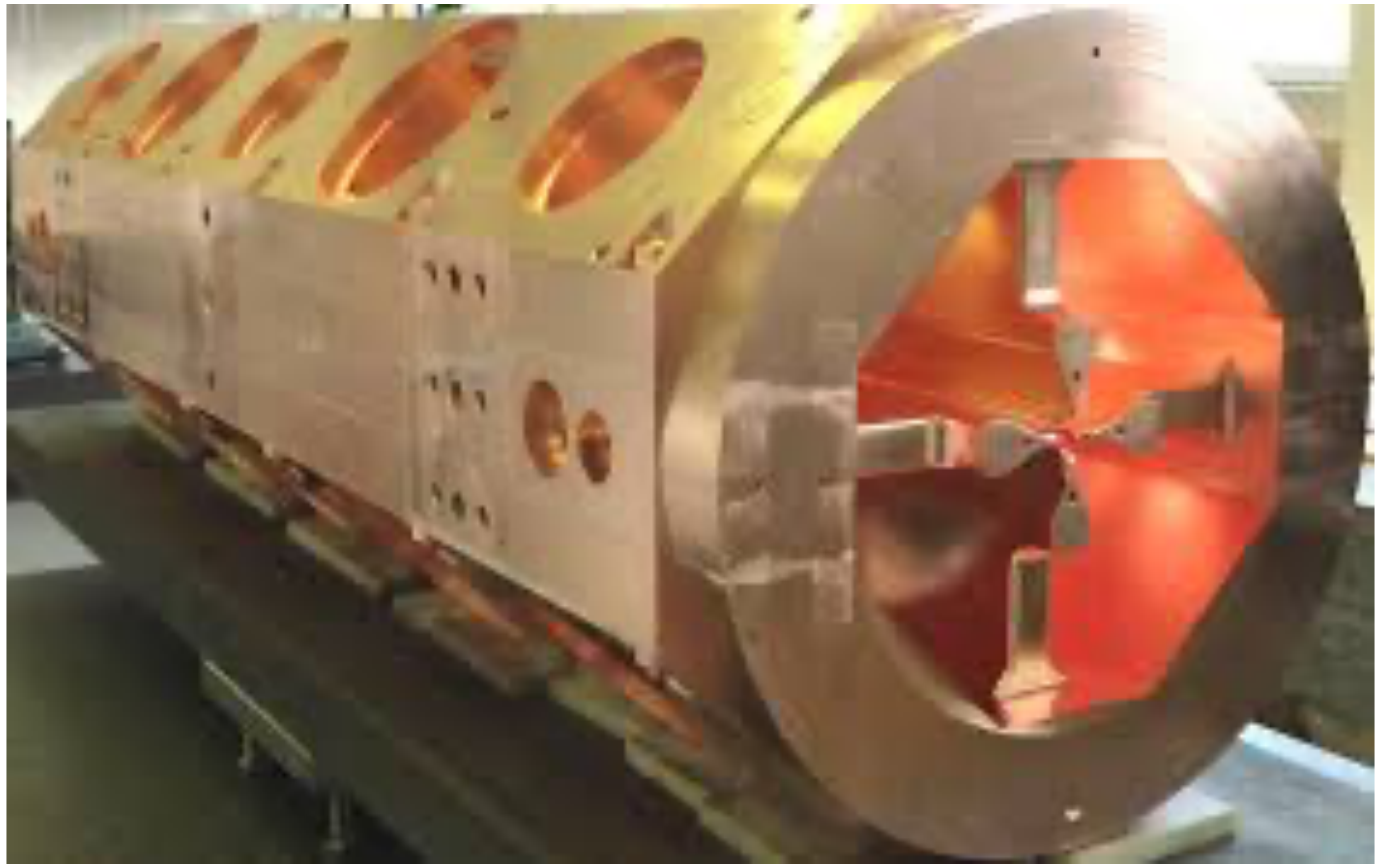


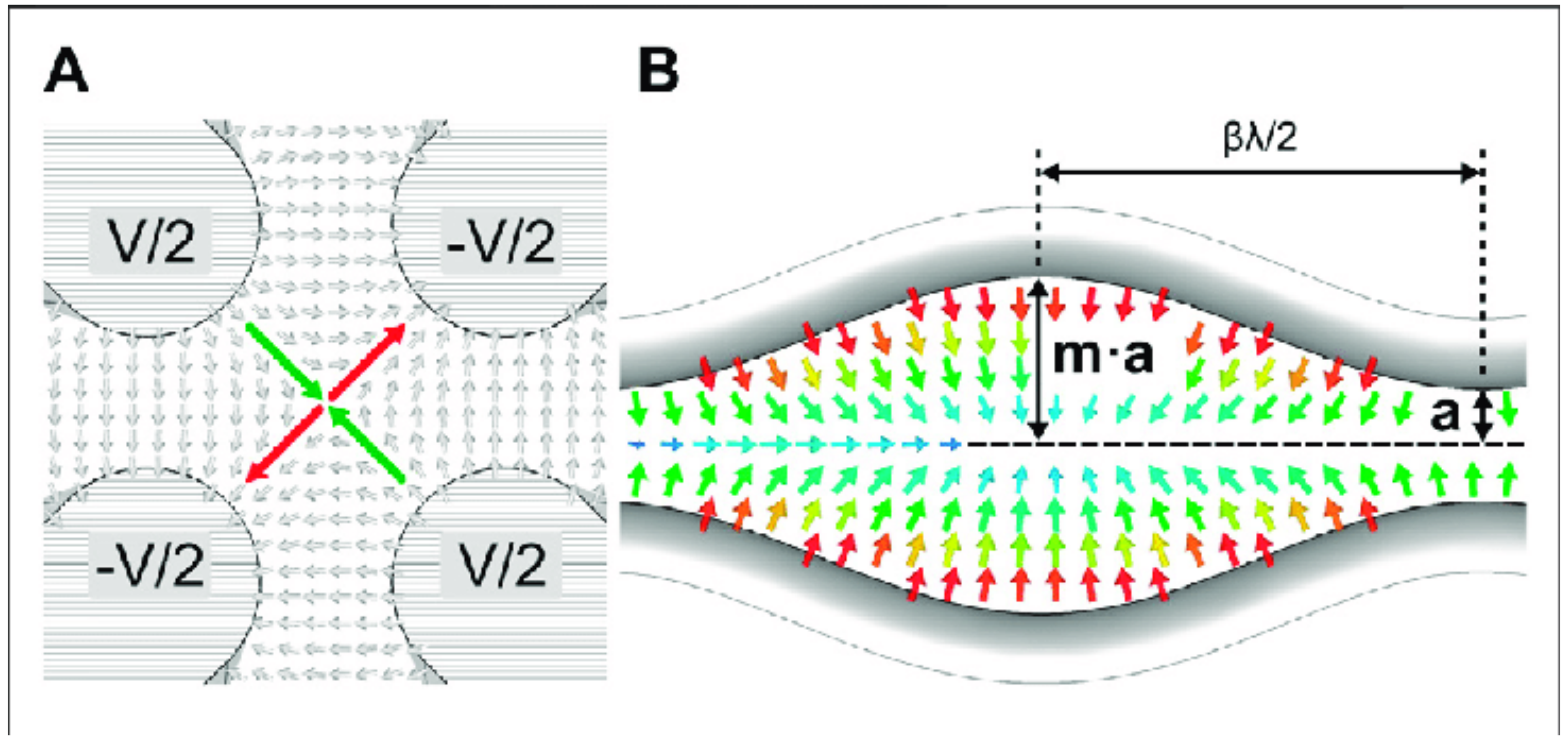


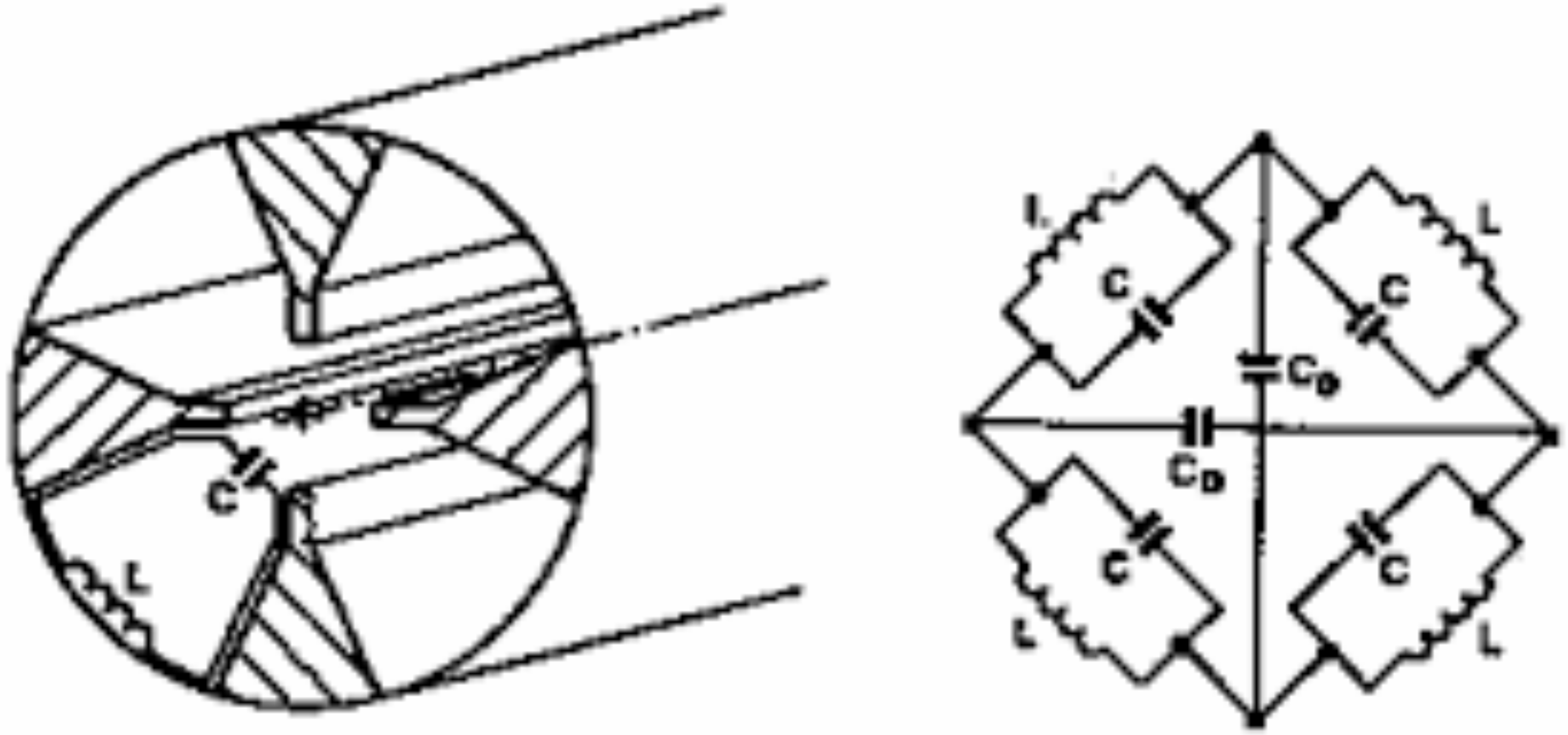


RFQ - Radio Frequency Quadruple

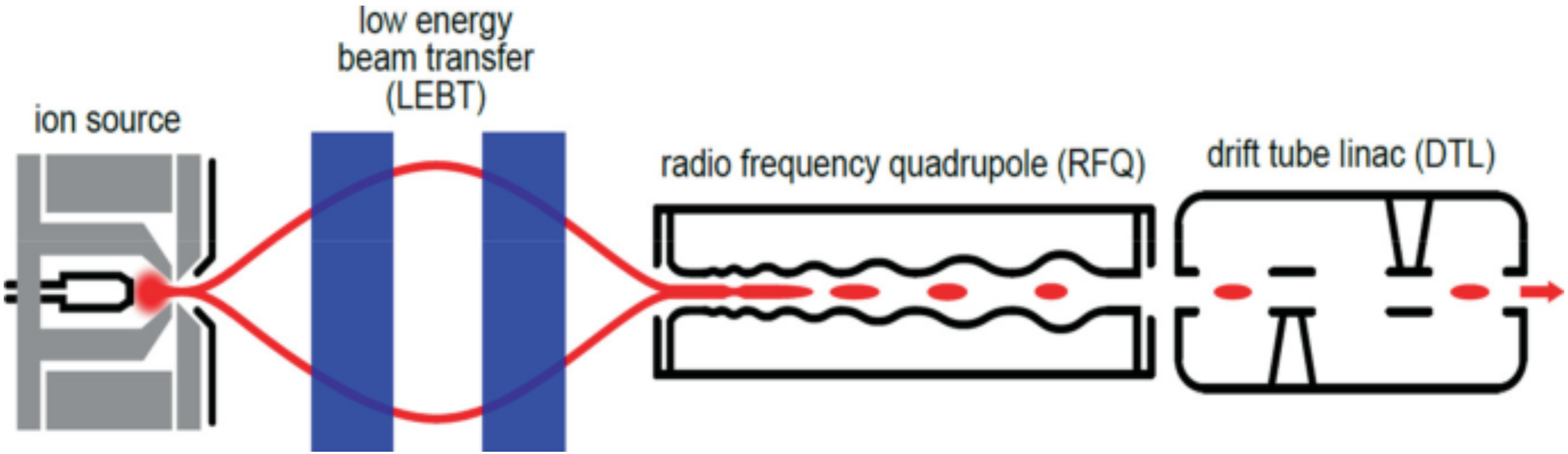








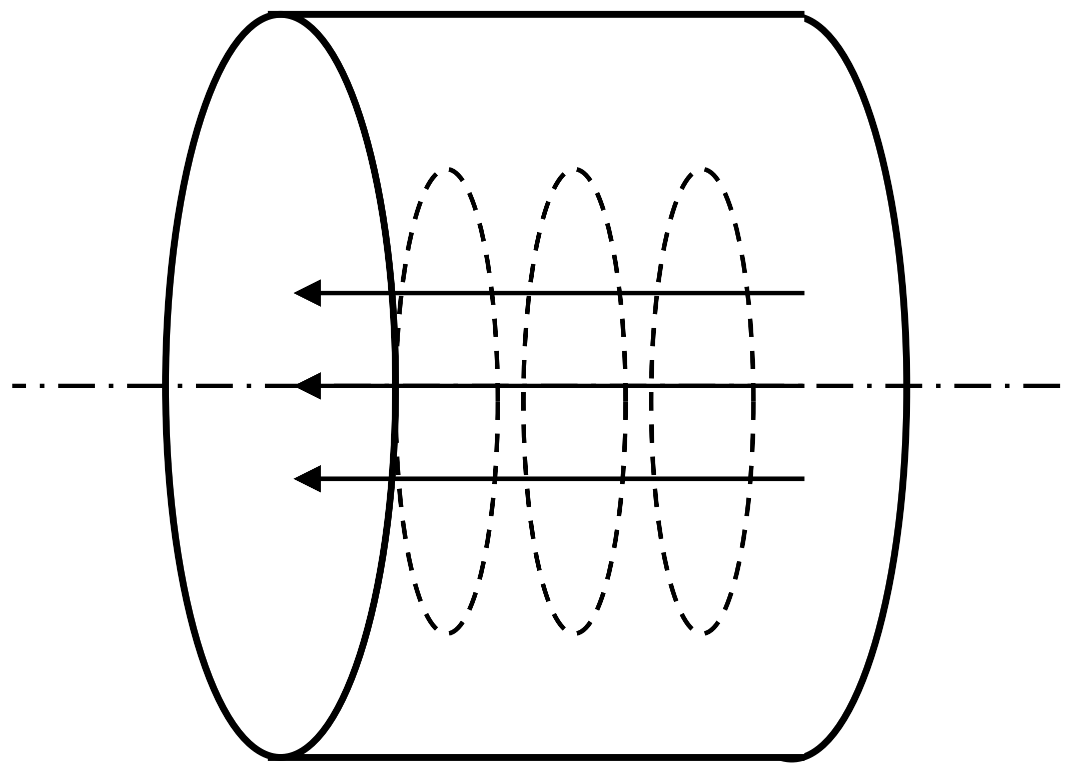
RFQ +DTL



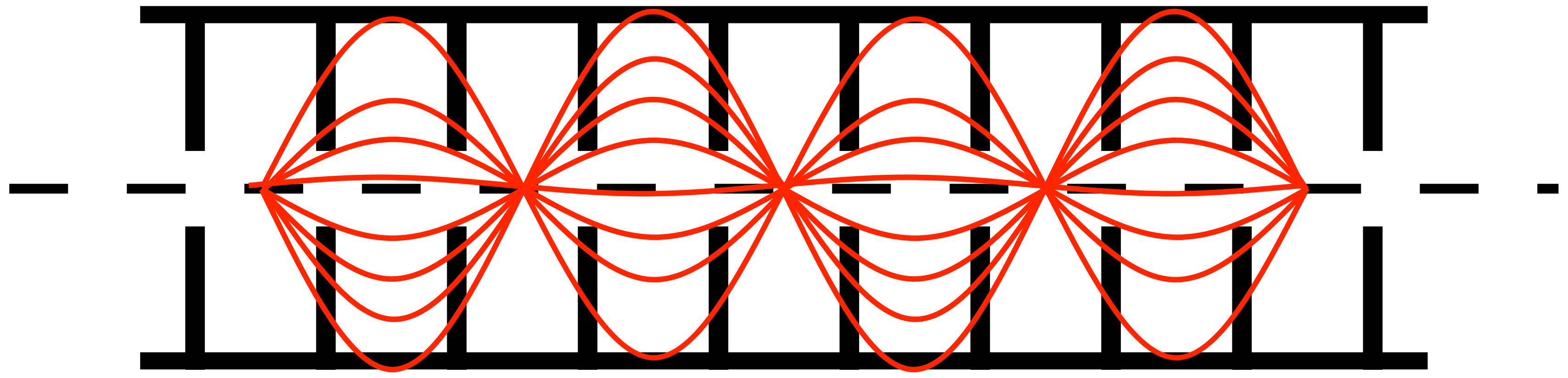
low energy
beam transfer
(LEBT)
ion source
radio frequency quadrupole (RFQ)
drift tube linac (DTL)

SW and TW Linac

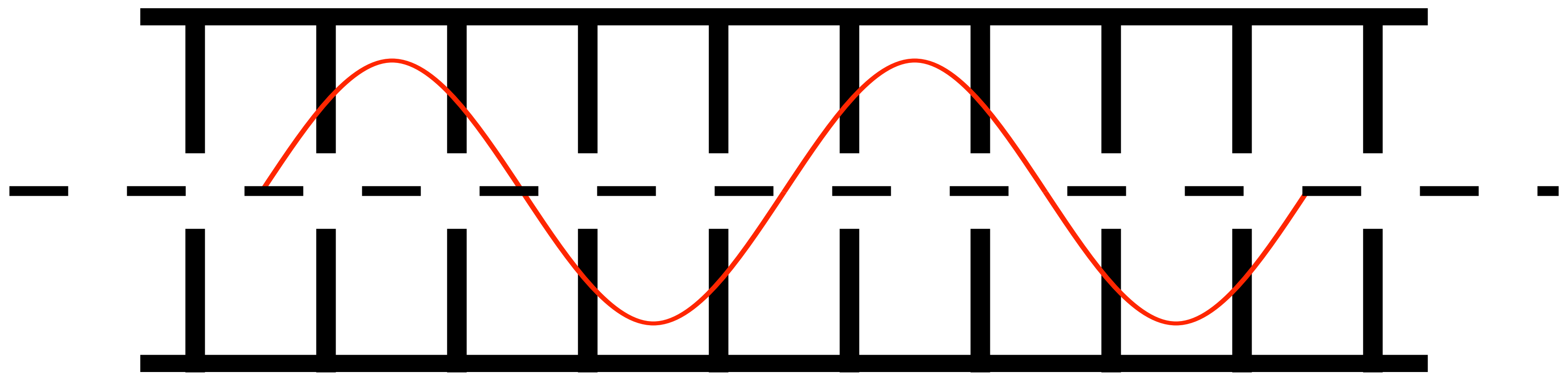
RF Cavity



TM010 mode

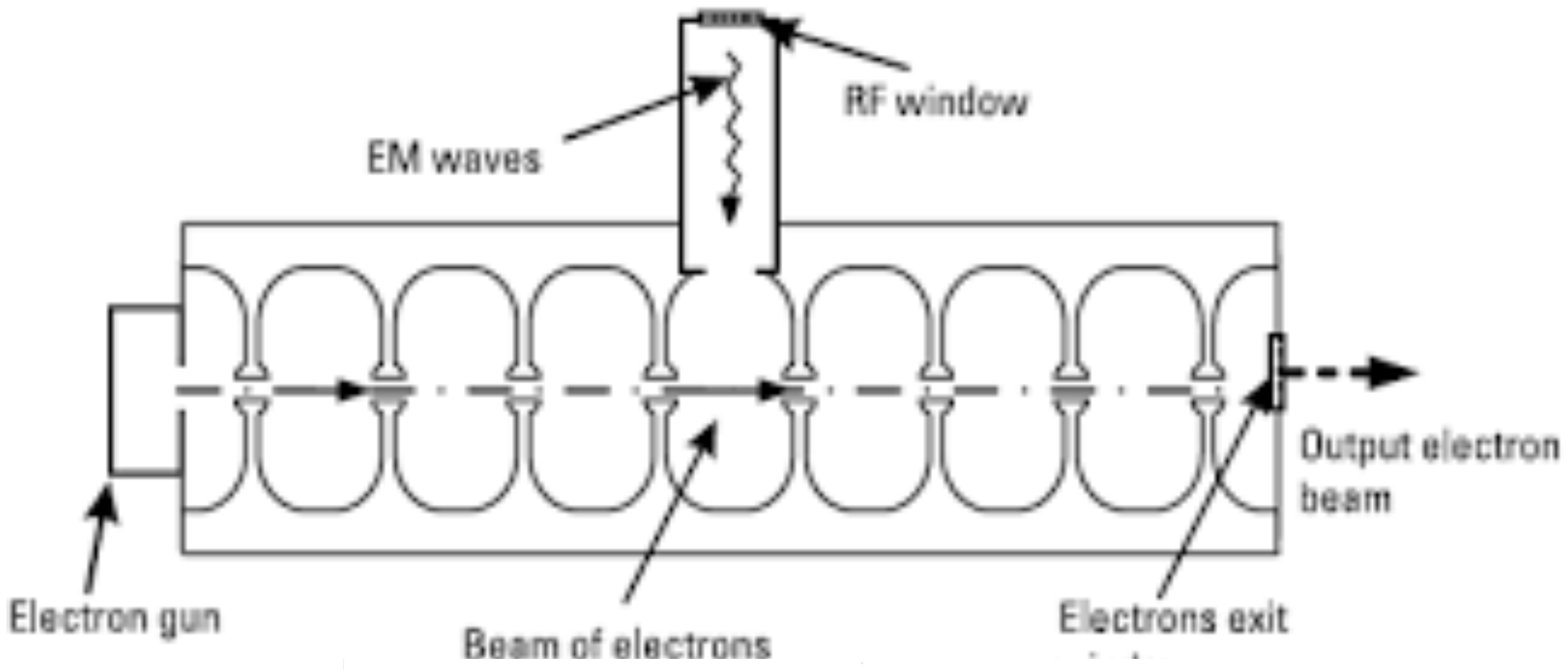


$\pi/2$ mode of standing wave
accelerating structures

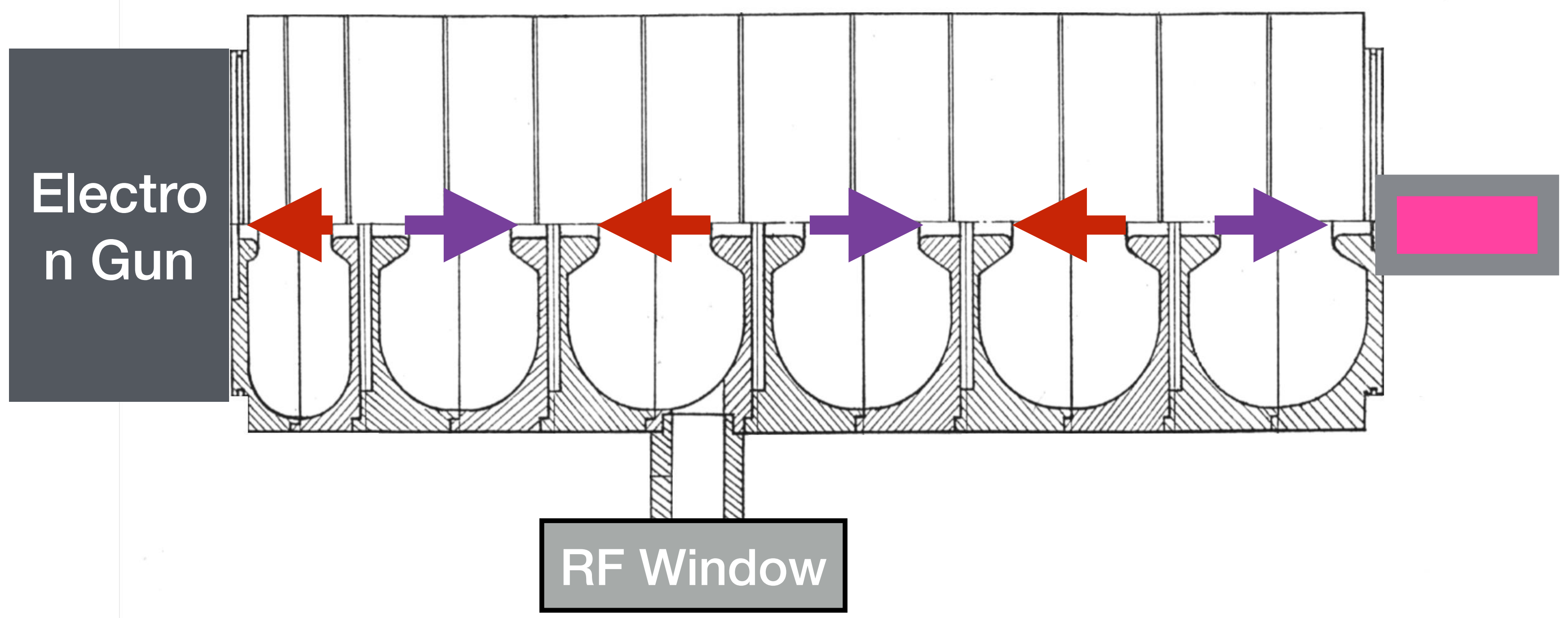


$\pi/2$ mode of traveling wave
accelerating structures

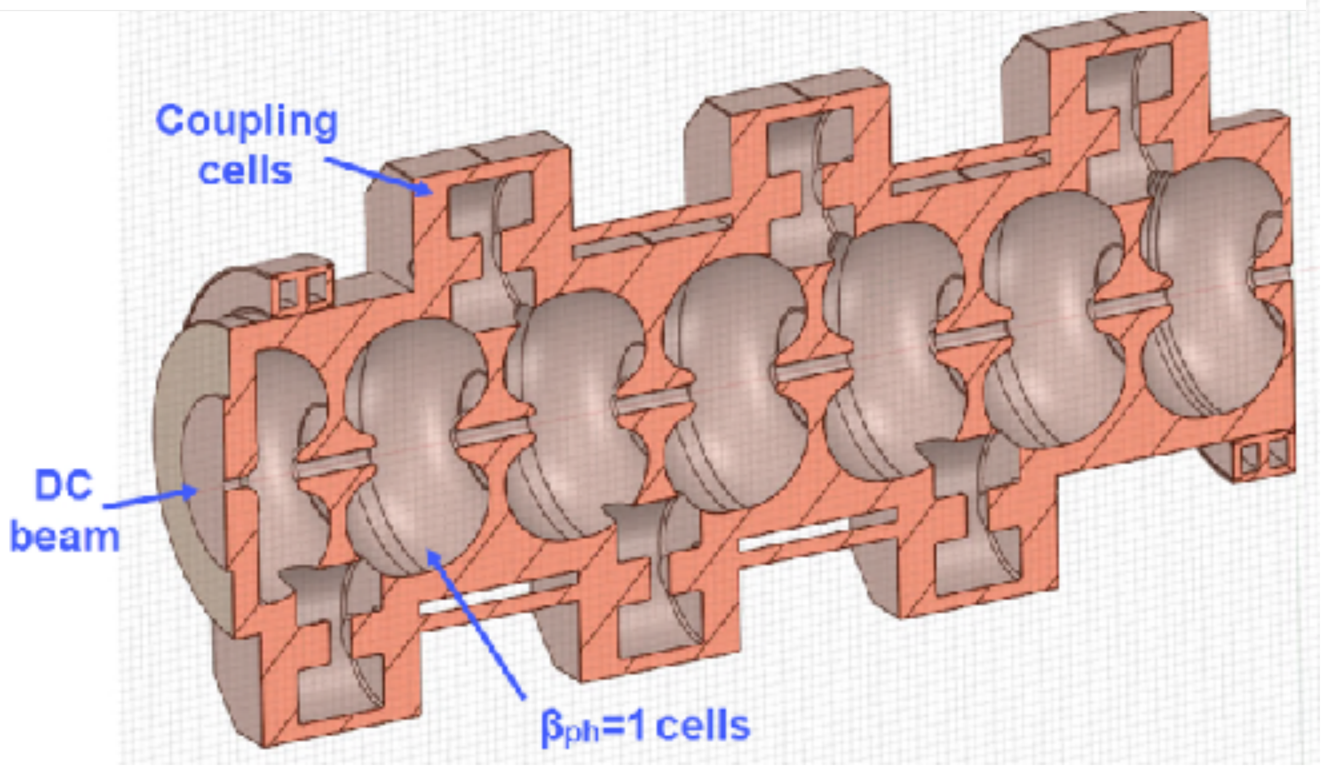
SW Linac



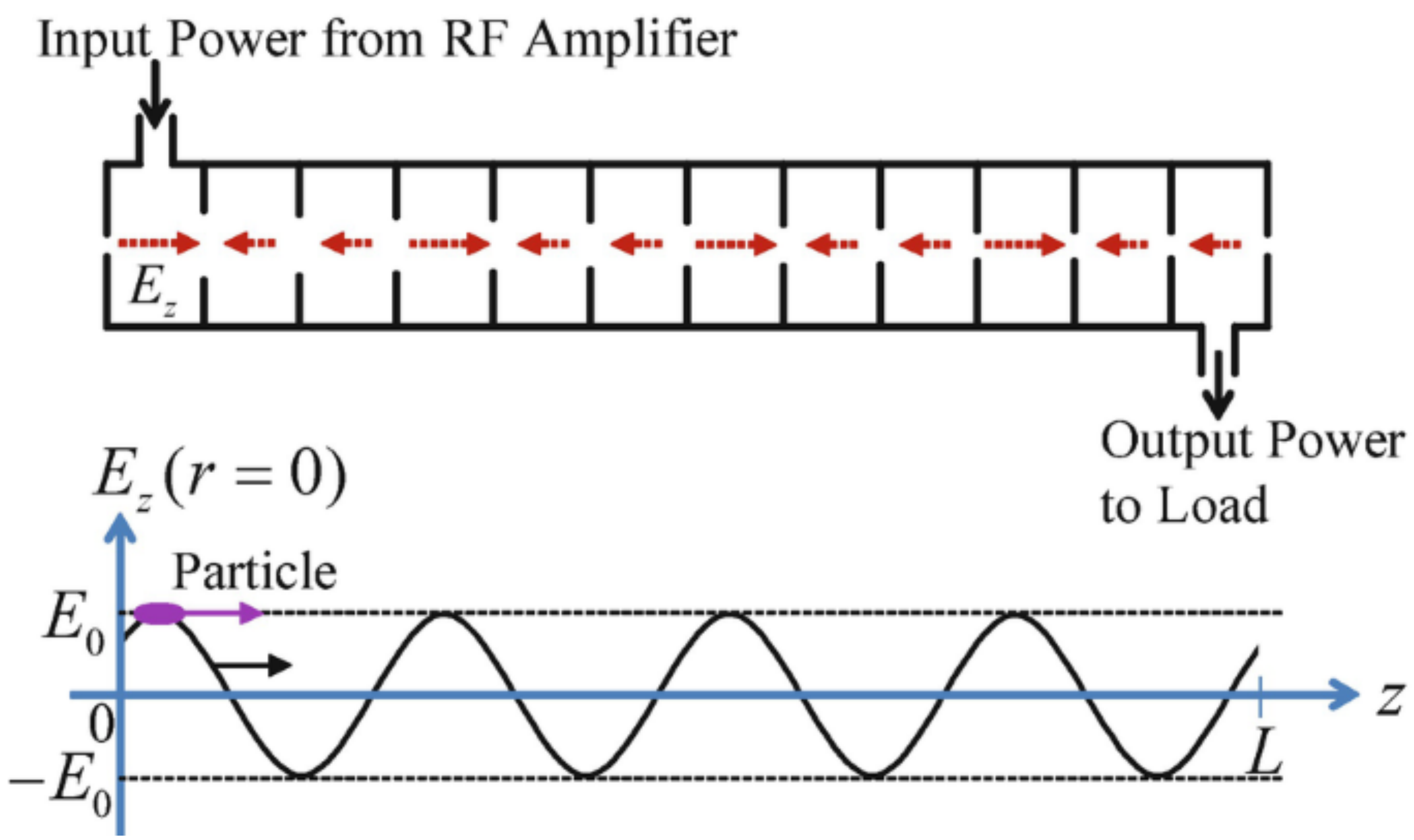
On-axis coupling bi-period SW structure

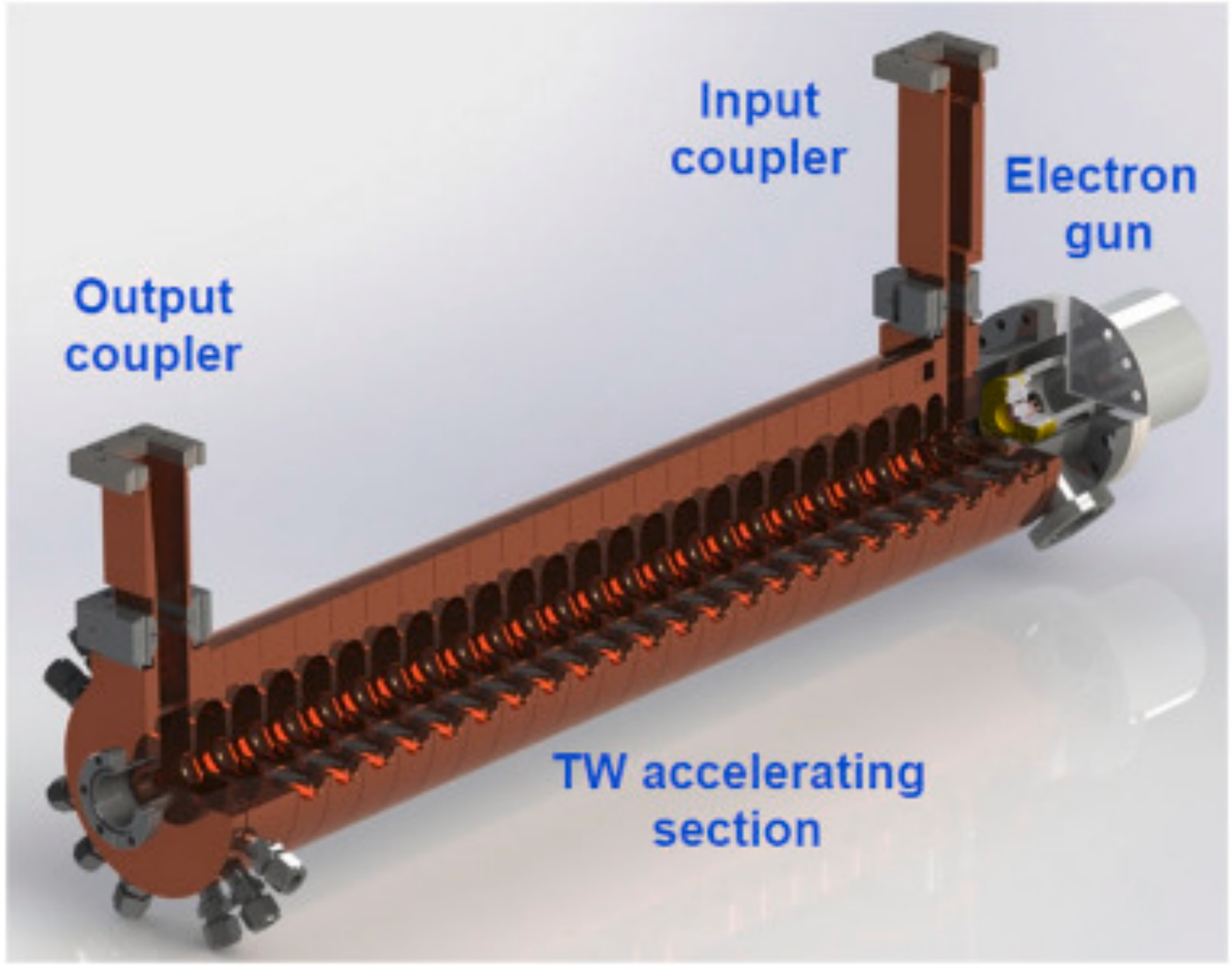


Side-coupling SW structure

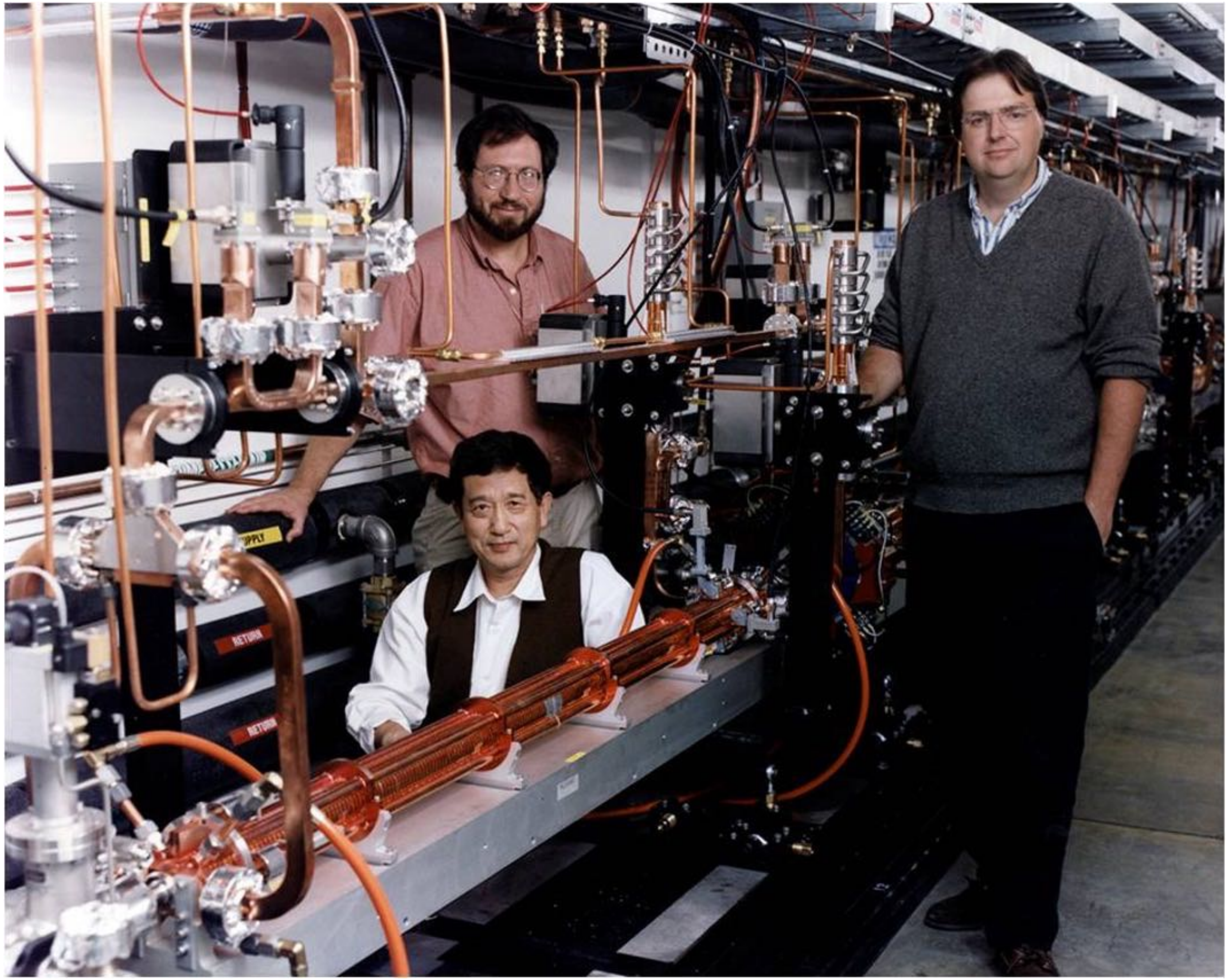


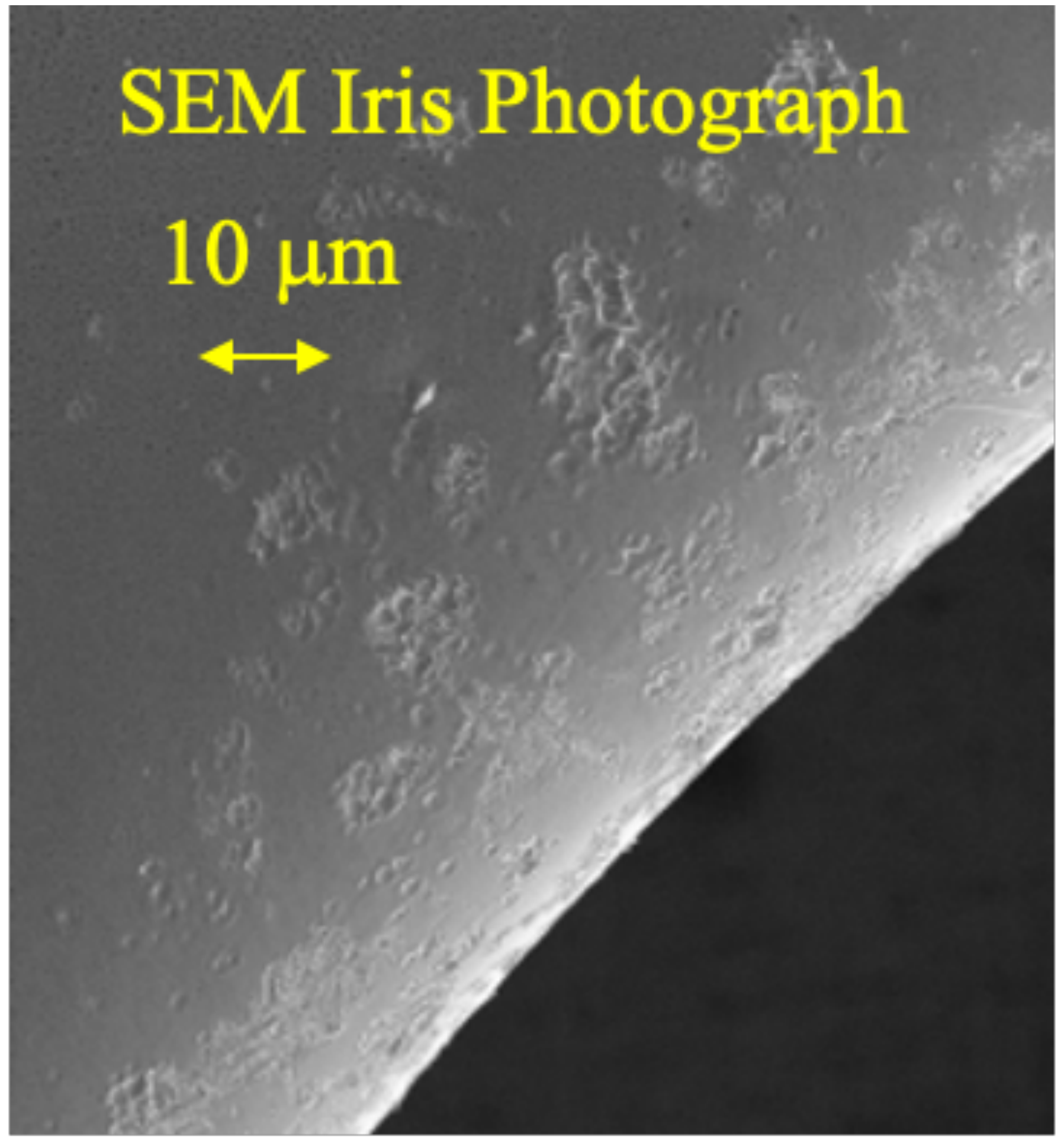
TW Linac





RF Vacuum Breakdown





The linac of PAL-XFEL

LINAC	
FEL radiation wavelength	0.1 nm (Hard X-ray)/1 nm (Soft X-ray)
Electron energy	10 GeV
Slice emittance	0.5 mm-mrad
Beam charge	0.2 nC
Peak current at undulator	3.0 kA
Pulse repetition rate	60 Hz
Electron source	Photo-cathode RF-gun
LINAC structure	S-band normal conducting

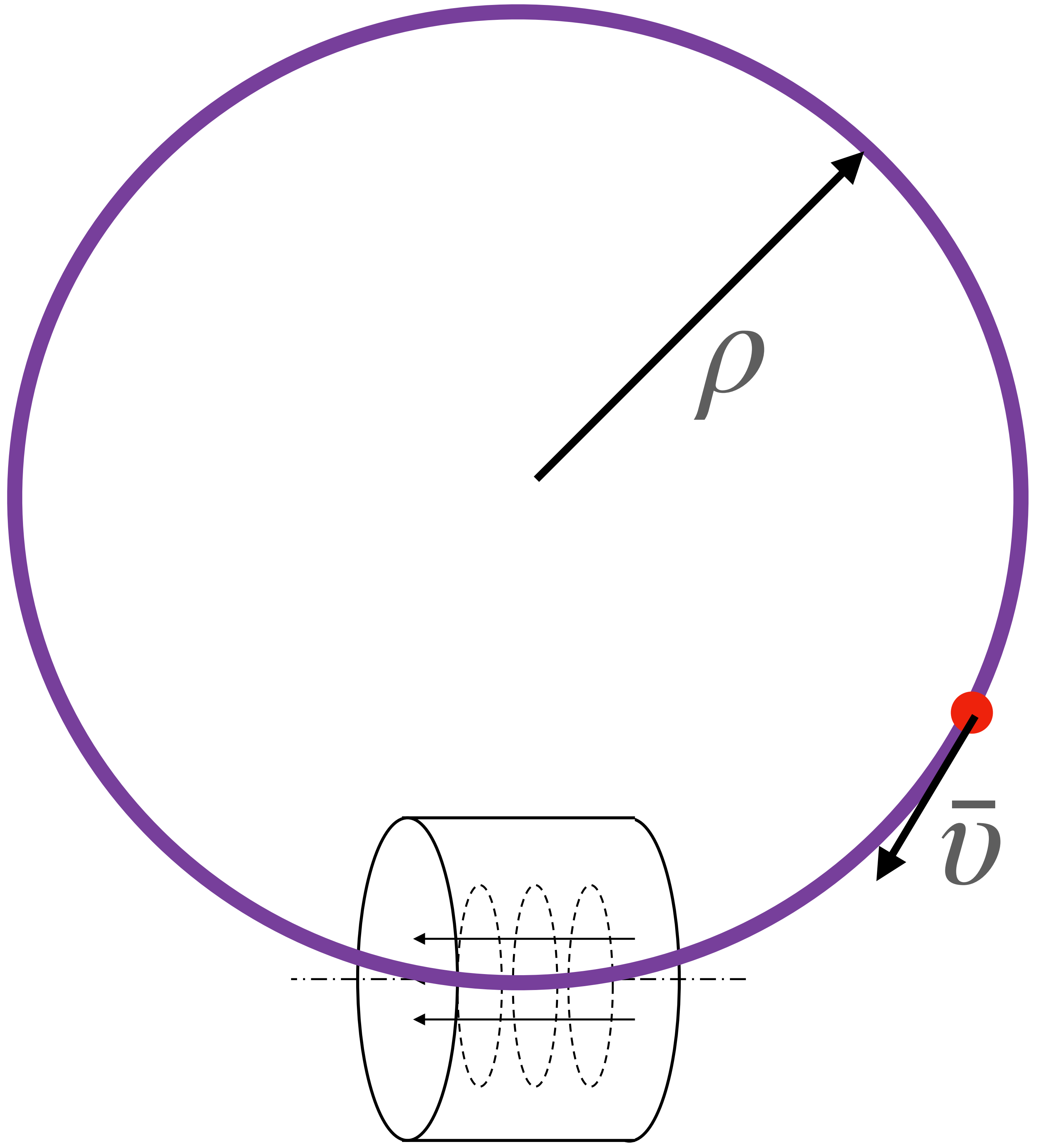
1.3.2 Circular Accelerators:

Cyclotron

- Cyclotron (Lawrence)
- Isochronous Cyclotron (Thomas)
- Synchrocyclotron
- SSC： Separated Sector Cyclotron
- FFAG: fixed-field alternating-gradient accelerators
- Microtron
- …

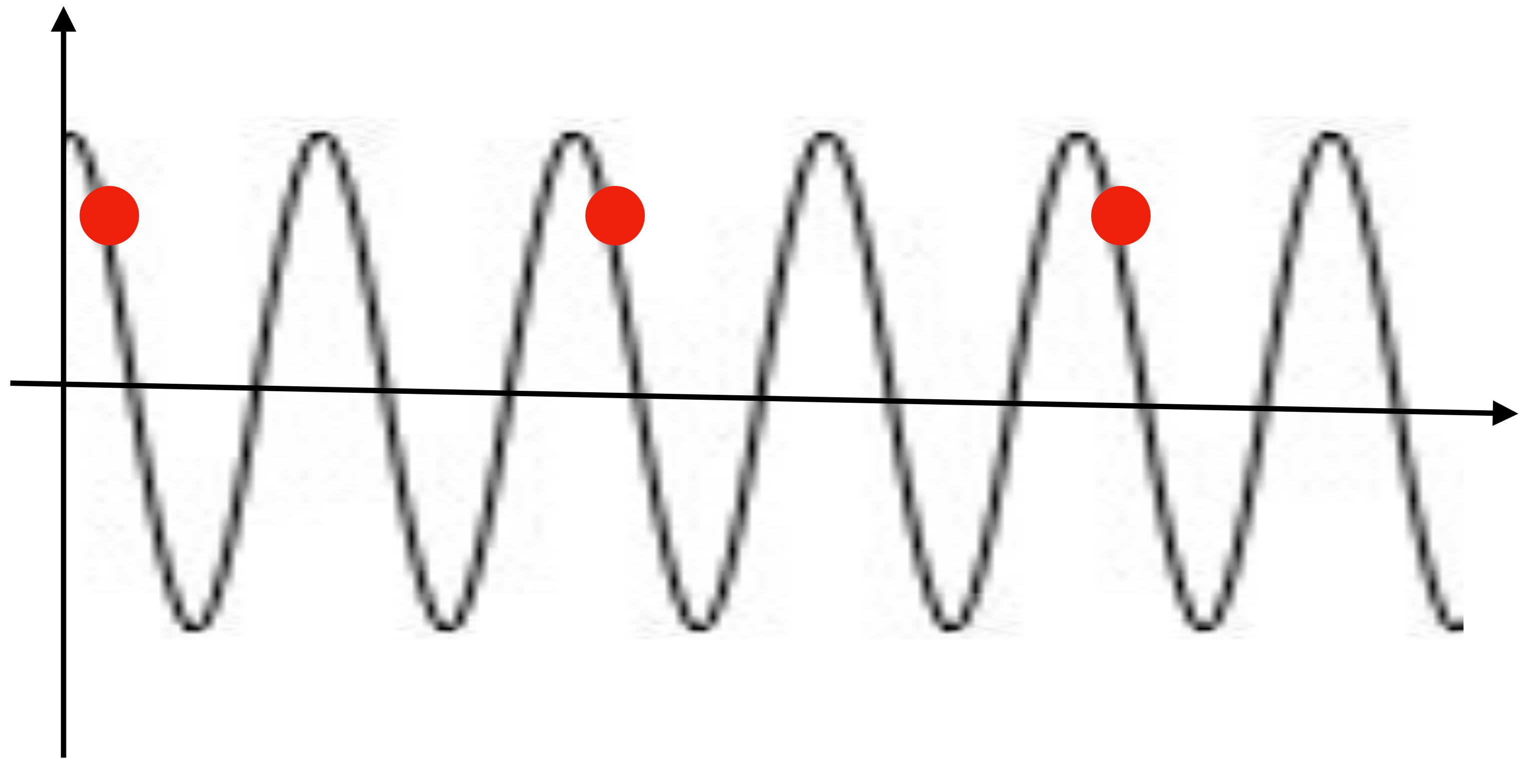
Synchrotron

- Weak-focusing & Strong-focusing: Alternative Gradient Focusing
- Electron synchrotron & Proton or heavy ion synchrotron
- Booster & Storage Ring
- …



$$E_1 = qV_0 \cos(\omega_{rf} t + \phi_s)$$

$k = 2$



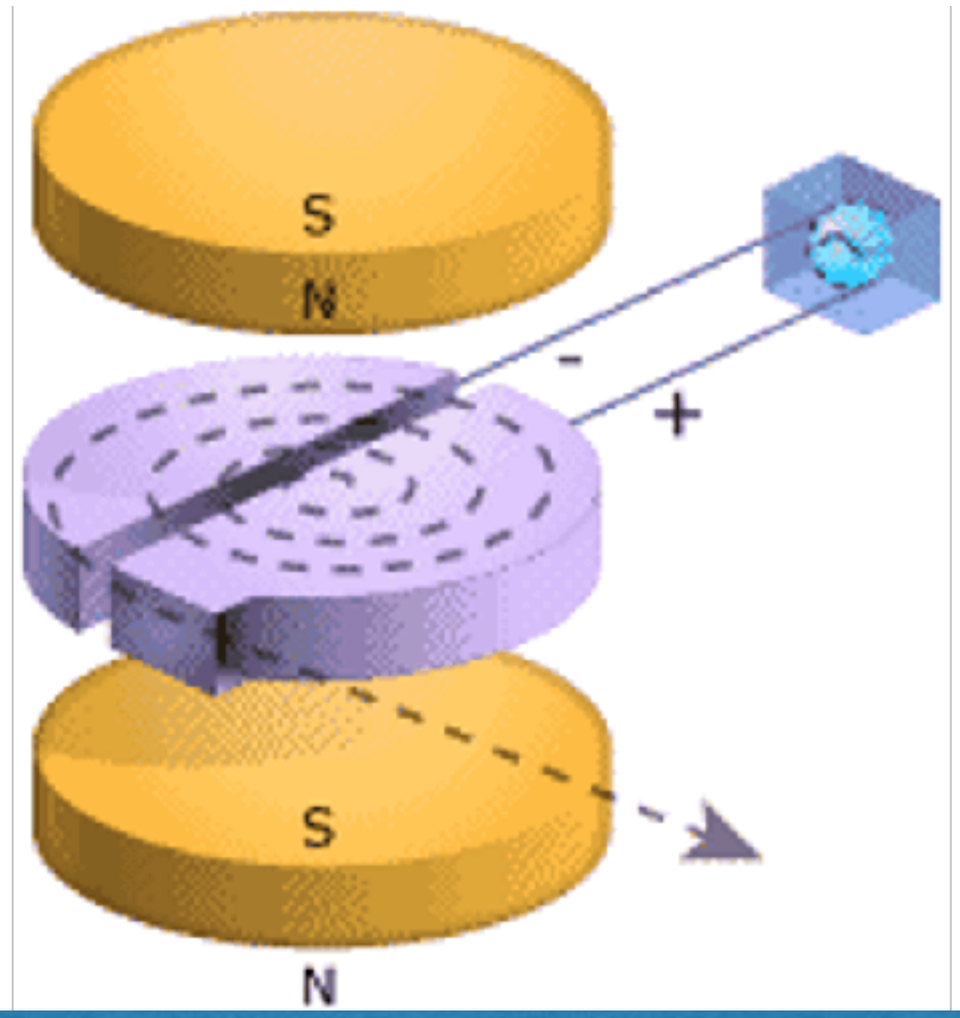
Synchronous condition : $\omega_{rf} = k\omega_s$ where k: integer

And $\omega_s = 2\pi / T_s = 2\pi \dfrac{\bar{v}}{C}$,

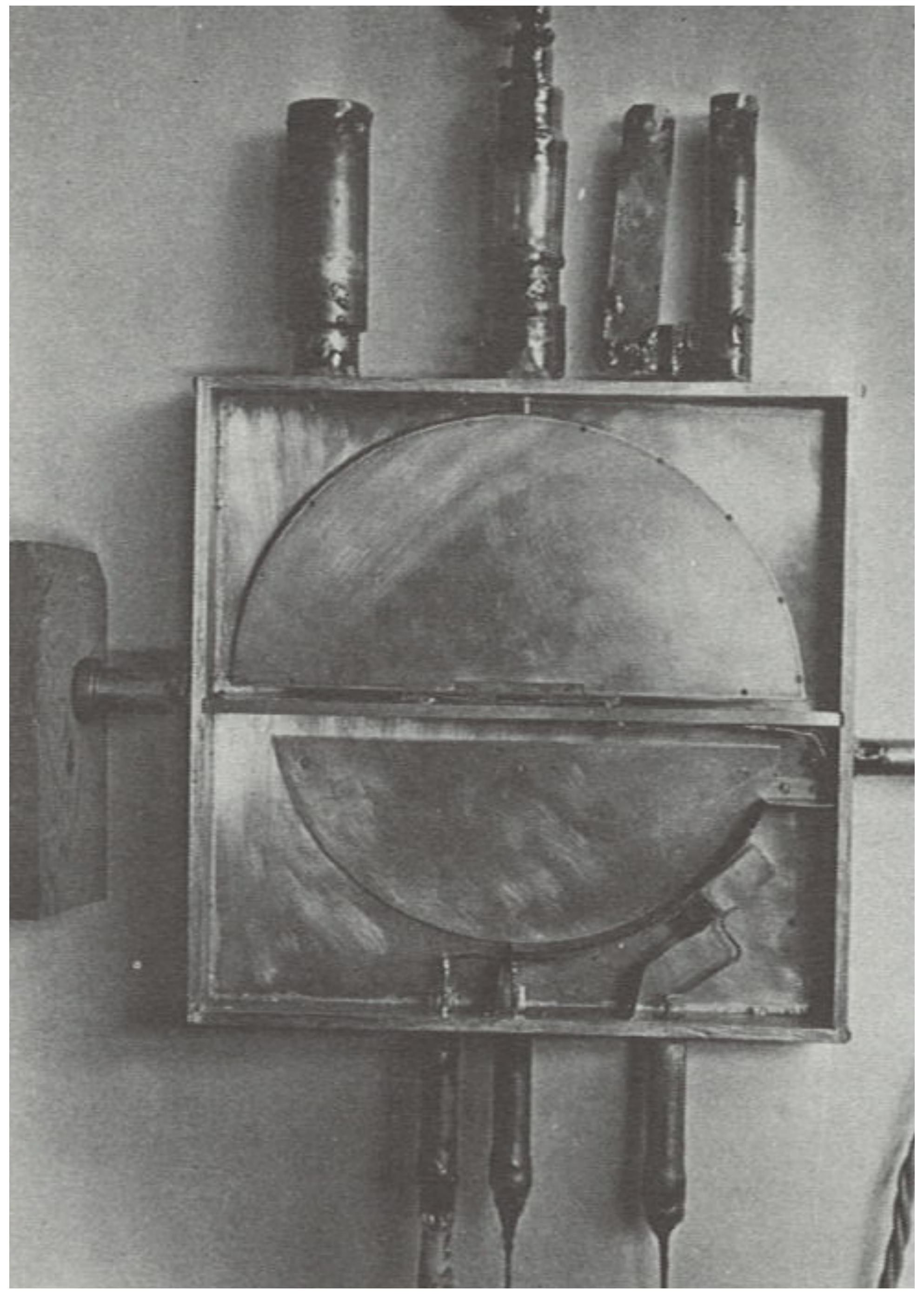
For a circle orbit: $\omega_s = \dfrac{\bar{v}}{\rho}$

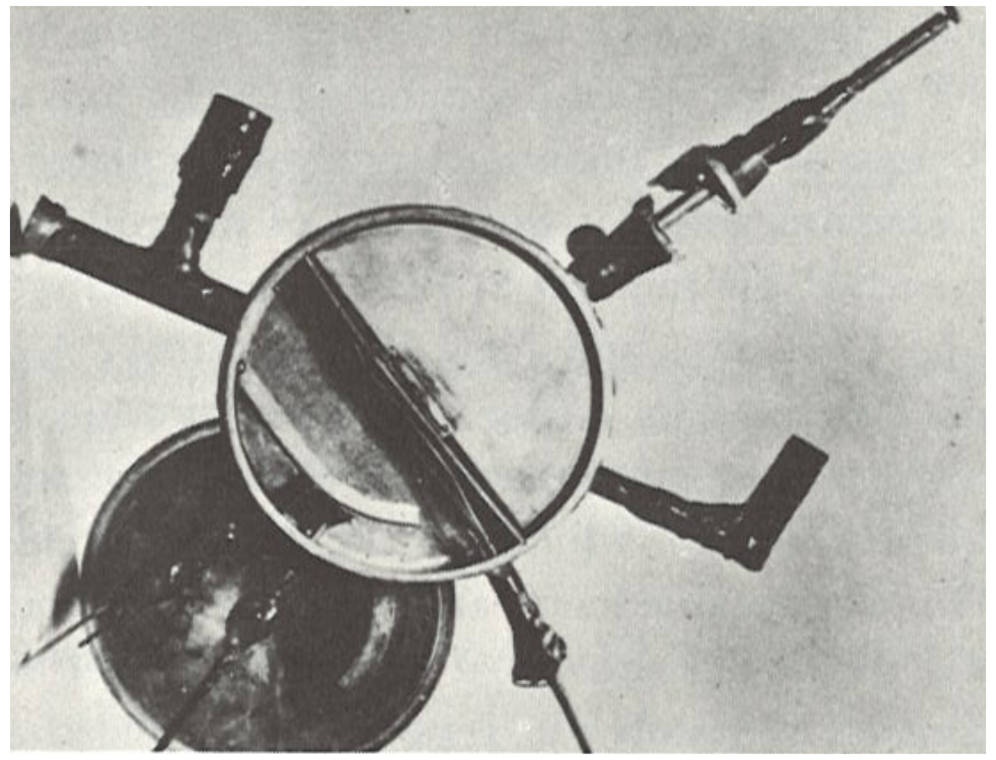
Cyclotron (Lawrence)

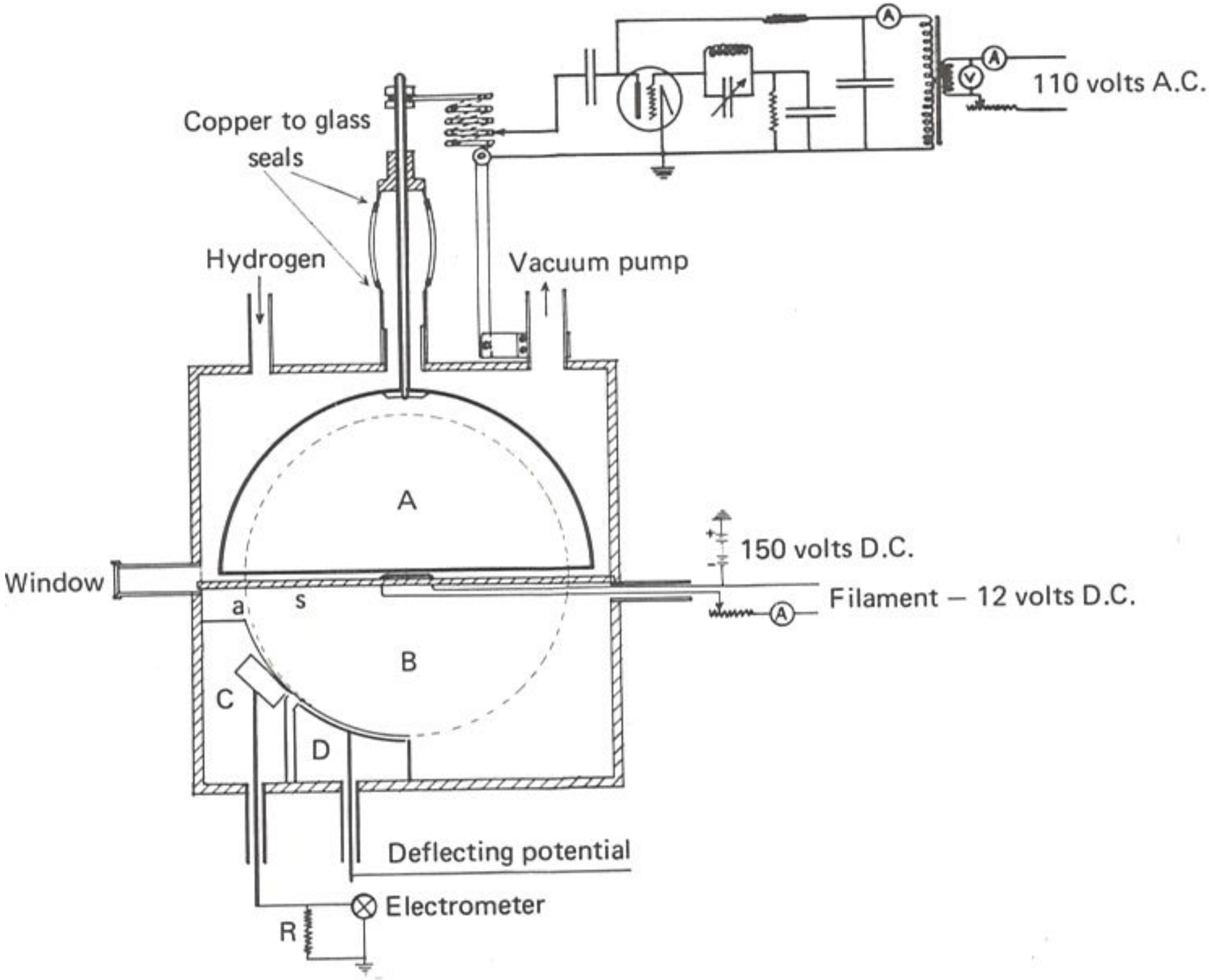
During 1931-1932, E. O. Lawrence and M.S.Livingston built the 1st cyclotron, which can accelerate H_2^+ ion to 0.5MeV and H_1^+ to 1.22MeV, with a up to 20 MHz 10kW power tube.



The original one at the Kensington Museum of Science in London







The one at Lawrence Hall of Science in Berkeley

- M. Stanley Livingston, "Particle Accelerators: A Brief History" , Harvard University Press: Cambridge , Massachusetts: 1969

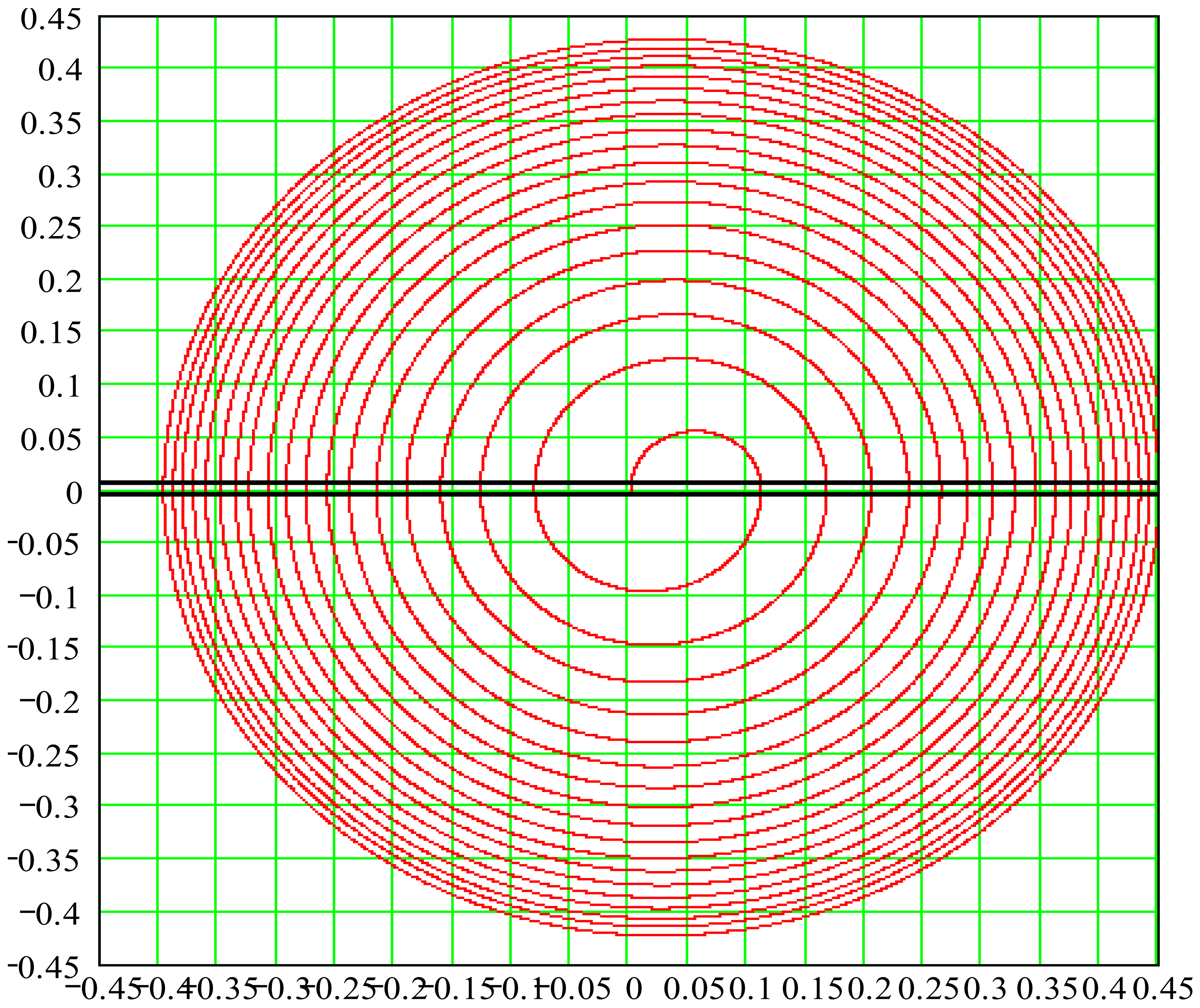
➡The Cyclotron Frequency:

$\omega_c = \frac{\bar{v}}{\rho} = \frac{ZeB}{m} \approx \text{const}.$, if the energy increase does not affect the particle mass a lot. So it is not satisfied for electrons.

$$f_c = \frac{\omega_c}{2\pi} = \frac{ZeB}{2\pi m_0} = \frac{Z \times 1 \times 1.6e^{-19}[C]B[Telsa]}{2\pi \times A \times 1.66e^{-27}[kg]} = 15.2[MHz]\frac{Z}{A}B[Tesla]$$

For a proton, Z=1,A=1, with 1Tesla magnetic fields, $f_c = = 15.2[MHz]$

➡The Cyclotron Particle Energy: $W = 48(B\rho)^2\frac{Z^2}{A}$



➡The Cyclotron Particle Orbit:

$$W = 48(B\rho)^2\frac{Z^2}{A} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{1}{2}\frac{\Delta W}{W}$$, with $\Delta W = 2ZV_0\cos\phi$,

$$\Rightarrow \Delta\rho = \frac{AV_0\cos\phi}{48ZB^2}\frac{1}{\rho}$$

➡Phase shift and energy limit of cyclotron:

Taking into account the special relativity, the particle mass is increasing during the acceleration. $m = \frac{m_0}{\sqrt{1-(v/c)^2}}$.

The cyclotron period is also increasing with the mass, and the accelerating phase is changing , if the RF frequency is fixed

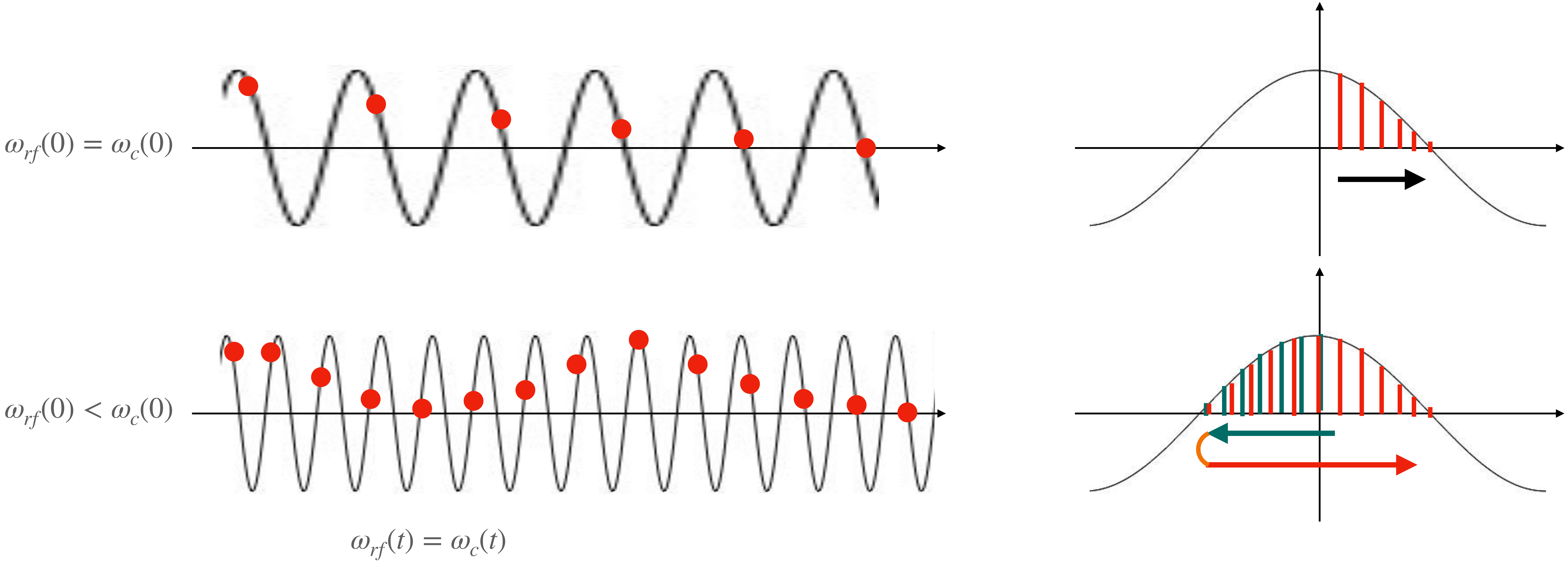
➡Phase shift and energy limit of cyclotron:

Taking into account the special relativity, the particle mass is increasing during the acceleration.

$$m = \frac{m_0}{\sqrt{1-(v/c)^2}}.$$

The cyclotron period is also increasing with the mass, and the accelerating phase is changing , if the RF frequency is fixed.

Finally, the phase will move to a decelerating phase. The particle begins to loss energy. That's the energy limit of cyclotron. And also the orbit difference of the neighbor-hood will be near zero. It is hard to extract the particle.

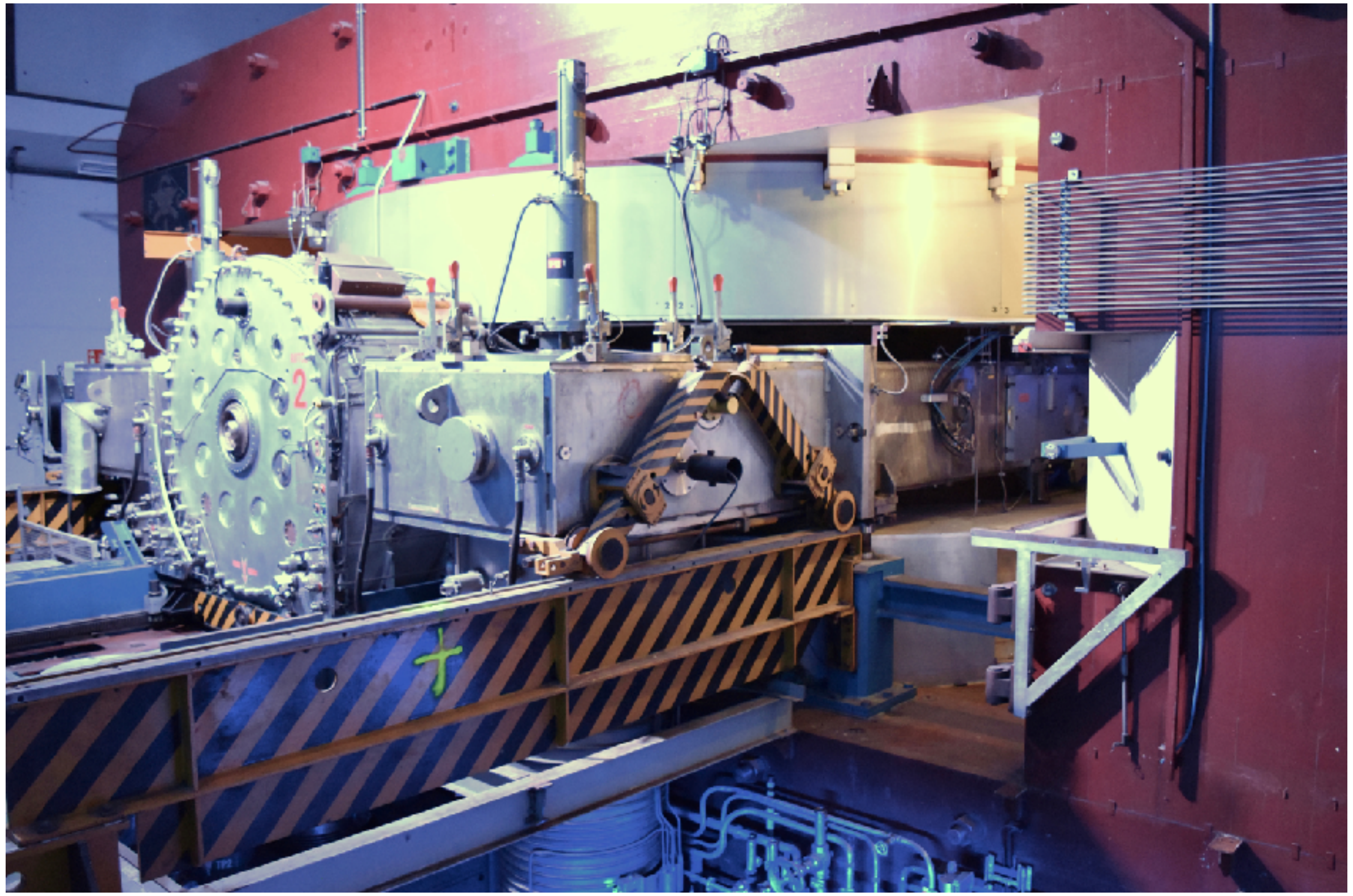


- Synchrocyclotron

$$\omega_{rf} = k\frac{ZeB}{m_0\gamma}$$

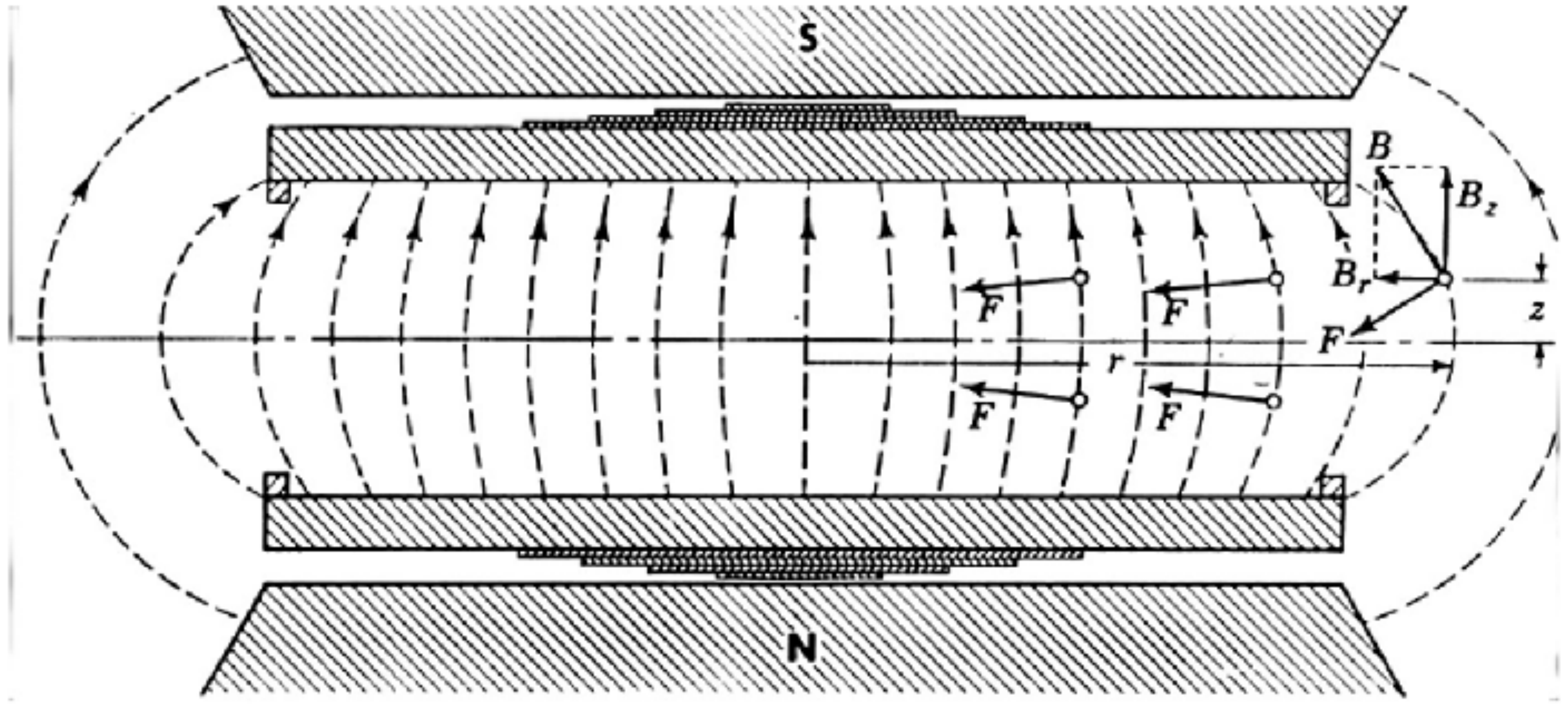
In 1946, the first Synchrocyclotron made by McMillan could produce 195 MeV deuterons and 390 MeV α-particles.

Synchro-Cyclotron (SC) at CERN with 15.7 metres (52 ft) in circumference started in 1954 and it achieved 600 MeV proton acceleration in August 1957, with the experimental program started in April 1958.



Transverse weak focusing:

Although the energy can be higher, the flux of particles was low until McMillan did something “strange”



$$\boldsymbol{B} = B_z\hat{e}_z + B_r\hat{e}_r \quad B_\theta = 0 \quad \frac{\partial B_z}{\partial \theta} = 0$$

From Maxwell’s Equations:

$$\nabla \cdot \boldsymbol{B} = 0, \ \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = 0$$

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0$$

$$\Rightarrow \hat{e}_r(\frac{1}{r}\frac{\partial B_z}{\partial \theta} - \frac{\partial B_\theta}{\partial z}) + \hat{e}_\theta(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}) + \hat{e}_z\frac{1}{r}(\frac{\partial}{\partial r}(rB_\theta) - \frac{\partial B_r}{\partial \theta}) = 0$$

If we assume the z-component of magnetic field at the plane z=0 is:

$B_z(r) = B_z(r_s)\frac{1}{r^n}$, r_s is the equilibrium orbit

$r = r_s + x$, with $\frac{x}{r_s} \ll 1$ and the linear approximation,

We will find

$$\ddot{x} + (1-n)\omega_s^2 x = 0$$
$$\ddot{z} + n\omega_s^2 z = 0$$

So to have a stable motion in z and r, we must let:

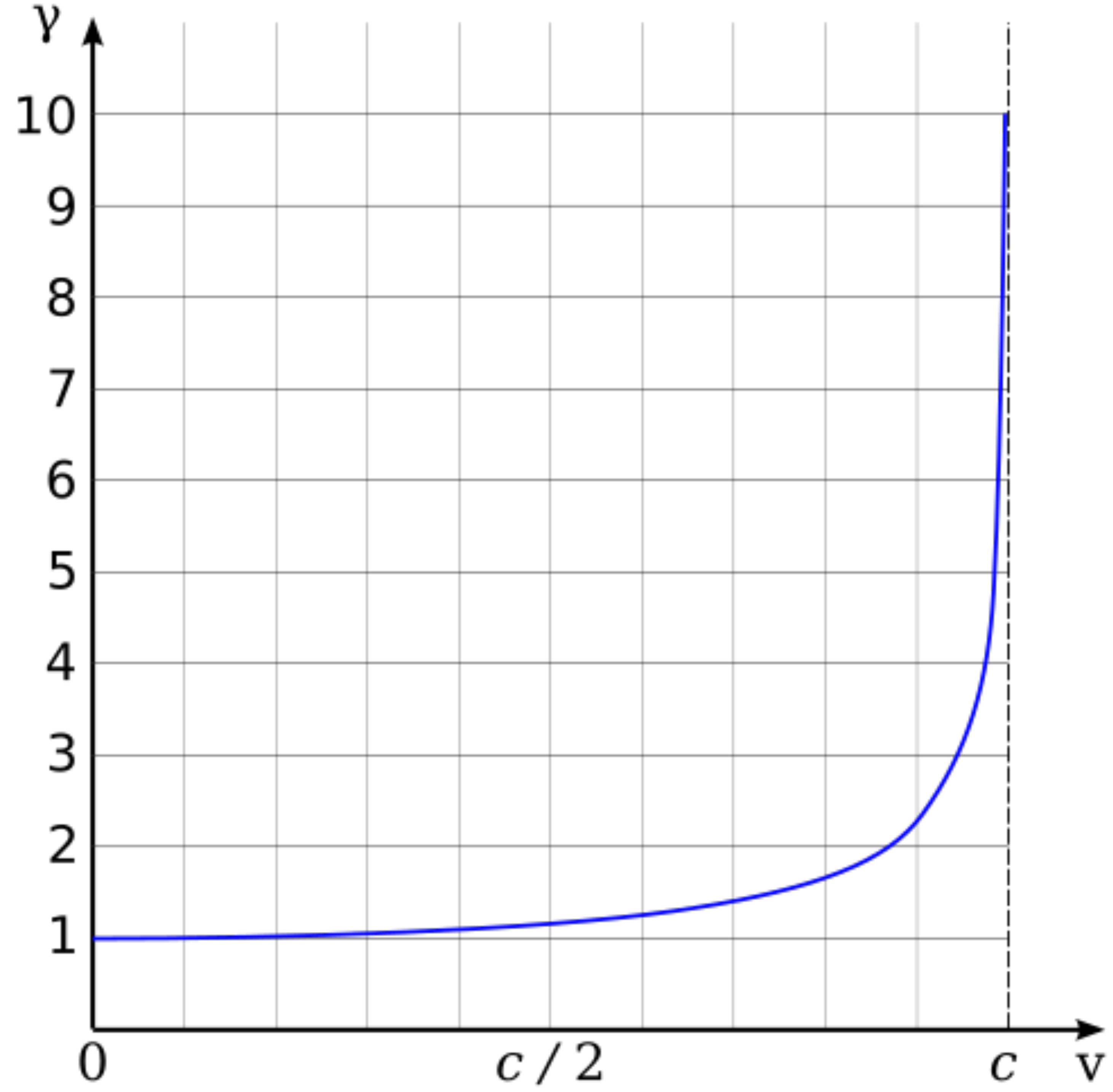
$$0 < n < 1$$

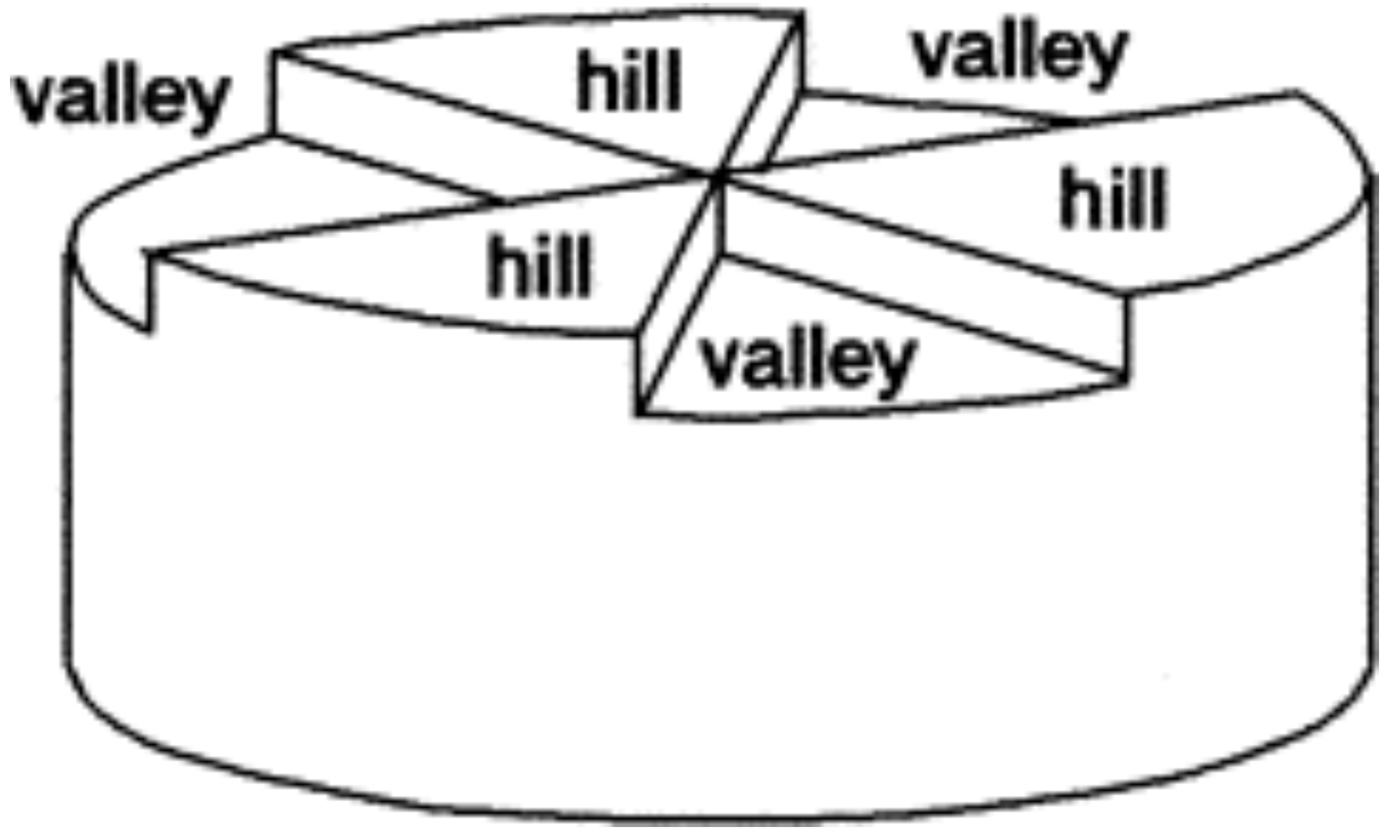
That means $B_z(r)$ must decrease radically, and not faster than $\frac{1}{r}$.

- Isochronous Cyclotron (Thomas)

$$B(r) = \frac{\omega_{rf} m_0 \gamma}{Zek}$$

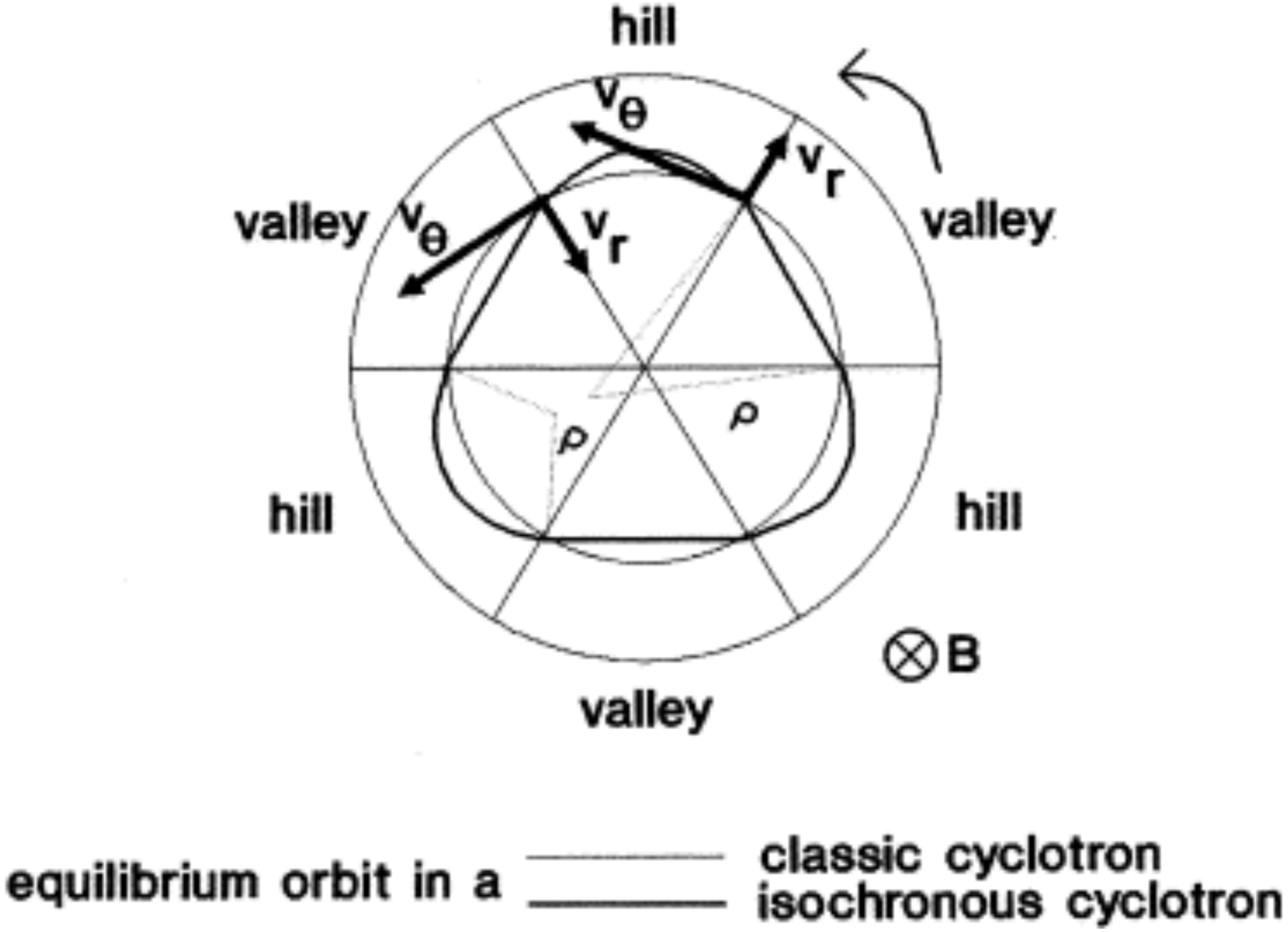
$$B(r) = \frac{m_0}{Zek\sqrt{(\frac{1}{\omega_{rf}})^2 - (\frac{r}{c})^2}}$$



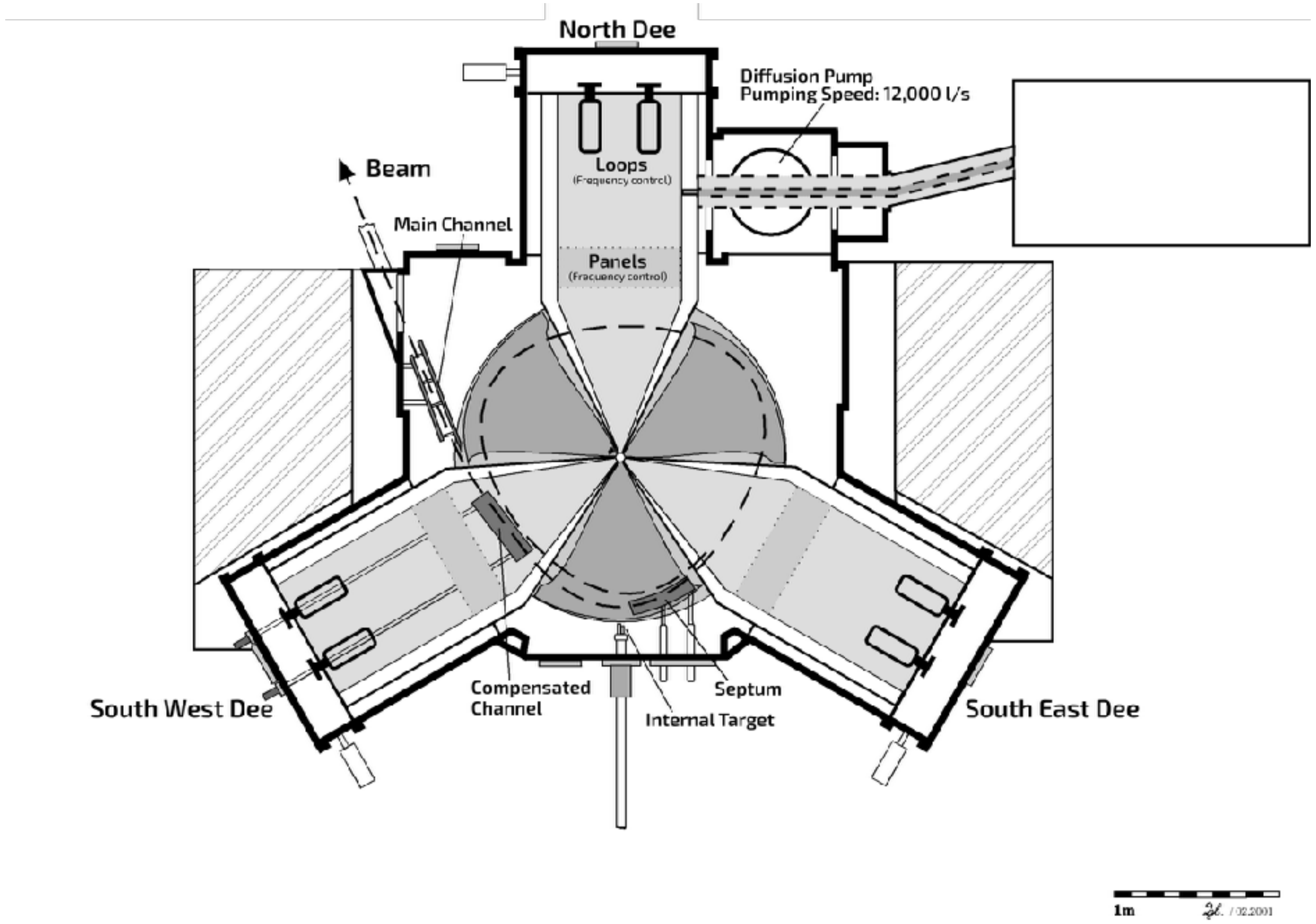


$$B_\theta \neq 0 \qquad \frac{\partial B_z}{\partial \theta} \neq 0$$

Yo keeping the stability at z direction, we need a new condition, rather than $n > 0$.

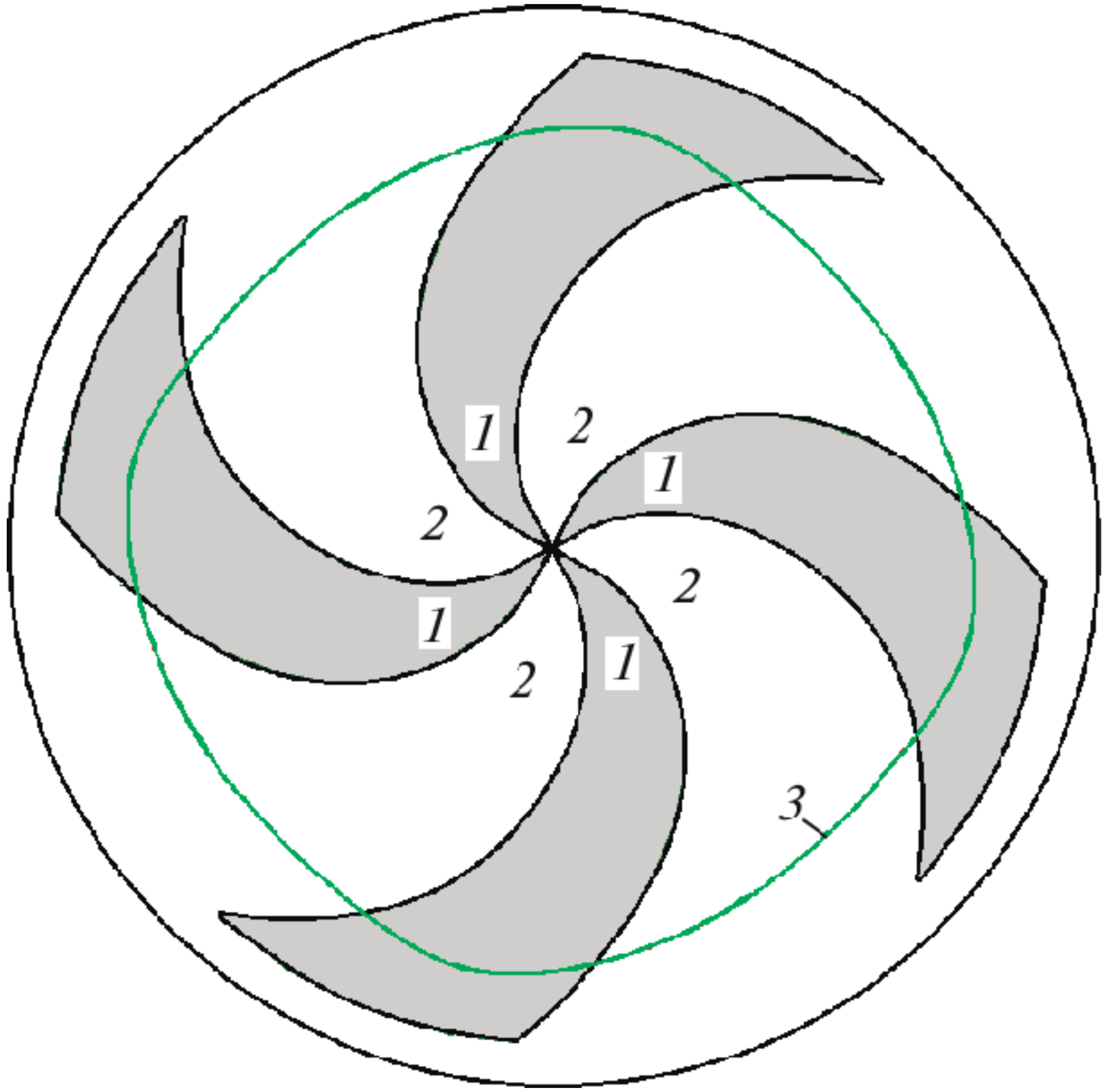


- Separated Sectors Cyclotron



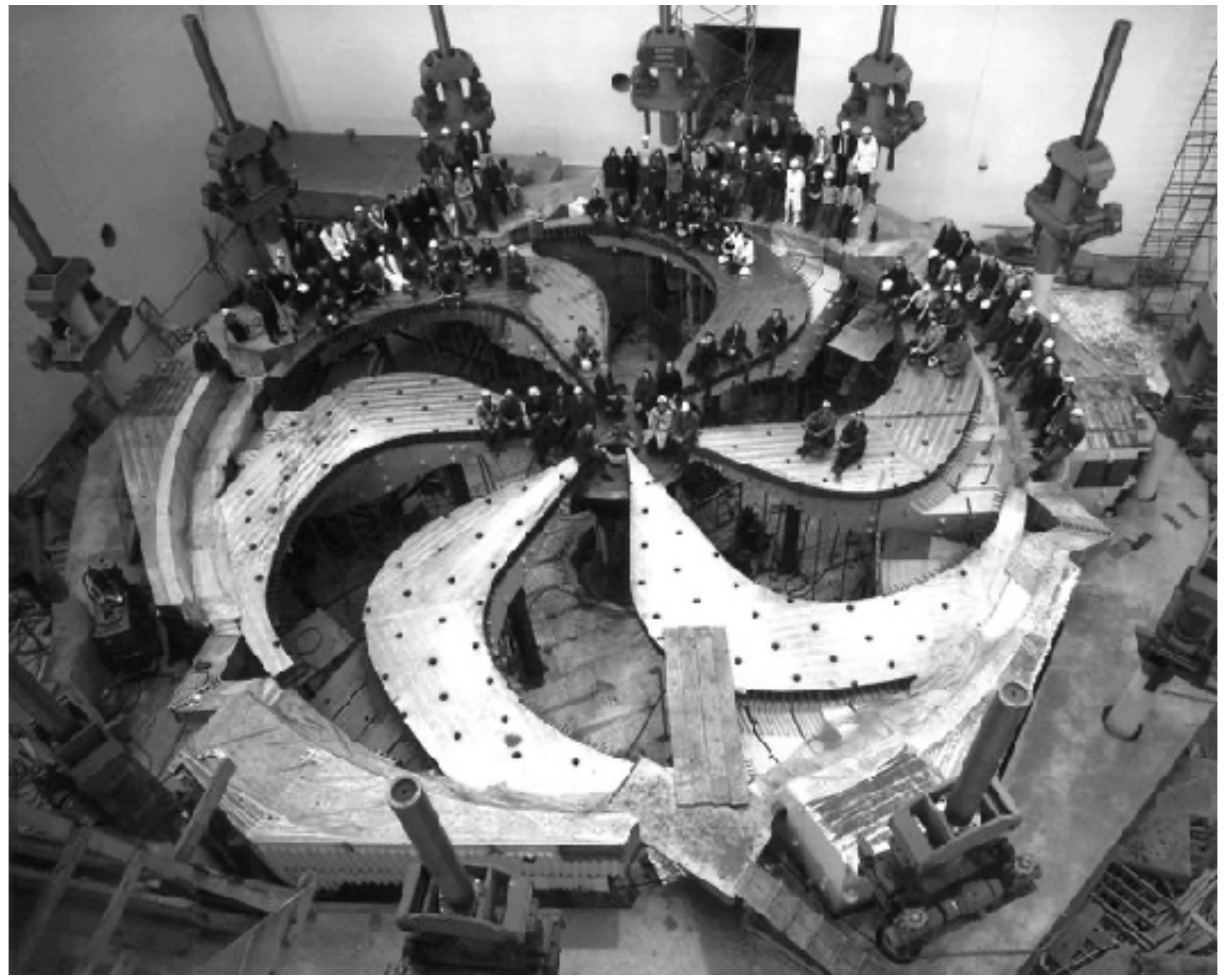
Horizontal cut through the Bonn Isochronous Cyclotron

- Spiral Sectors Cyclotron

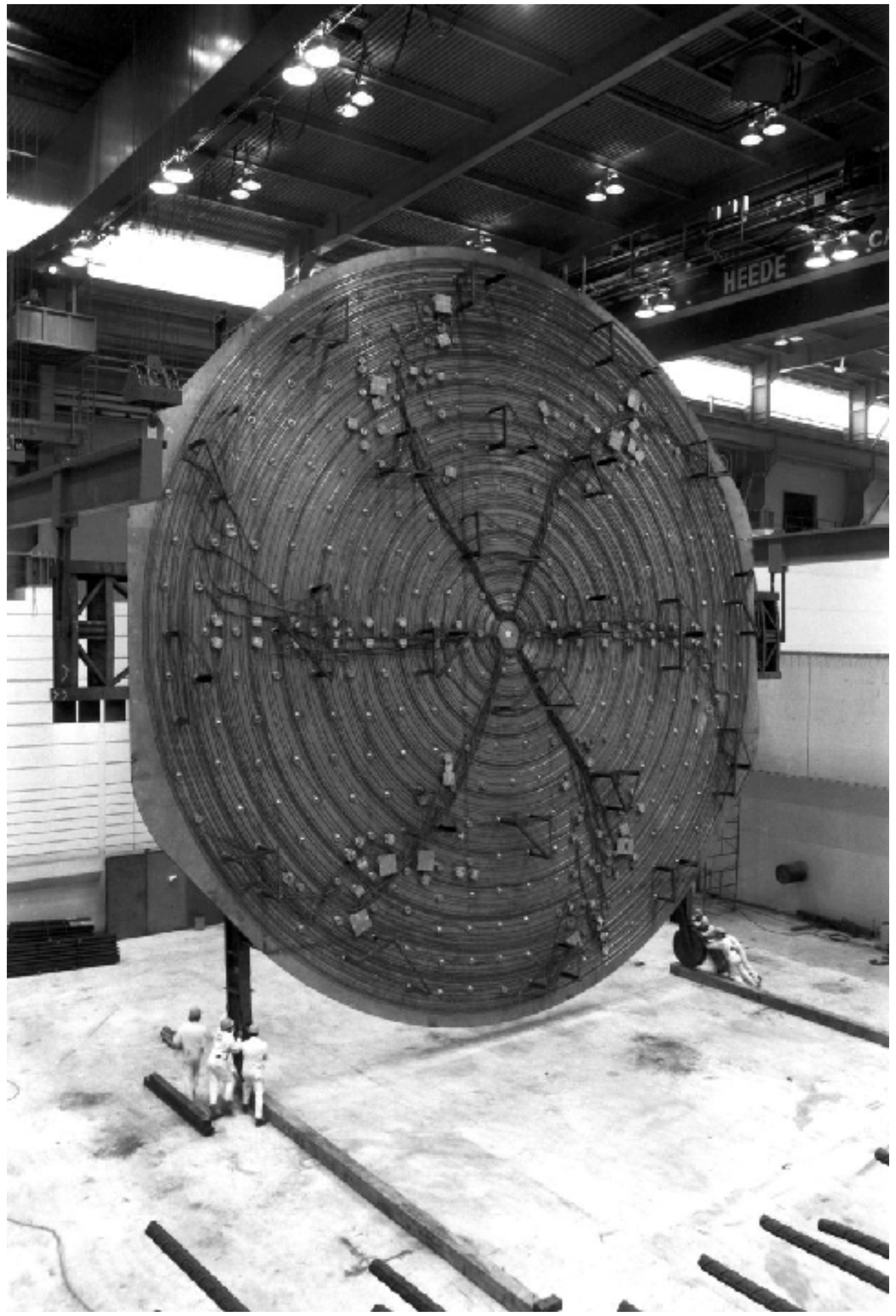


First 500 MeV Proton Beam from the TRIUMF Cyclotron, 1974

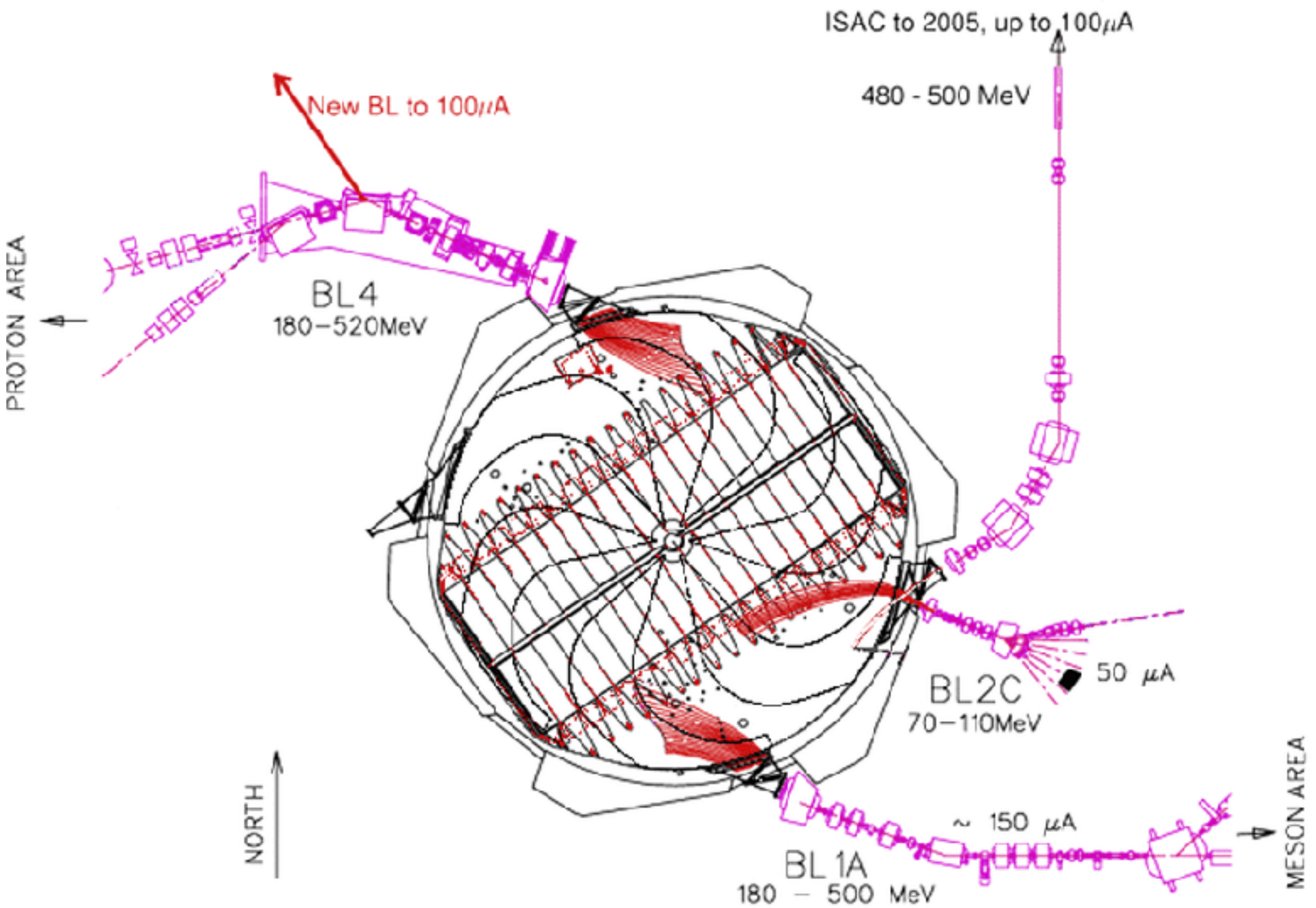
TRIUMF's field goes from 0 to about 320 inches radius with maximum beam radius of 310 inches,



January 1972: TRIUMF staff gather on the lower six sectors of the cyclotron magnet.

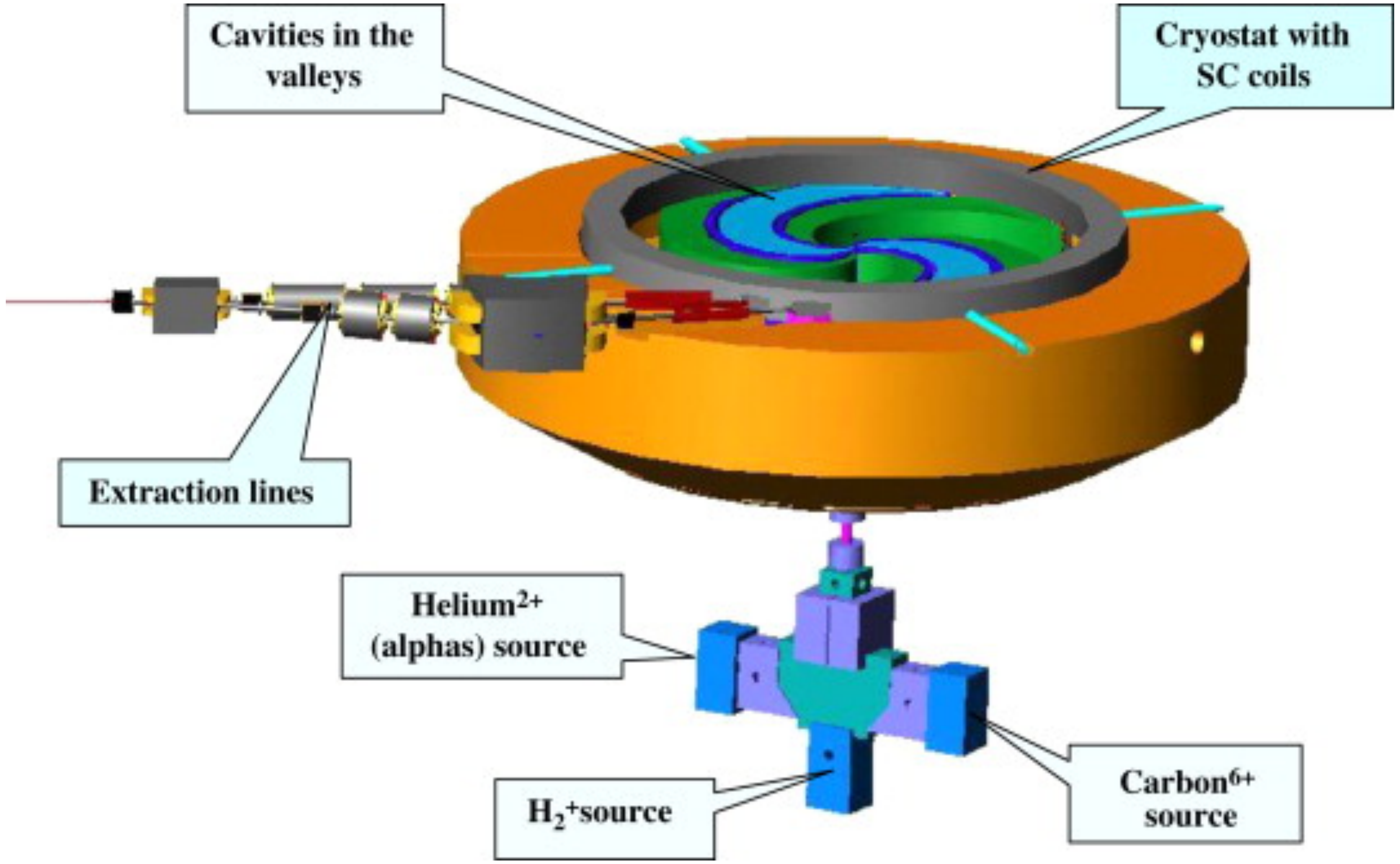


February 1972: The base of the cyclotron vacuum tank is turned over, following installation of the trim coils and cooling coils

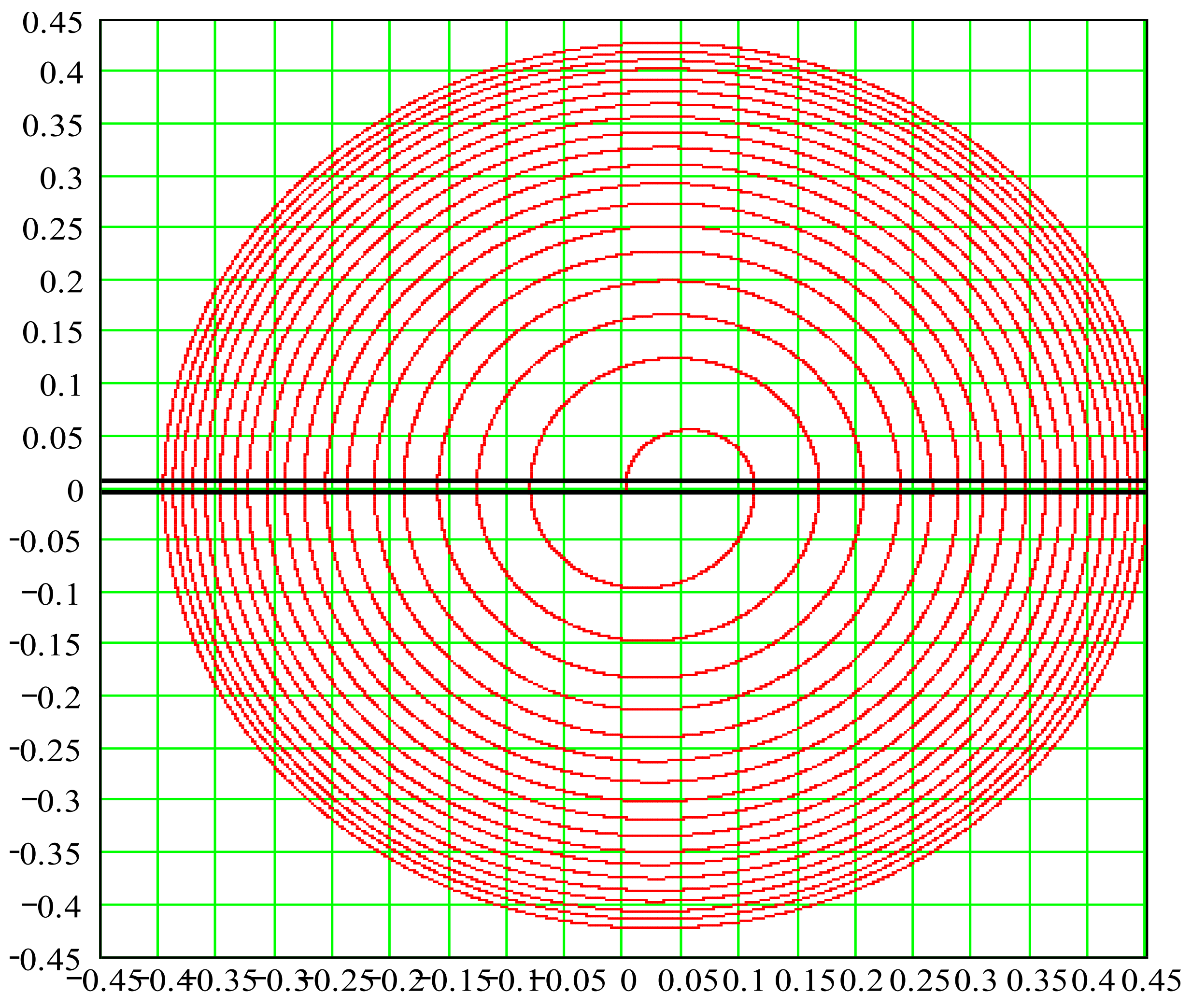


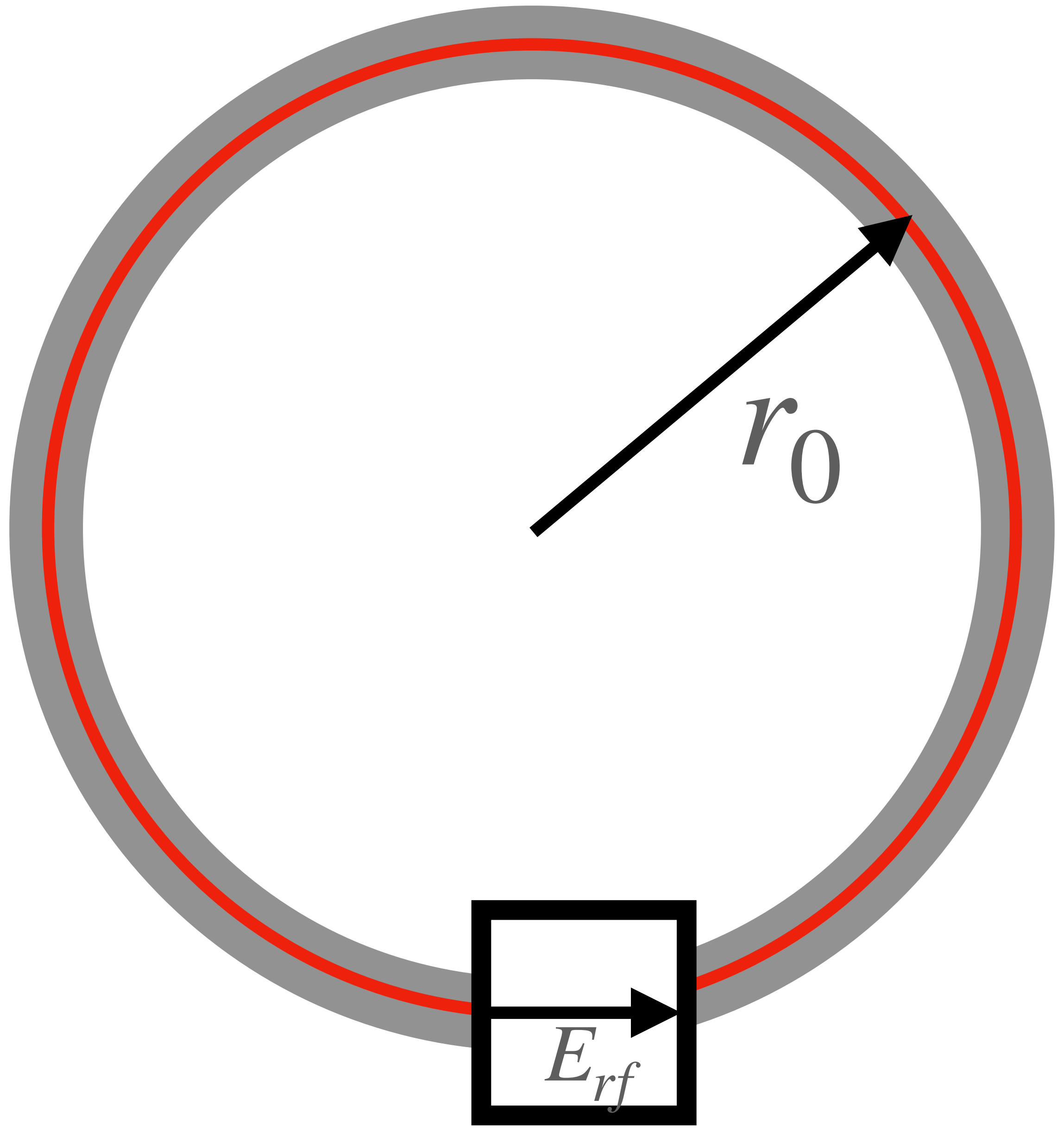
Acceleration of H – ions in the TRIUMF cyclotron and stripping extraction allows to provide multiple beams in parallel at variable energy

The cyclotron can be much smaller with superconducting coil to generate much higher magnetic fields



Synchrotron





$$B(t)\,|_{r=r_0} = \frac{m_0\omega_s(t)\gamma(t)}{Ze}$$

$$\omega_{rf}(t) = k\omega_s(t) = k\frac{v(t)}{r_0}$$

Machine Statistics

Diameter: 75 feet

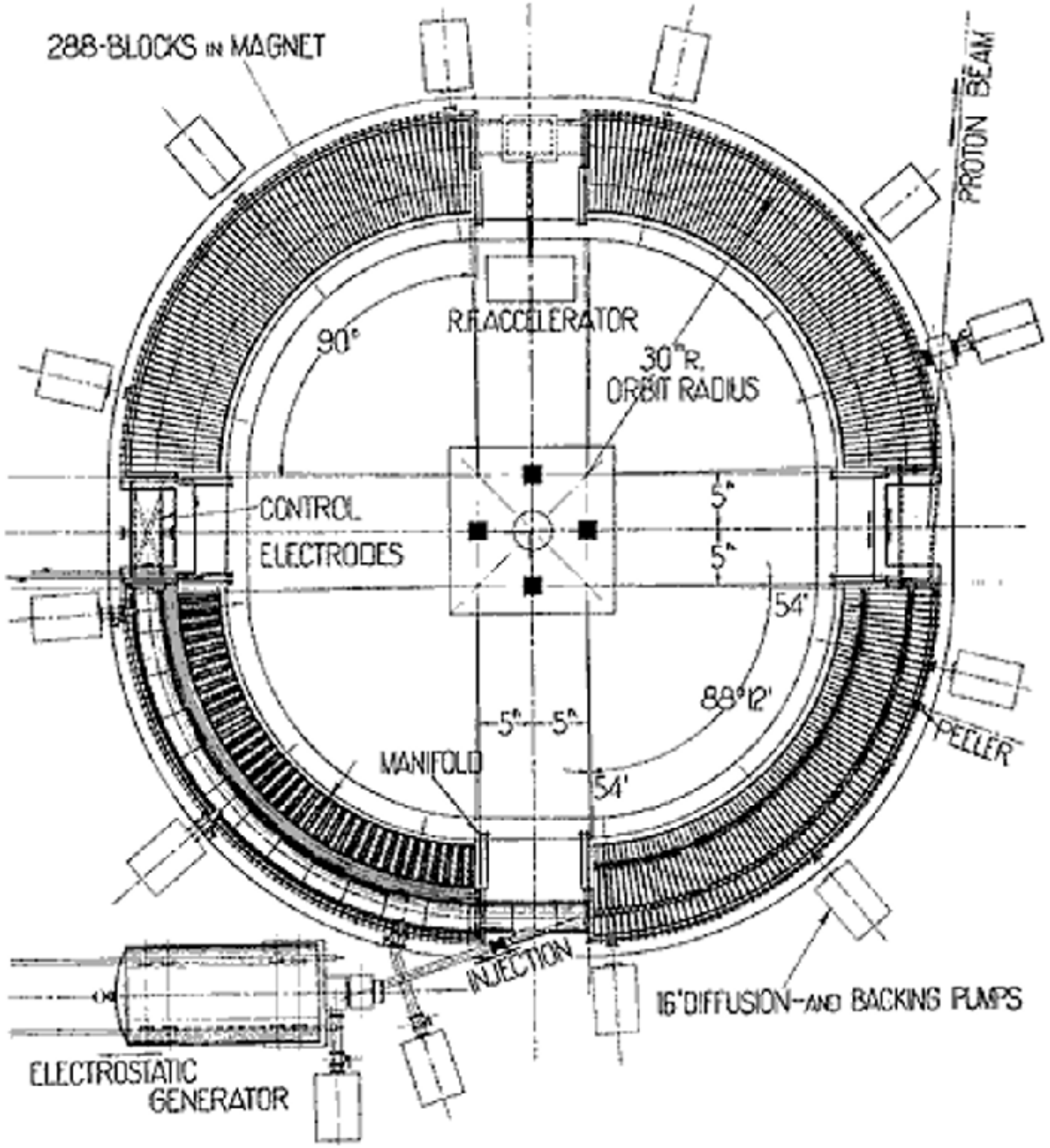
Total magnets in machine: 288

Magnet size: 8 feet high and 8 feet wide.

Machine composition: 1,700 tons of steel, 70 tons of copper.

Four magnet quadrants, each with a radius of 30 feet, separated by four 10-foot straight sections

Proton injection source: 3.6 million electron volt (MeV) Van de Graaff electrostatic generator.



Cosmotron－3GeV

•Electron synchrotron & Proton or heavy ion synchrotron

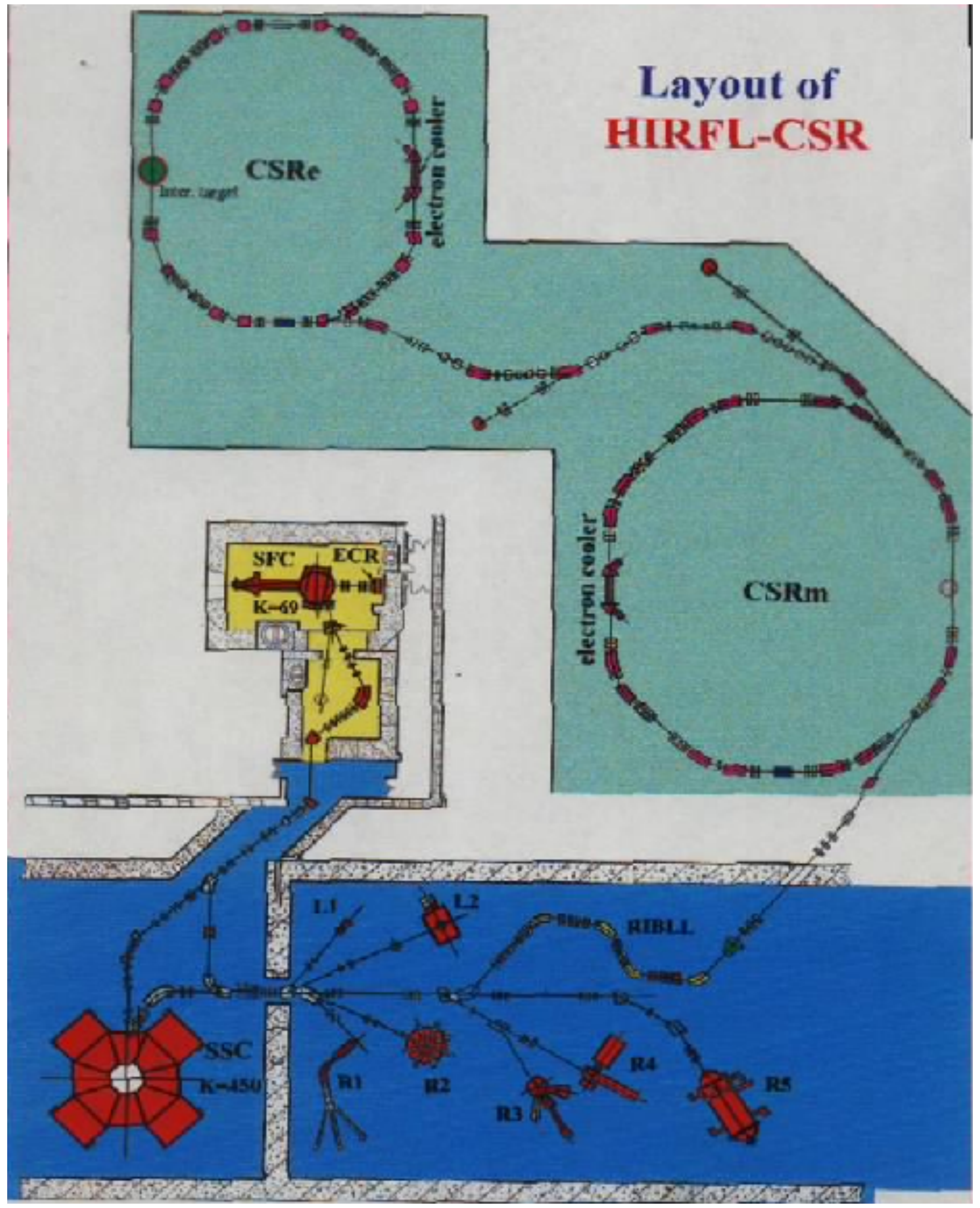
For electron, $E_0 = m_0c^2 = 0.511\text{MeV}$, it is quite easy to get a speed of almost that of light, $\beta \approx 1, v \approx c$, which is a almost constant. $\omega_{rf}(t) = k\omega_s(t) = k\dfrac{v(t)}{r_0} \approx k\dfrac{c}{r_0} = \text{constant}.$

For Proton or heavy ion, its mass is several thousand times of that of an electron. $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = 1 + \dfrac{W}{E_0}$.

Its speed is not near that of light. The rf frequency must changing along its energy increasing.

$$\omega_{rf}(t) = k\omega_s(t) = k\frac{v(t)}{r_0}$$

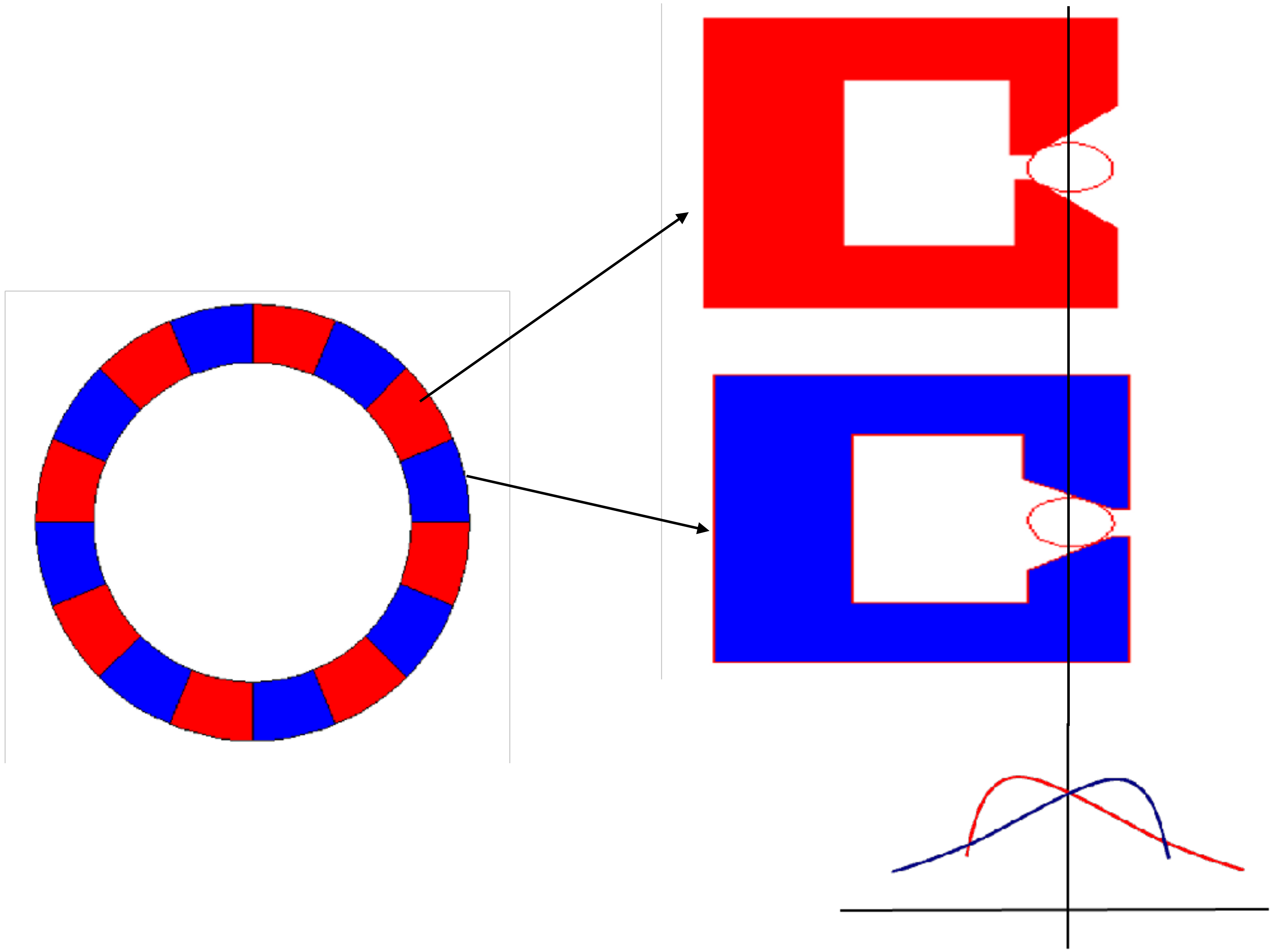
HLS-I

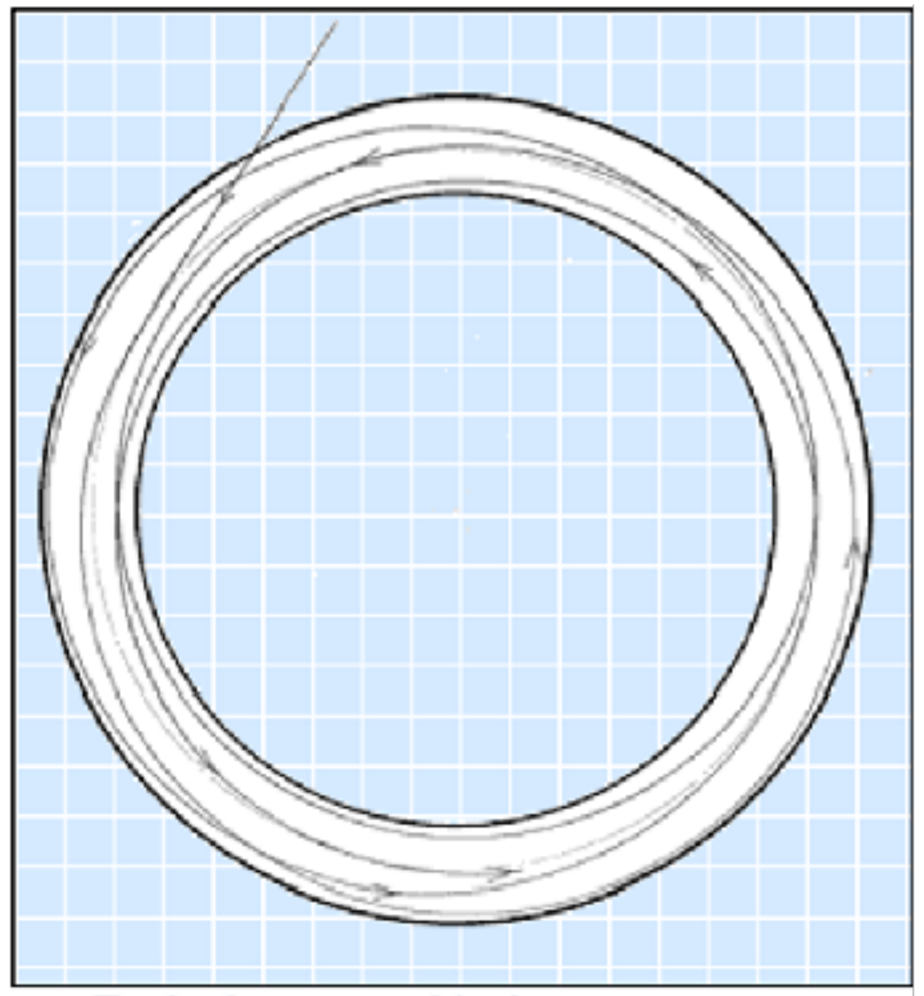


• Strong-focusing: Alternative Gradient Focusing

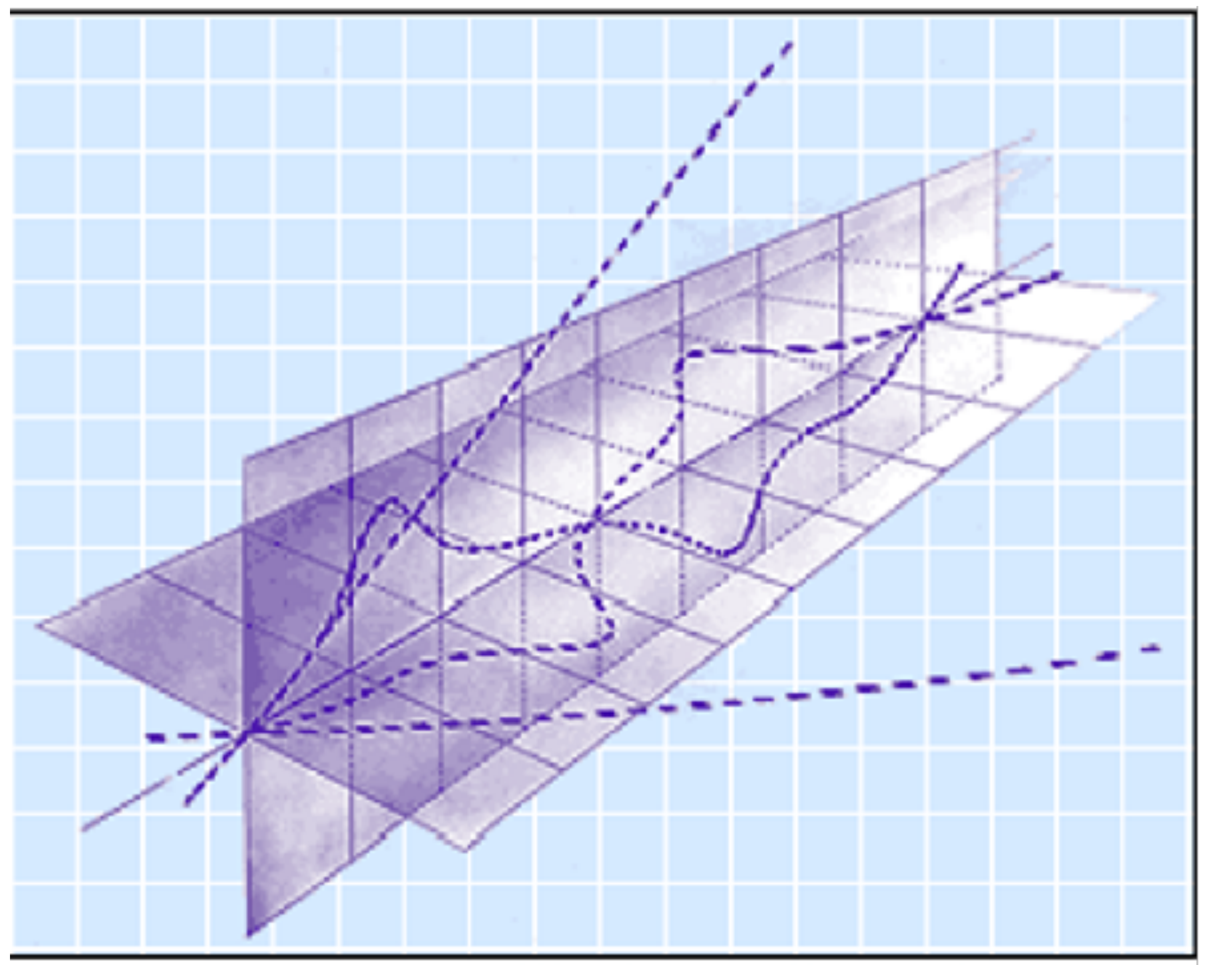
Weak focusing: $B_z(r) = B_z(r_s)\frac{1}{r^n}, \quad 0 < n < 1$

Strong focusing: $n(\theta) \gg 1, \quad \text{and}, \quad n(\theta) \ll -1$ alternatively.

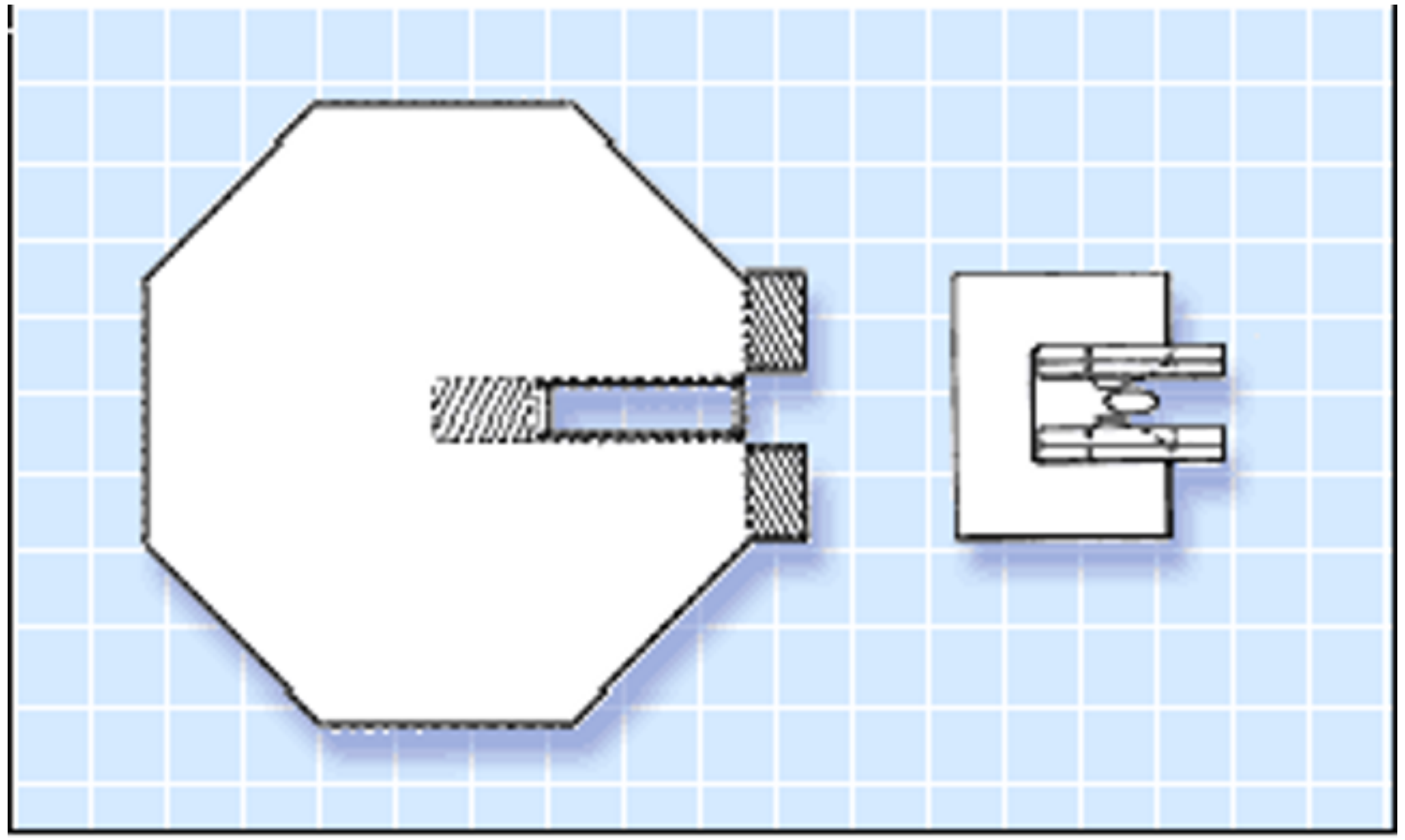




Typical proton orbits in a constant gradient or "weak-focusing" proton synchrotron

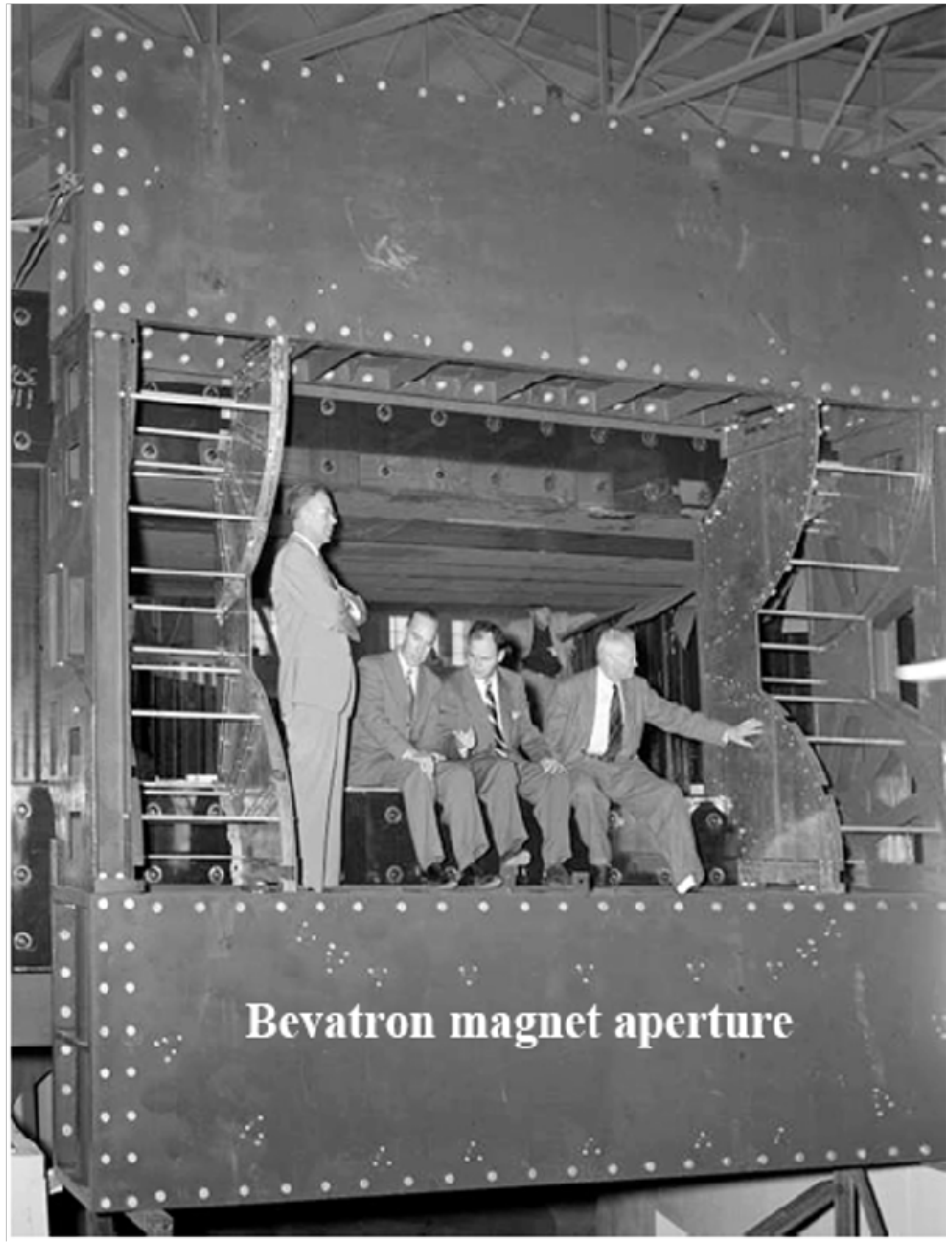


Typical proton orbits in an alternating gradient or "strong-focusing" proton synchrotron

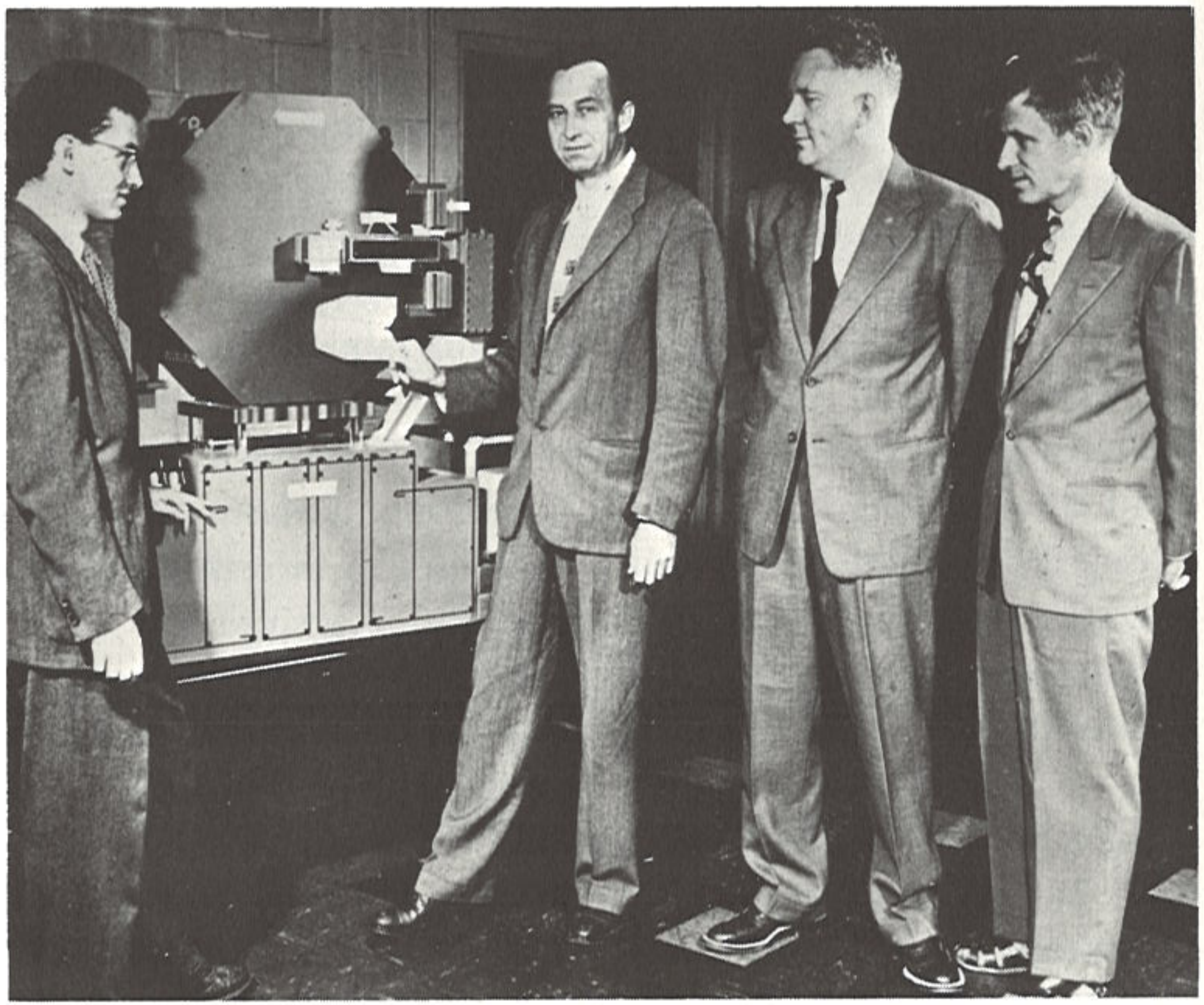


Size comparison between the Cosmotron's weak-focusing magnet (L) and the AGS alternating gradient focusing magnets

The vacuum chamber of the
6 GeV Bevatron could fit whole physicists



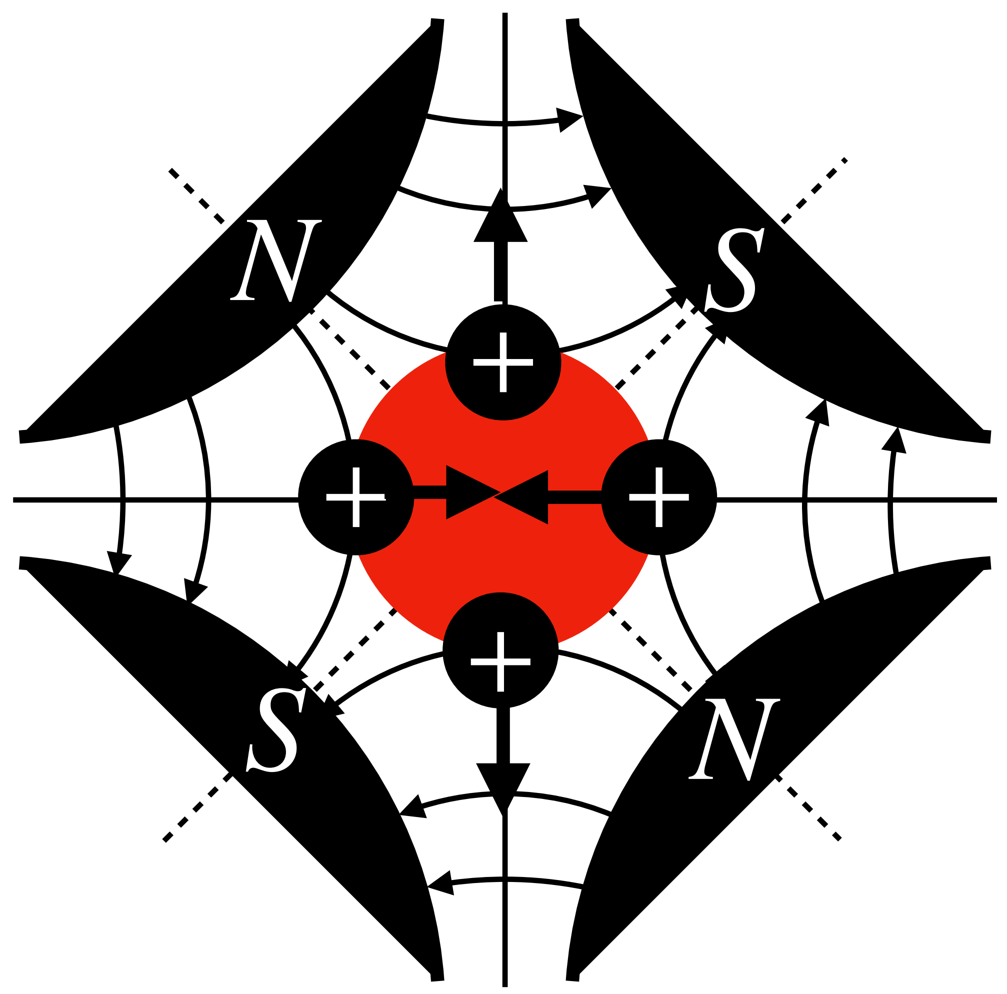
E.D.Courant, M.S.Livingston, H.S. Snyder and J.P.Blewett
demonstrating the relative cross sections of the Cosmotron magnet
and a speculative alternating-gradient magnet of very large gradient.



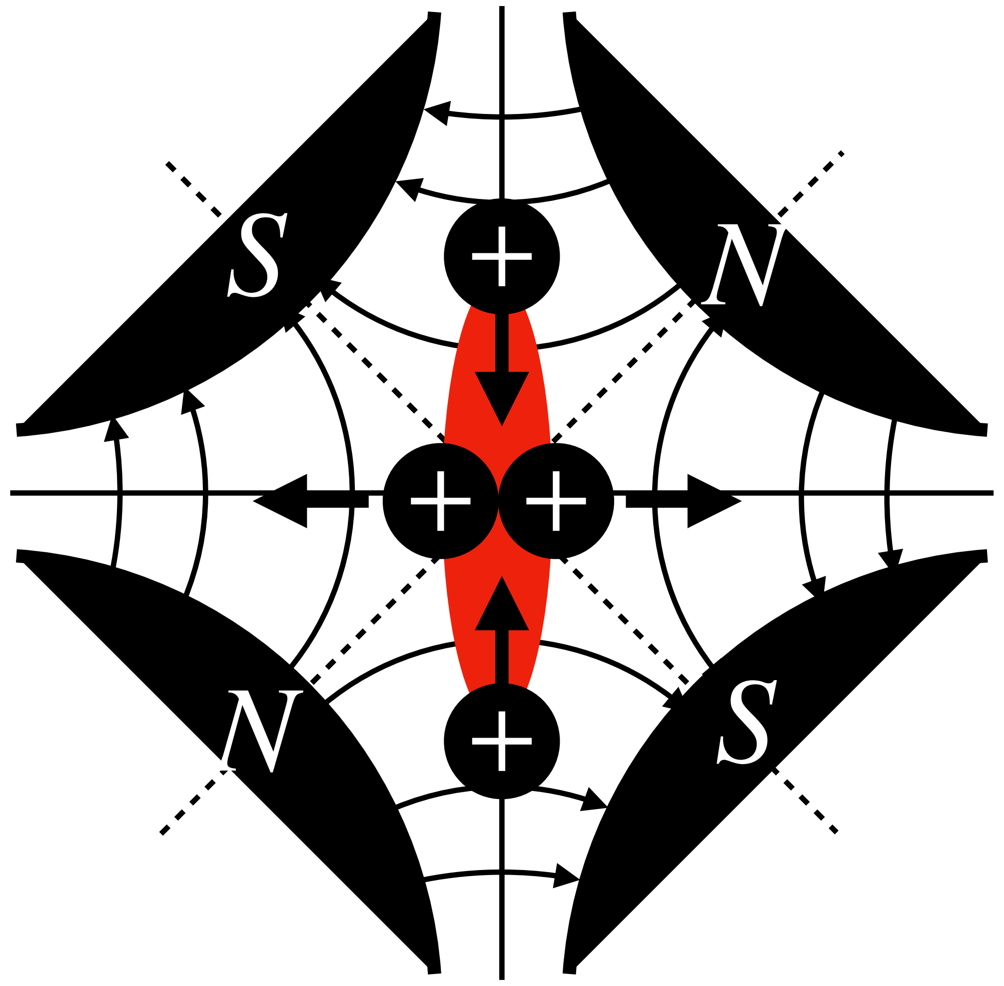
- M. Stanley Livingston, "Particle Accelerators: A Brief History", Harvard University Press: Cambridge, Massachusetts: 1969

•Magnet quadruple

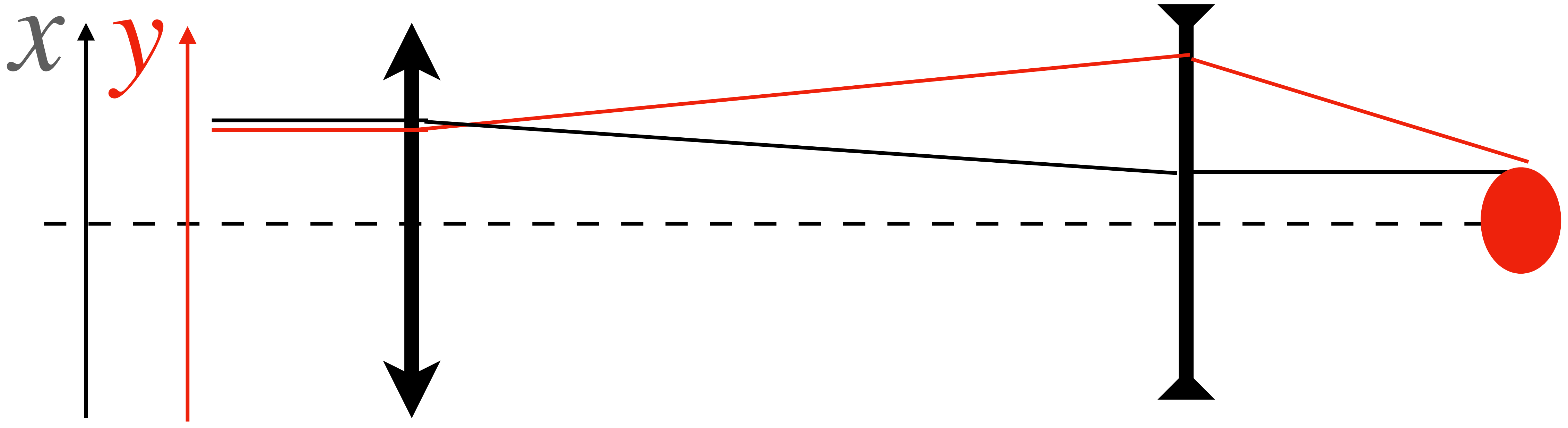
The surfaces of the poles of quadruple are hyperbola :$xy = \text{const}$.



$$B_x = ky$$
$$B_y = kx$$



$$B_x = -ky$$
$$B_y = -kx$$



Thin len approximation:

Thin-len focusing , defocusing quad. and a drift of length d:

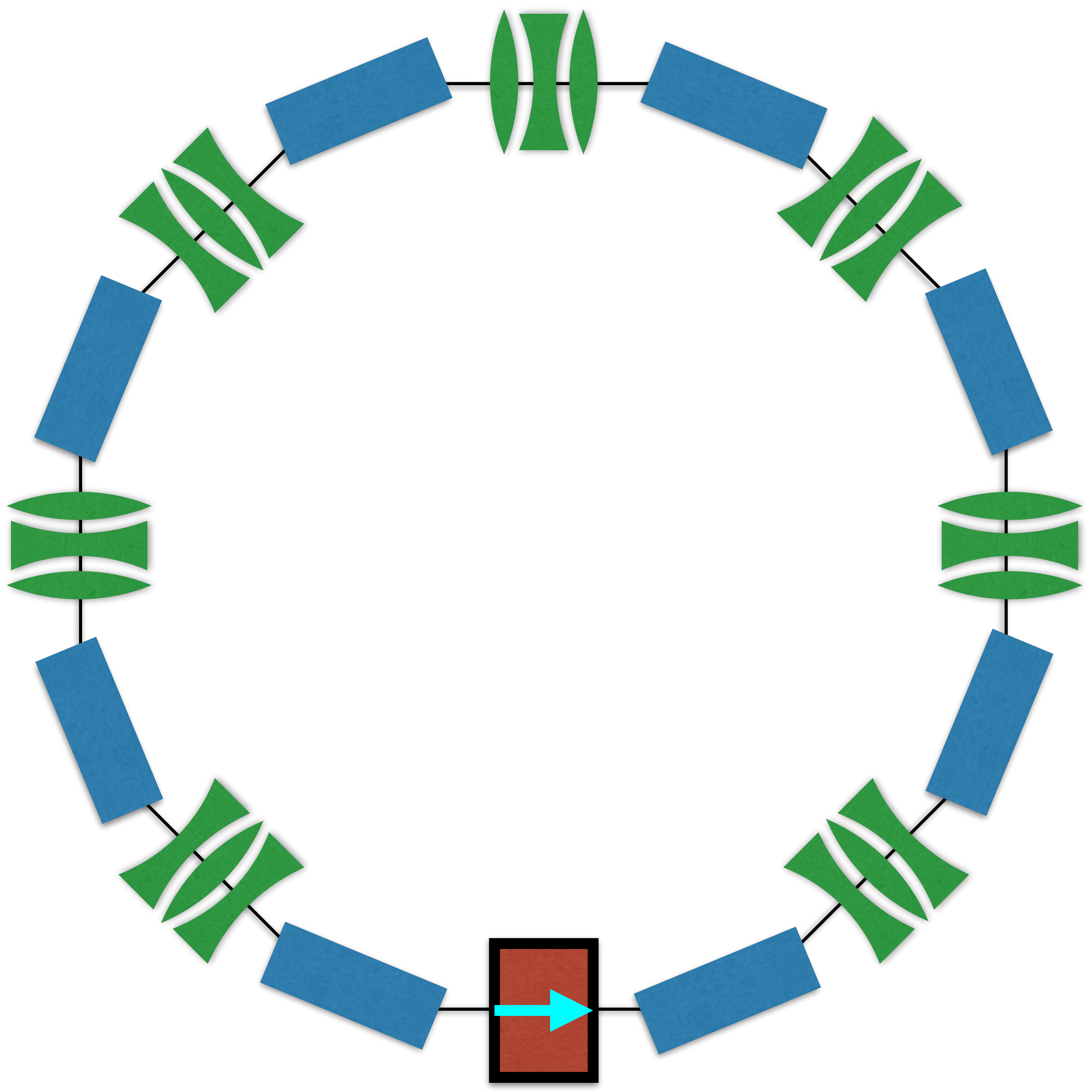
$$M_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 1 \end{bmatrix}, M_d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{f} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 1 \end{bmatrix} M_{drift} = \begin{bmatrix} 1 & d & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For a FODO cell:

$$M = M_{drift} M_d M_{drift} M_f = \begin{bmatrix} 1 - \frac{d}{f} - \frac{d^2}{f^2} & 2d + \frac{d^2}{f} & 0 & 0 \\ -\frac{d}{f^2} & 1 + \frac{d}{f} & 0 & 0 \\ 0 & 0 & 1 + \frac{d}{f} - \frac{d^2}{f^2} & 2d + \frac{d^2}{f} \\ 0 & 0 & -\frac{d}{f^2} & 1 - \frac{d}{f} \end{bmatrix}$$

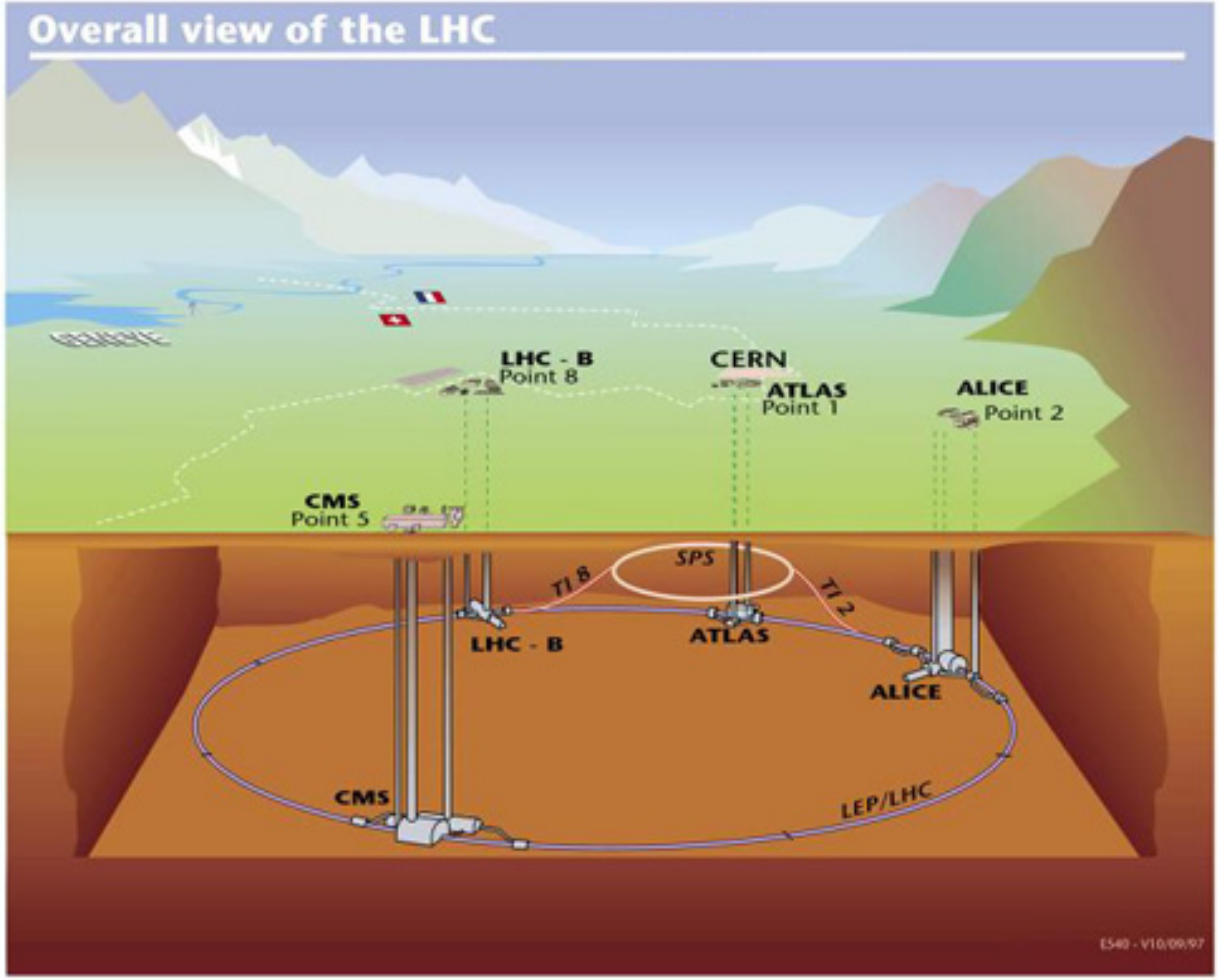
$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix} = M \begin{bmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{bmatrix}$$

Synchrotron Accelerators

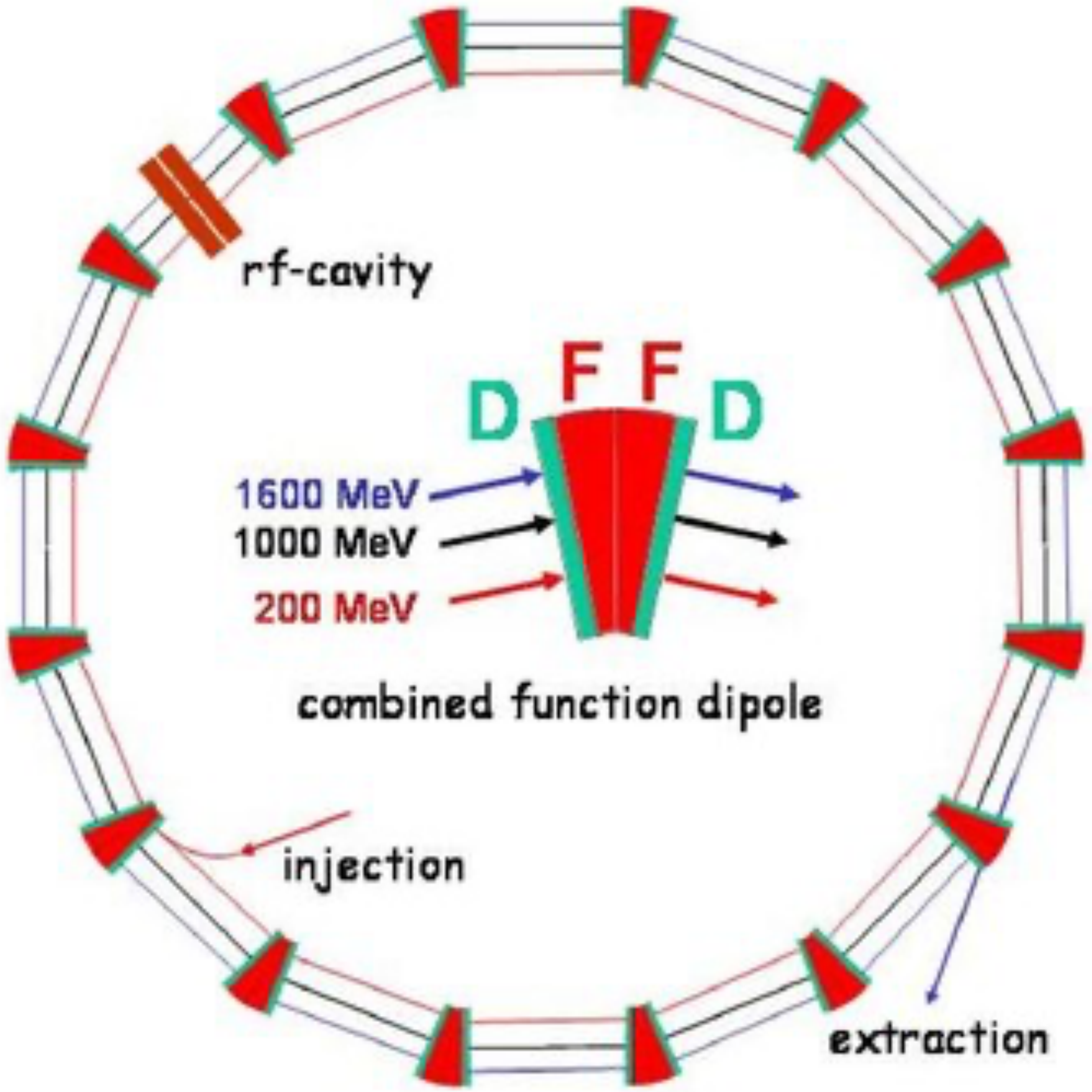


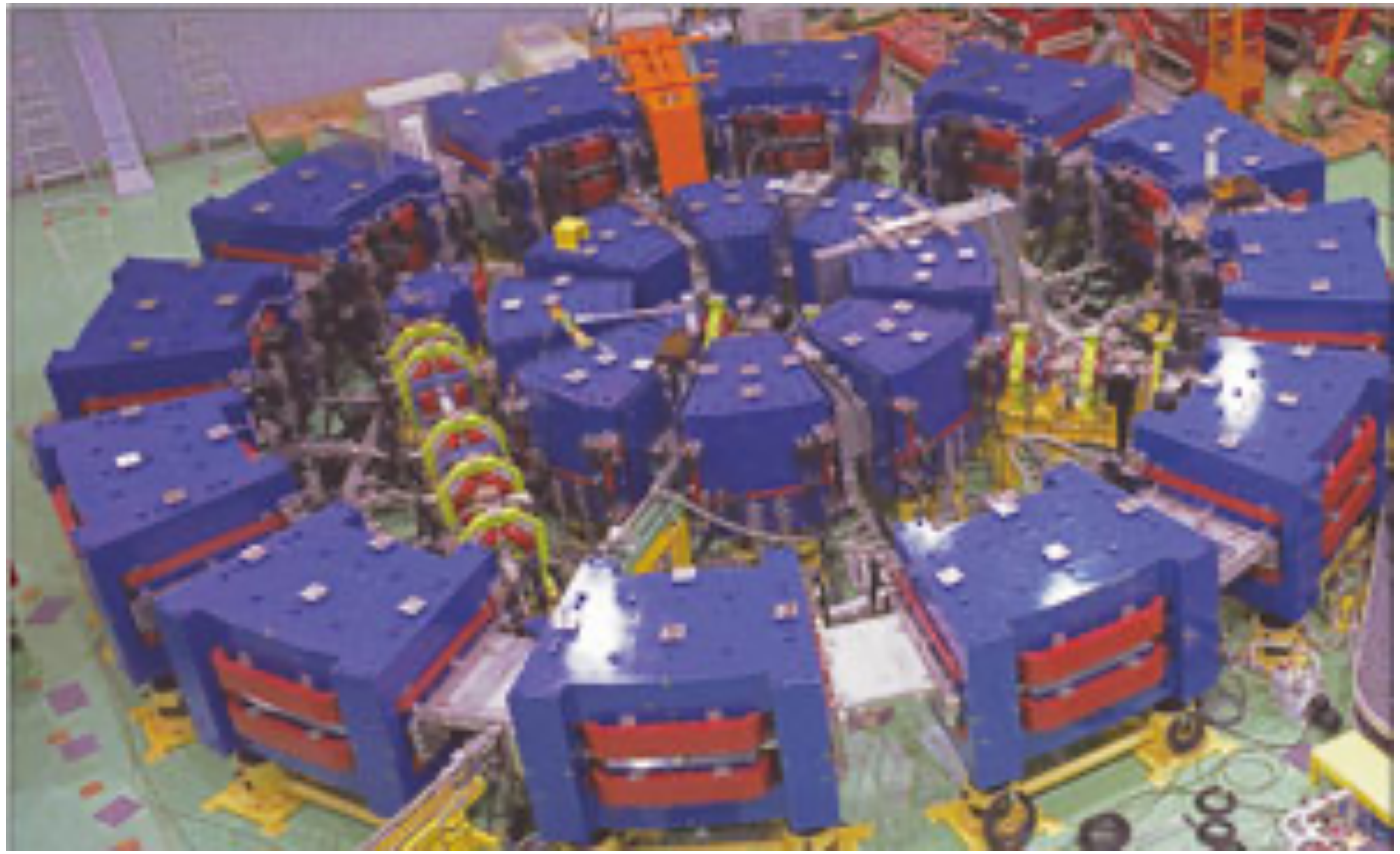
Large Hadron Collider (LHC)

http://home.web.cern.ch,2015



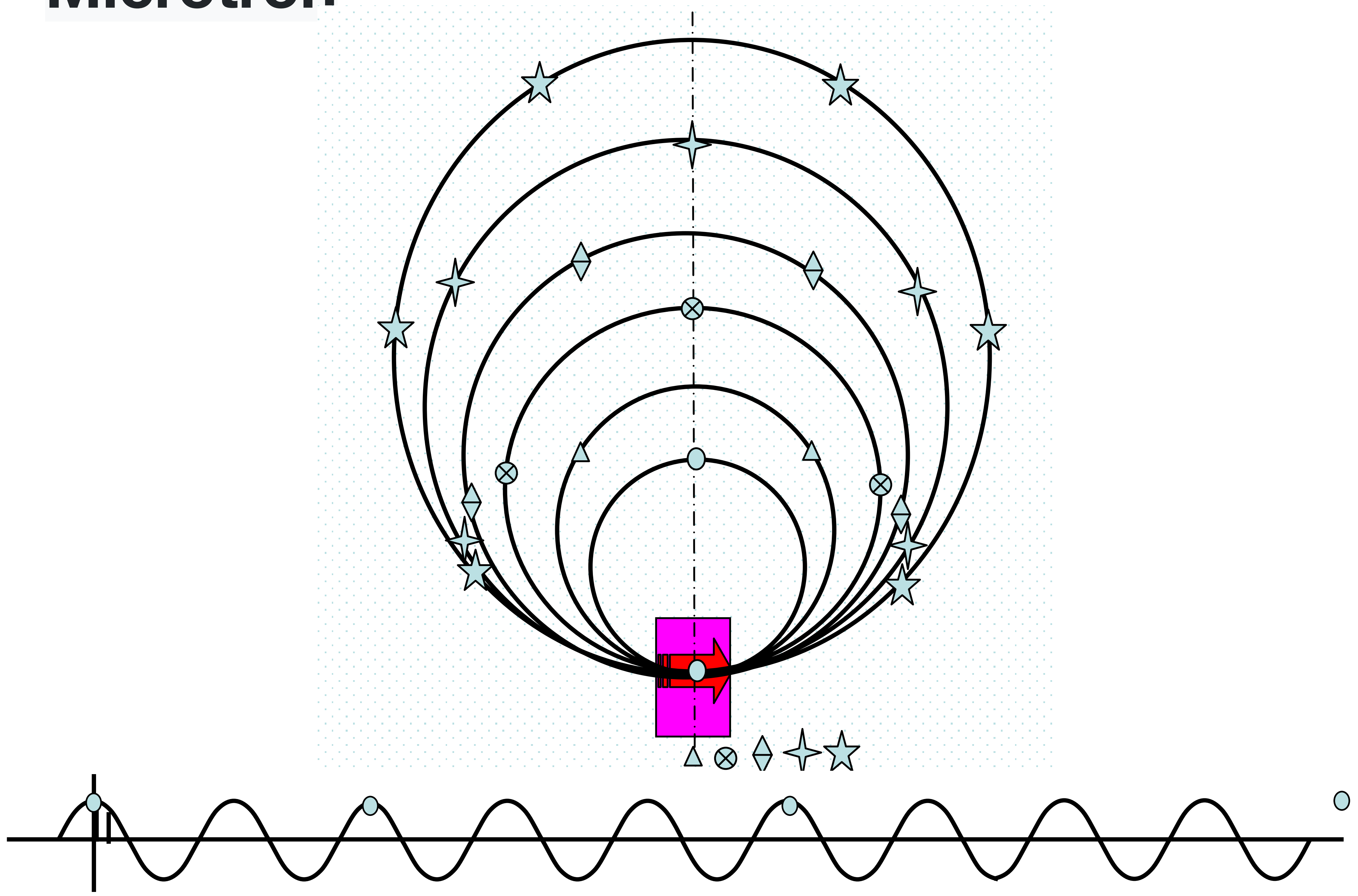
- **FFAG accelerator**

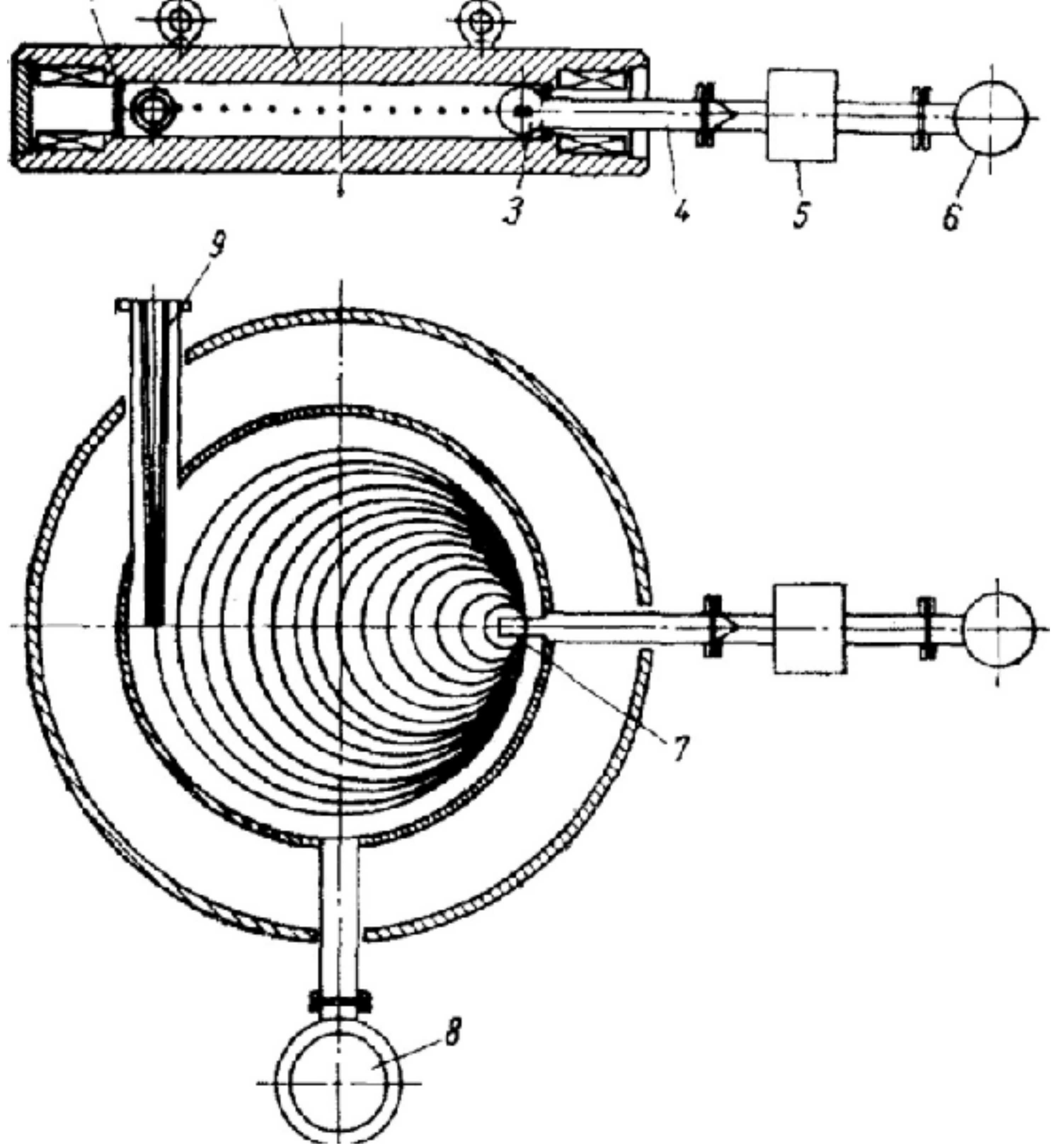




The proton FFAG accelerator developed at Kyoto University Research Reactor Institute (KURRI), Japan, for a basic experiment on accelerator-driven systems. A third injector ring is located outside this image frame.

- **Microtron**

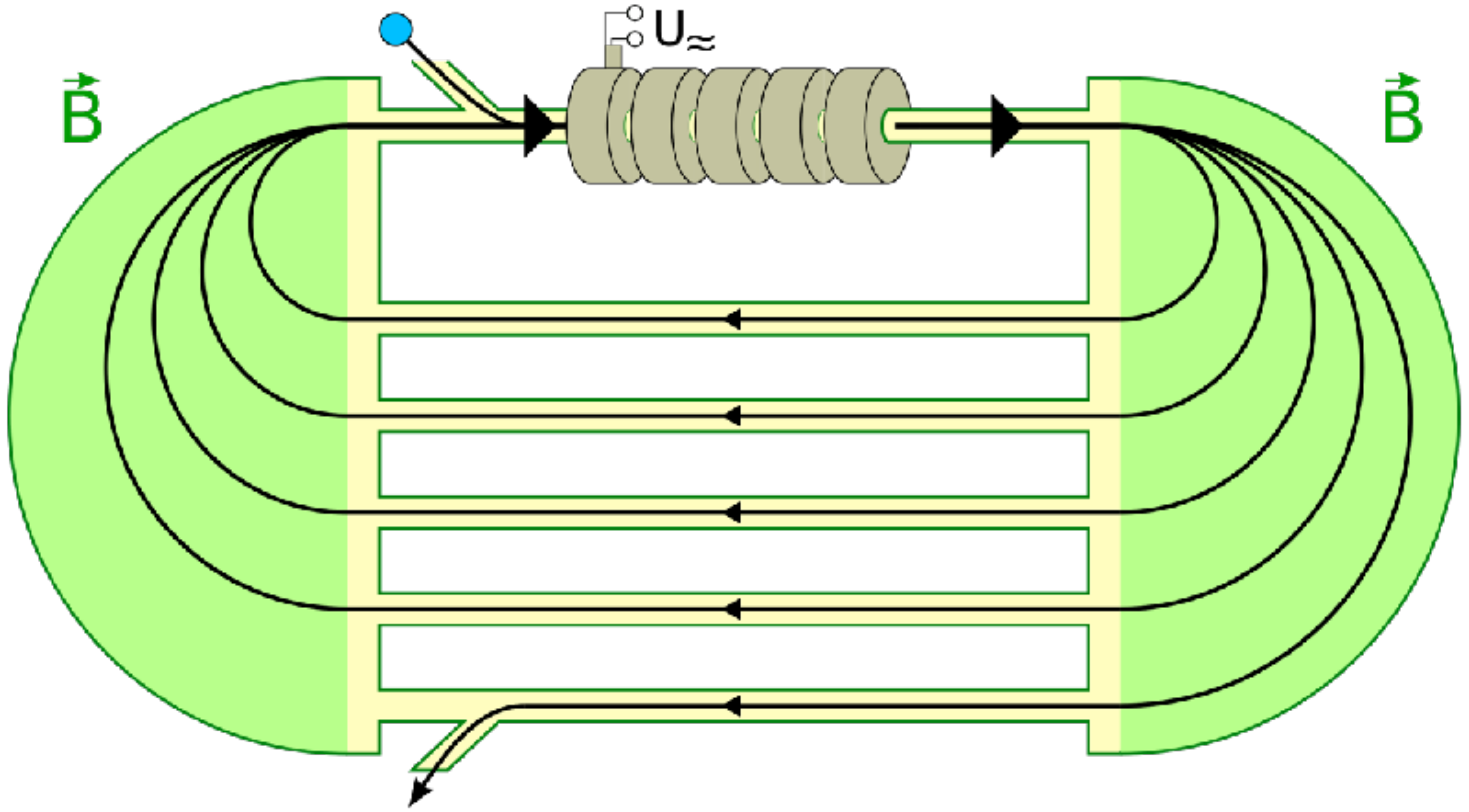




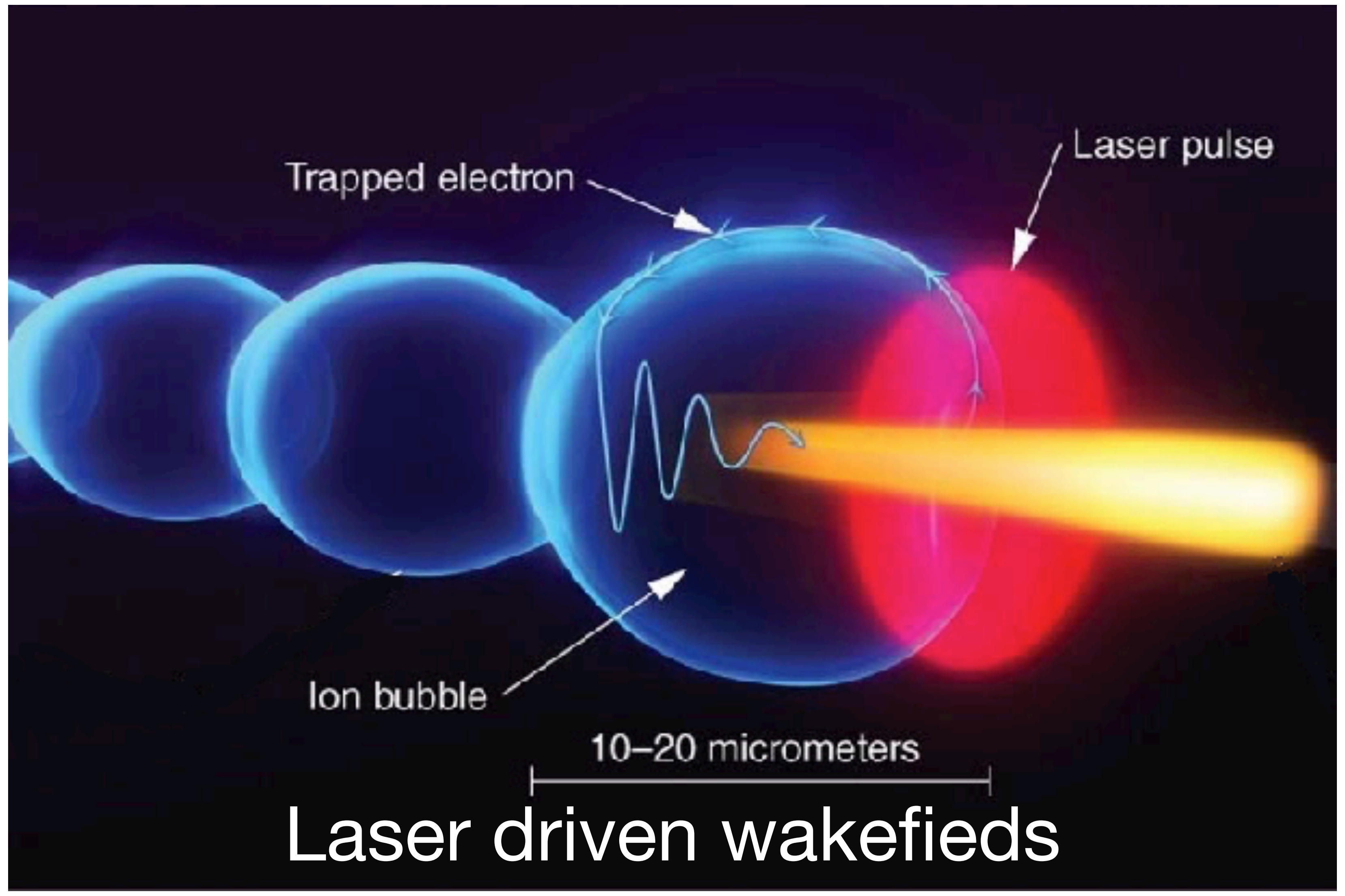
$$\omega_{rf} = k_N \omega_{sN} = \text{constant} \quad k_N = k_1 + (N-1)\Delta k$$

The synchronous condition is achievable for electrons,

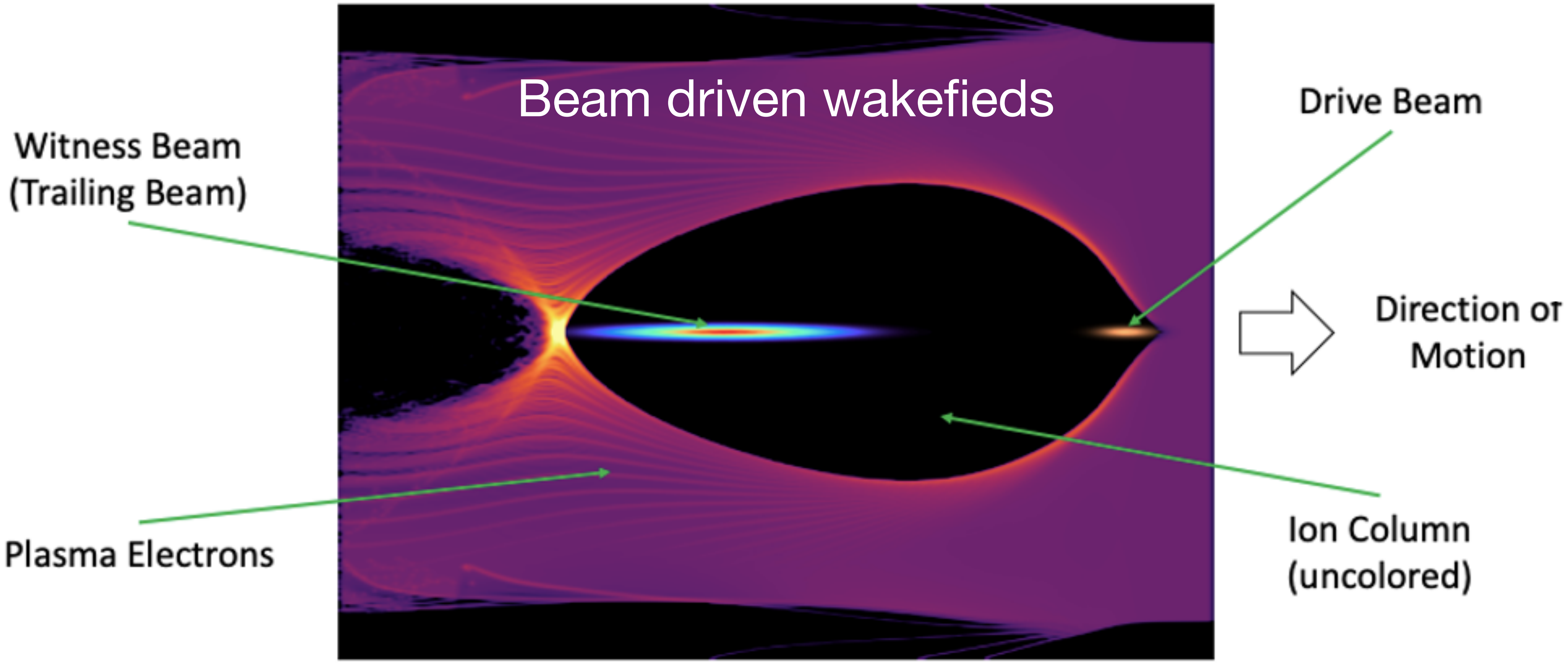
for its E0=0.511MeV, $\Delta k = \dfrac{\Delta E}{E_0} = \text{integer.}$



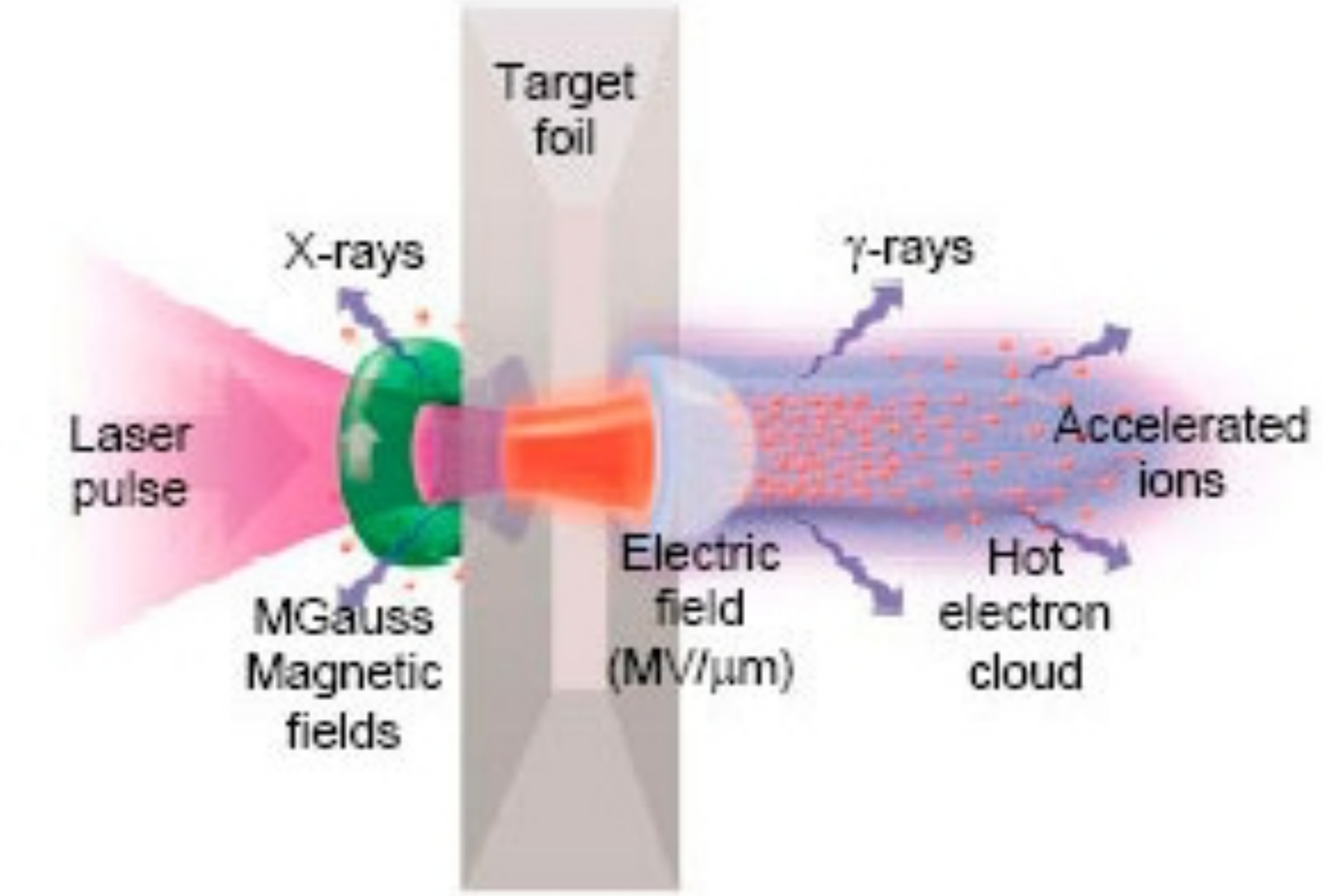
1.4 Plasma Wake-Field Acceleration



Laser driven wakefieds



Beam driven wakefieds



Laser plasma ion acceleration

Livingston Chart

Each line represents one kind of accelerators. The end of each line is not mean that kind of accelerator disappears after that time, but no even higher energy accelerators of that kind from then.

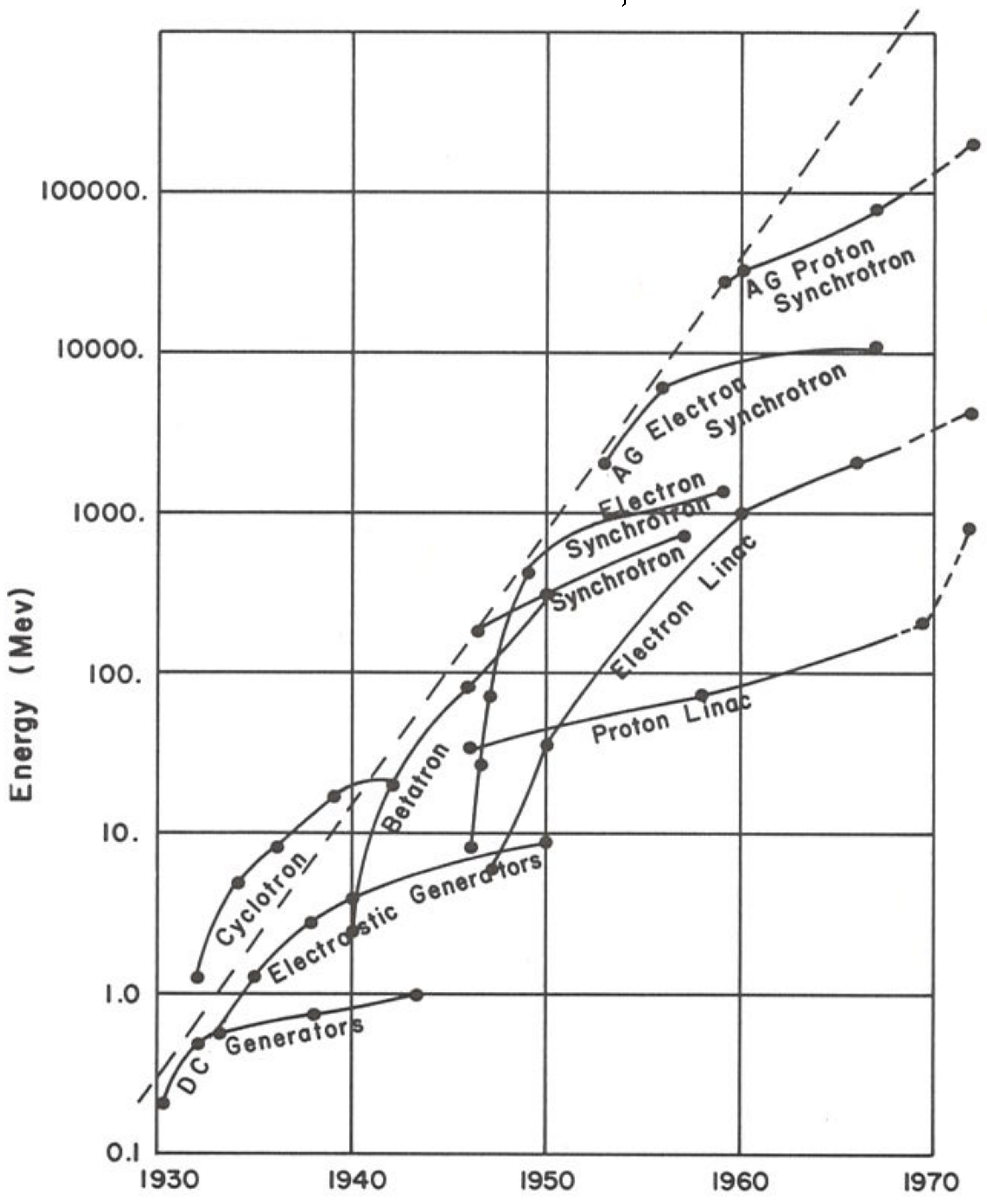
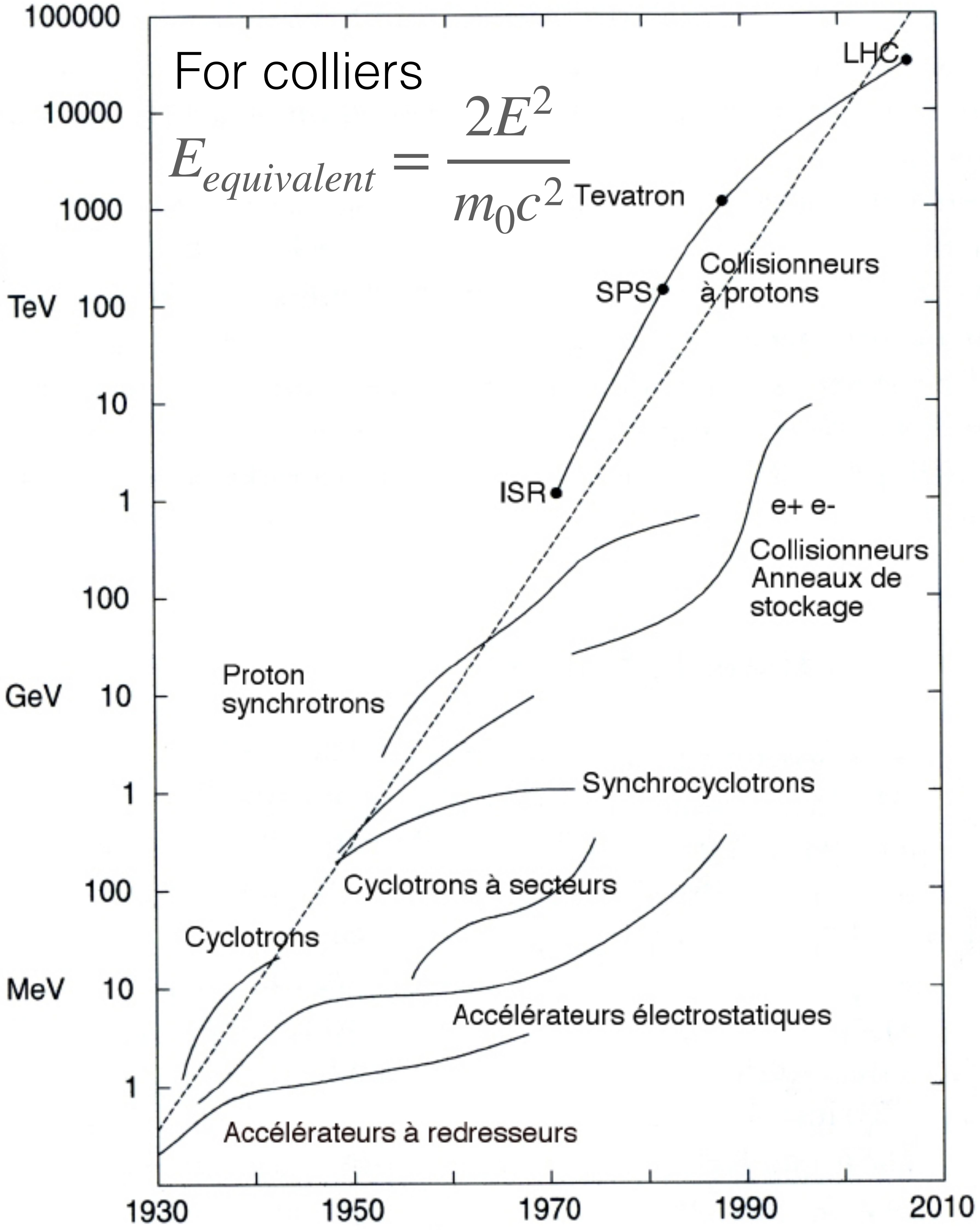


FIG. 34. Energies achieved by accelerators from 1932 to 1968. The linear envelope of the individual curves shows an average tenfold increase in energy every 6 years.



- M. Stanley Livingston, "Particle Accelerators: A Brief History", Harvard University Press: Cambridge, Massachusetts: 1969